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Cournot Fire Sales in Real and Financial Markets
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**Abstract**

In standard Walrasian macro-finance models, pecuniary externalities due to fire sales lead to excessive borrowing and insufficient liquidity holdings. We investigate whether imperfect competition (Cournot) improves welfare through internalizing the externality and find that this is far from guaranteed. Cournot competition can overcorrect the inefficiently high borrowing in a standard model of levered real investment. In contrast, Cournot competition can exacerbate the inefficiently low liquidity in a standard model of financial portfolio choice. Implications for welfare and regulation are therefore sector-specific, depending both on the nature of the shocks and the competitiveness of the industry.

Key words: fire sales, pecuniary externalities, overinvestment, liquidity, financial regulation, macroprudential regulation
1 Introduction

The macro-finance literature has taken great interest in fire-sale externalities. Canonical models show that such pecuniary externalities lead to overinvestment in risky capital (e.g. Lorenzoni, 2008) and overinvestment in illiquid assets (e.g. Allen and Gale, 2004) because perfectly competitive agents do not internalize how their ex-ante choices will affect fire-sale prices after adverse shocks. In practice, however, agents may not be perfectly competitive. Industry concentration has increased substantially over recent decades, both in the real and in the financial sector (see e.g. Gutiérrez and Philippon, 2018, and Corbae and Levine, 2018, respectively). These increased concentrations raise the possibility that firms do internalize price impacts in asset markets.

In the standard macro-finance models, pecuniary externalities would be mitigated if agents internalized their effects on prices: agents would invest in less capital (i.e. borrow less) or invest in fewer illiquid assets (i.e. hold more liquidity), so that asset prices are higher when bad aggregate shocks occur. In this paper, we show that this mitigating effect of imperfect competition is not robust to simple modifications of the standard macro-finance models. Instead of being mitigated, the inefficiencies can be over-corrected or even exacerbated, depending on whether fire sales occur in real or financial sectors, and whether the shocks are purely aggregate or have an idiosyncratic component.

We consider two settings that can be interpreted as a real sector with risky production that is funded with debt and a banking sector that invests in illiquid projects to issue liquid deposits. What distinguishes the two settings is the reason that fire sales can occur in each. Fire sales in the first setting (“real sector”) occur due to bad productivity shocks hitting the asset side of levered institutions, forcing them to sell to second-best users in order to repay debts. Fire sales in the second setting (“financial sector”) occur due to liquidity shocks hitting the liability side of liquidity transforming institutions, forcing them to sell illiquid assets to cash-strapped buyers. Thus, our analysis covers two standard macro-finance models — a model of productivity shocks with borrowing constraints, and a model of liquidity shocks with illiquid assets. In both of these settings, a social planner would choose less investment, leading to higher asset prices (less severe fire sales).

To these standard setups, we introduce the following crucial modifications: (i) “Cournot behavior” of agents, i.e. internalizing the marginal impact an agent’s ex-ante balance sheet decisions have on ex-post asset prices, and (ii) a combination of both aggregate and idiosyncratic risk. When fire sales occur because some agents receive bad shocks, then other agents receive good shocks and are therefore in a favorable position to buy fire-sale assets. Agents strategically consider how their ex-ante choices will affect ex-post prices, both
when they receive bad shocks and contribute to fire sales, and when they receive good shocks and benefit from fire sales. Our settings nest the standard macro-finance variants of these models, and we show that, in the standard setting, Cournot mitigates the externalities.

However, the strategic considerations of potential buyers and sellers have important consequences when the nature of idiosyncratic and aggregate risk differ from the standard formulations. The social planner considers how initial decisions will affect fire-sale prices *in the aggregate*, and then equalizes the marginal utilities for buyers and sellers. Cournot agents consider separately how their initial decisions will affect prices *when they are a buyer* and *when they are a seller*. In contrast to the social planner perspective, these price impacts are different and therefore weigh differently the respective marginal utilities in a Cournot agent’s first-order condition.

Cournot competition can therefore overcorrect or exacerbate the inefficiency, depending on the particular sector and on the nature of aggregate and idiosyncratic risk. As a result, to study the effect of industry concentration within macro-finance models of fire sales, there is no alternative but to go into specifics and understand the subtleties within different classes of models. Accordingly, we make small, relevant modifications that leave the direction of the externality intact while the “direction” of strategic Cournot behavior may differ. It may seem strange to analyze two models within a single paper, but the mechanics of how the Cournot optimization operates depends on the specific setting. The two models we consider are general versions of two of the most important models in the macro-finance literature, and thus they provide the appropriate representative settings to study the effects of industry concentration on fire sales.

First, standard models of fire sales due to productivity shocks and borrowing constraints typically consider “pure aggregate risk” so that all agents receive a bad shock at the same time (e.g. Lorenzoni, 2008). Bad shocks force firms to sell capital to repay debts, pushing down asset prices, and requiring even more sales in order to raise funds. If firms borrowed less initially, then fire sales would be smaller, and less capital would be reallocated to inefficient users. Hence, the standard model features *over-investment* in capital in the Walrasian equilibrium. To this standard setup we introduce idiosyncratic productivity risk in the bad state, so that some firms have good productivity and can buy up capital at cheap prices. With Cournot competition, firms know that when they receive bad shocks they will sell capital, and so they strategically would like to hold less capital to minimize the price impact. Firms also know that when they receive good shocks they will buy capital, and they would like to purchase capital at lower prices, which they would do by having fewer funds available to buy capital — which occurs by holding less capital. So
whether a buyer or a seller, firms strategically would like to have invested in less capital in either case. As a result, the Cournot equilibrium can feature under-investment relative to the constrained efficient level chosen by the social planner because shocks to capital determine the funds available to repay debts or buy new capital. We discipline our model by matching some key empirical moments, and show that it is empirically plausible for Cournot competition to over-correct the pecuniary externality, so that market power leads to under-investment in capital.

Second, standard models of fire sales due to liquidity transformation typically consider idiosyncratic liquidity shocks that cannot be adequately insured as a result of incomplete markets (e.g. Allen and Gale, 2004). Investors receiving liquidity shocks are forced to sell illiquid assets, and thus their consumption is a function of the asset price. If investors held fewer illiquid assets, the interim asset price would be higher, providing better insurance to investors selling assets because of liquidity shocks. Hence, the standard model features over-investment in illiquid assets in Walrasian equilibrium. In our model, investors know that holding fewer illiquid assets will push up the asset price, which is good when they are sellers but bad when they are buyers. The relative weight given to the two states depends on the marginal effect of holding more liquidity on the prices in the two states. Importantly, whether the price is more responsive to the buyers’ liquid asset holdings (demand) or the sellers’ illiquid asset holdings (supply) depends on the level of the asset price. When the price is sufficiently low, investors have a greater strategic incentive to push down the price (to buy at cheap prices when they are buyers). As a result, fire sales are more extreme, and Cournot competition can lead to even lower asset prices. We find that it is empirically plausible for Cournot competition to exacerbate the pecuniary externality rather than mitigate it, so that market power leads to over-over-investment in illiquid assets.

In summary, while Cournot competition can mitigate the inefficiency arising from fire sales in both settings, in the setting representing the real sector, Cournot can instead reverse the inefficiency and in the setting representing the financial sector, Cournot can also exacerbate the inefficiency. Given these different results, it is worth clarifying why the nature of the fire sales in the two settings are so different. First, it may seem trivial to point out that in either setting, the price impact of additional funds used to buy assets, or of additional assets for sale, depends on the total supply of assets and funds in the market. But in the setting with liability-side shocks, the supply of funds and of illiquid assets is completely determined by the initial investment decision. The value of liquid assets is not affected by whether or not an agent receives a liquidity shock. Hence, what matters for price impacts of buying and selling are initial liquidity holdings, which are endogenously determined by the aggregate risk of agents receiving liquidity shocks. In contrast, in the
setting with asset-side shocks to capital productivity, both the supply of funds to buy capital and the supply of capital for sale do depend on the productivity shock. A firm with high productivity has more funds to buy capital — and the more capital initially invested, the more a high productivity shock increases funds available to buy capital. In the same way, firms with low productivity sell more capital to repay debts because productivity is low — and the more capital initially invested, the more a low productivity shock requires selling capital to repay debts. The source of the shock — asset-side or liability-side — has very different implications for the supply of assets for sale and of funds available to buy those assets.

**Related literature.** The literature on generic inefficiency arising from pecuniary externalities dates to Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986), which provide justifications for policy interventions when private agents do not internalize their effects on prices. Dávila (2015) and Dávila and Korinek (2018) provide recent analysis of pecuniary externalities in macro-finance models with borrowing constraints, showing that terms of trade and collateral externalities are distinct, as are the issues of efficiency and amplifications. Stein (2013) is an example of policy thinking based on academic insights.

Closely related to the literature on pecuniary externalities are the papers on fire sales and limits to arbitrage: Shleifer and Vishny (1992), Gromb and Vayanos (2002), Shleifer and Vishny (2011). All of these papers on pecuniary externalities share the feature that inefficiencies arise because price-taking agents do not internalize how their portfolio decisions affect prices, affecting risk sharing and borrowing capacities.

Our paper relates to the literature on over-investment, which includes Caballero and Krishnamurthy (2001), He and Kondor (2016), and Lorenzoni (2008). Other recent work has considered the possibility of under-investment due to market power. In particular, Gutiérrez and Philippon (2017, 2018) document that investment is low based on Tobin’s Q and that the shortfall is related to industry concentration. In theoretical work, Kurlat (2019) shows that the canonical over-investment result can also be reversed if the microfoundation for fire sales is based on adverse selection as opposed to slow moving capital or other constraints on potential buyers.

The literature on liquidity provision includes Diamond and Dybvig (1983), Bhattacharya and Gale (1987), Jacklin (1987), and Allen and Gale (2004). Recently, Farhi et al. (2009) and Geanakoplos and Walsh (2018) study inefficient liquidity provision with private trades in financial markets. These papers study how incomplete markets lead to under-provision of liquidity (typically, though different specifications of shocks can lead to over-provision).
A few macro-finance papers consider the implications of firms internalizing their effect on prices. Corsetti, Dasgupta, Morris, and Shin (2004) consider how the presence of a large trader affects the likelihood of currency crises, as small traders take into account strategically the behavior of the large trader (small traders are more aggressive). Dávila and Walther (2019) consider the leverage decisions of large and small banks when banks internalize how their leverage and size affect bailout probabilities (small banks use more leverage in the presence of large banks). In a complementary paper to ours, Neuhann and Sockin (2020) consider Cournot agents in a model with investment and complete Arrow–Debreu markets, and study how market power leads to distortions in risk sharing and investment decisions. In contrast, we consider models with incomplete markets where pecuniary externalities have welfare effects, and study the impact of market power on the (in)efficiency of equilibrium allocations.

Diamond and Rajan (2011) study how anticipating potential future firesales can affect asset markets today, reducing buyers’ willingness to pay and, in turn, sellers’ willingness to sell. Gale and Yorulmazer (2013) argue that costly bankruptcy and incomplete markets cause inefficient liquidity hoarding. Malherbe (2014) argues that liquidity provision can exacerbate adverse selection. Perotti and Suarez (2002) highlight the incentive to be the “last bank standing.” Kuong (2016) shows that the pecuniary externality leads to interactions between firms’ borrowing and risk-taking decisions. Finally, Morrison and Walther (2020) consider how market discipline and systemic risk interact in a competitive setting with both aggregate and idiosyncratic risk where banks may be in a position to buy or sell assets.

2 Cournot in a leverage trade-off model

We consider separately a standard model of fire sales due to productivity shocks (Section 2) and a standard model of fire sales due to liquidity shocks (Section 3); Appendix A shows how the two models can be combined to capture an economy with both a real and a financial sector. The setting for our model with productivity shocks is similar to Lorenzoni (2008). In this model, the key choice is the ex-ante scale of debt-funded investment in productive but risky capital i.e. a leverage trade-off. Ex post, more leverage, allowing more investment, is preferred if hit by a good productivity shock, while less leverage, with less debt to repay, is preferred if hit by a bad productivity shock. The model is therefore representative of firms with limited internal funds that need to borrow to invest. The canonical result in this type of model is that a pecuniary externality leads to inefficiently high bor-
rowing in the Walrasian equilibrium — the “inefficient credit booms” of Lorenzoni (2008). The (constrained) social planner internalizes the externality and chooses lower borrowing which leads to higher fire-sale prices and increases welfare. We show that internalizing the pecuniary externality through Cournot behavior can not only mitigate but over-correct the standard inefficiency by leading to underinvestment even compared to the social planner.

**Technology and preferences.** There are three periods, \( t = 0, 1, 2 \). There are two types of agents, \( 2N \) firms and \( 2N \) households.\(^1\) Households are risk neutral with deep pockets and do not discount consumption. Firms consume at \( t = 2 \) and have risk-averse utility \( v(c) \) with \( \lim_{c \to 0} v'(c) = \infty \). Capital can be irreversibly produced from consumption goods at unit cost and is perfectly durable. Firms have access to a linear production technology using capital in each period. Capital \( k_i \) held by firm \( i \) at \( t = 0 \) produces \( A_i k_i \) consumption goods at \( t = 1 \), where \( A_i \) is uncertain. Period 2 functions as a continuation value so we assume that every unit of capital held at \( t = 1 \) produces one unit of consumption at \( t = 2 \).\(^2\) Firms are each endowed with \( n > 0 \) units of capital at \( t = 0 \) and can borrow at a rate of \( R \geq 1 \); for simplicity, we assume that borrowing is risk free.\(^3\) We assume that \( \mathbb{E}[A_i] > R \) so firms will leverage to invest. Denoting borrowing by \( d_i \geq 0 \), firm \( i \)'s balance sheet at \( t = 0 \) satisfies \( k_i = n + d_i \).

Households have access to an inferior production technology that yields \( F(k) \) consumption goods at \( t + 1 \) for capital holdings \( k \) at \( t \), with \( F(k) = a \log(1 + k) \). This technology implies that households buy capital to produce if the capital price is below \( a \). To ensure that households only buy capital following a fire sale at \( t = 1 \), we suppose that \( a < 1 \).

In this setup, allowing firms to hold liquid assets in addition to capital is equivalent to having firms simply hold less debt. Accordingly, we will consider firms’ investment and borrowing decisions and discuss over-borrowing or over-investment, though the reader should understand that over-borrowing is equivalent to under-provision of liquidity (i.e., holding too few liquid assets).\(^4\)

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\(^1\)We do not need the number of firms to equal the number of households. We only need the number of households to be proportional to the number of firms to ensure that the economy properly scales as \( N \) varies. An even number of firms is necessary to ensure that they can be split into to equal-sized groups.

\(^2\)Modeling firms as risk-averse with linear production is a tractable way to generate a motive for insurance. We could also model firms as risk-neutral with curvature in their production technology (see Holmström and Tirole, 1998).

\(^3\)We take \( R \) as exogenous but can think of it as an outside option available to impatient lenders (see Appendix A).

\(^4\)In this case, letting \( \ell \) denote investments in liquid assets (e.g., cash), the budget constraint would be \( q_0 k + \ell = n + d \). It is easy to verify that consumption in each state, as well as quantities of assets sold/purchased, are just a function of \( d - \ell \), and so ignoring liquidity holdings is equivalent to folding liquidity holdings into
Uncertainty. There are two aggregate states in the economy at $t = 1$, a good state and a bad state, with probabilities $\alpha$ and $1 - \alpha$, respectively. In the good state, all firms have productivity $\overline{A} > R$ and are therefore able to repay their debt. In the bad state, half of the $2N$ firms, randomly selected, have low productivity $A_L$ and the other half have high productivity $A_H$ with $A_H > A_L$ and low average productivity:

$$A = \frac{1}{2}A_L + \frac{1}{2}A_H < R.$$ 

We mainly consider the case $A_H > R > A_L$ but also discuss the case $R > A_H > A_L$ below. We assume that firms cannot borrow more in the bad state at $t = 1$, so a firm $i$ with low productivity has a cash shortfall $A_L k_i - R d_i < 0$, forcing it to sell capital. A firm $j$ with high productivity have a cash surplus $A_H k_j - R d_j > 0$ allowing it to buy capital. Additionally, the low average productivity $A$ ensures that households, in addition to firms with high productivity, will buy capital in the bad state.

Capital market. We assume that firms may behave strategically in the initial period $t = 0$ but are price takers at the interim date $t = 1$. This is to clearly contrast the Cournot decision at $t = 0$ with the social planner decision at $t = 0$ by keeping other periods unchanged. However, we show in Appendix B that allowing for strategic behavior at $t = 1$ would not affect our results. In the bad aggregate state, a firm $i$ with low productivity sells capital to repay debts and supplies

$$z_{iL} = \frac{R d_i - A_L k_i}{q} = \frac{(R - A_L) k_i - R n}{q},$$

while a firm $j$ with high productivity buys capital and demands

$$x_{jH} = \frac{A_H k_j - R d_j}{q} = \frac{R n + (A_H - R) k_j}{q}.$$ 

Households are perfectly competitive and their demand for capital is $x_{hh} = a/q - 1$.\(^5\) Thus, market clearing for capital, $\sum z_{iL} = \sum x_{jH} + \sum x_{hh}$ with $N$ low types, $N$ high types the debt in our baseline analysis.

\(^5\)Households’ demand is the solution to $\max_{x_{hh}} \{a \log(1 + x_{hh}) - qx_{hh}\}$ with first-order condition $a (1 + x_{hh})^{-1} = q$.
Table 1: Summary of aggregate states and firm consumption.

<table>
<thead>
<tr>
<th>Aggregate state</th>
<th>Prob.</th>
<th>Prod. shock</th>
<th>Consumption</th>
<th>Capital price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good state</td>
<td>$\alpha$</td>
<td>$A$</td>
<td>$\tau_i = R_n + (A + 1 - R) k_i$</td>
<td>$\bar{q} = 1$</td>
</tr>
<tr>
<td>Bad state</td>
<td>$1 - \alpha$</td>
<td>$A_L$ (prob. 1/2)</td>
<td>$c_{iL} = \frac{R_n}{q} - \frac{R - A_L - q}{q} k_i$</td>
<td>$q &lt; a &lt; 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_H$ (prob. 1/2)</td>
<td>$c_{jH} = \frac{R_n}{q} + \frac{A_H - R + q}{q} k_j$</td>
<td></td>
</tr>
</tbody>
</table>

and $2N$ households, implies that the price of capital in the bad state is

$$q = a + R_n + \sum_{j \in H} \frac{(A_H - R) k_j}{2N} - \sum_{i \in L} \frac{(R - A_L) k_i}{2N}. \quad (1)$$

**Payoffs.** In the good state, all firms have the same productivity and firm $i$’s consumption at $t = 2$ is given by

$$\bar{c}_i = (\bar{A} + 1) k_i - R d_i = R_n + (\bar{A} + 1 - R) k_i.$$

In the bad state, a low-productivity firm $i$ sells capital, resulting in consumption

$$c_{iL} = k_i - \frac{R d_i - A_L k_i}{q} = \frac{R_n}{q} - \frac{R - A_L - q}{q} k_i,$$

while a high-productivity firm $j$ buys capital, resulting in consumption

$$c_{jH} = k_j + \frac{A_H k_j - R d_j}{q} = \frac{R_n}{q} + \frac{A_H - R + q}{q} k_j.$$

Table 1 summarizes the payoffs across states and times. Accordingly, the expected utility of firm $i$ at $t = 0$ is given by

$$V(k_i) = \alpha v(\bar{c}_i) + (1 - \alpha) \left( \frac{1}{2} v(c_{iH}) + \frac{1}{2} v(c_{iL}) \right)$$

$$= \alpha v\left( R_n + (\bar{A} + 1 - R) k_i \right)$$

$$+ (1 - \alpha) \left( \frac{1}{2} v\left( \frac{R_n}{q} + \frac{A_H - R + q}{q} k_i \right) + \frac{1}{2} v\left( \frac{R_n}{q} - \frac{R - A_L - q}{q} k_i \right) \right). \quad (2)$$

### 2.1 Walrasian equilibrium

We first consider the allocation arising in the Walrasian equilibrium (WE), to which we then compare the social planner (SP) allocation and the allocation arising in the Cournot equilibrium (CE). In the Walrasian equilibrium, all firms act as price takers with respect
to the $t = 1$ price of capital $q$ when choosing their level of borrowing at $t = 0$ to maximize their expected utility from (2). Taking $q$ as exogenous, the first-order condition of a firm in the Walrasian equilibrium is

$$\alpha \left( \overline{A} + 1 - R \right) v'(\overline{e}) + \frac{1 - \alpha}{2} \left( \frac{A_H - R + q v'(c_H)}{q} - \frac{R - A_L - q v'(c_L)}{q} \right) = 0. \quad (3)$$

The first term is the benefit of more capital in the good state, where everyone receives a high productivity shock and holding more capital yields a net return $\overline{A} + 1 - R > 0$. The second term is the benefit or cost of more capital in the bad state, depending on whether the firm receives a high or a low productivity shock. Holding more capital benefits a firm in the bad state if it has high productivity since it allows for more profitable purchases of fire-sold capital, yielding a net return $(A_H - R)/q + 1$, but hurts a firm with low productivity since it forces more costly sales of capital, yielding a net return $-(R - A_L)/q + 1$.

### 2.2 Social planner

We now consider the allocation chosen by the social planner who maximizes firm utility while being constrained to a choice of investment (and implied borrowing) at $t = 0$ just like the firms themselves. To enable a direct comparison to the firms’ first-order condition (3), we do not explicitly consider household welfare in the social planner’s problem.\(^6\)

We replicate the standard (and intuitive) result that the Walrasian equilibrium invests in too much capital. The social planner chooses a single level of capital for all firms to maximize their expected utility from (2) but takes into account the effect on the equilibrium price of capital (1) which, setting $k_i = k_j = k$ for all $i$ and $j$, simplifies to

$$q = a + Rn - \left( R - \frac{1}{2} (A_H + A_L) \right) k = a + Rn - (R - A) k.$$  

\(^6\)Including household welfare would only strengthen our result of Cournot agents overcorrecting the externality since it would reduce the social planner’s incentive to mitigating fire sales which benefit households.
Compared to the Walrasian equilibrium first-order condition (3), the social planner’s first-order condition considers the impact of capital holdings on the price:

$$\alpha (\bar{A} + 1 - R) v'(c) + \frac{1 - \alpha}{2} \left( \frac{A_H - R + q}{q} v'(c_H) - \frac{R - A_L - q}{q} v'(c_L) \right)$$

$$\quad + \frac{1 - \alpha}{2} \left( \frac{dc_H}{dq} v'(c_H) + \frac{dc_L}{dq} v'(c_L) \right) \frac{dq}{dk} = 0 \quad (4)$$

A higher level of capital decreases the fire-sale price, $dq/dk = -(R - \bar{A}) < 0$, which is good for high types who buy capital and have consumption decreasing in $q$ (for sufficiently small $n$),

$$\frac{dc_H}{dq} = -\frac{(A_H - R) k + Rn}{q^2} < 0,$$

but bad for low types who sell capital and have consumption increasing in $q$,

$$\frac{dc_L}{dq} = \frac{(R - A_L) k - Rn}{q^2} > 0.$$

Thus, the social planner trades off the gain to high types against the loss to low types (marginal-utility weighted). We can simplify the term in parentheses in (4), capturing the trade-off, as

$$\frac{dc_H}{dq} v'(c_H) + \frac{dc_L}{dq} v'(c_L) = \frac{1}{q} \left( -x_H v'(c_H) + z_L v'(c_L) \right).$$

By assumption, capital sales by low types exceed capital purchases by high types, i.e., $x_H < z_L$. Furthermore, high consumption exceeds low consumption and so marginal utility of high types is less than that of low types. Hence, $x_H v'(c_H) < z_L v'(c_L)$ and the additional social planner term in (4) is negative so that the social planner chooses a lower level of capital.

**Proposition 1** (Standard inefficiency of Walrasian equilibrium). *The pecuniary externality leads to inefficiently high investment in the Walrasian equilibrium, $k^{WE} > k^{SP}$.*

This is the standard result as shown by Lorenzoni (2008); the social planner holds less capital, which reduces fire sales, increasing the asset price in the bad state and increasing production since less capital is sold to low-productivity households.
2.3 Cournot equilibrium

In the Cournot equilibrium, firms take into account the effect of their own investment choice at $t = 0$ on the equilibrium price at $t = 1$, i.e. they maximize their expected utility from (2) subject to (1). A Cournot firm’s first-order condition therefore also contains a price-effect term but, in contrast to the social planner, the Cournot firm considers separately the price effect it has as a high or low type:

\[ a \left( \bar{A} + 1 - R \right) v'(\tau) + \frac{1 - \alpha}{2} \left( \frac{A_H - R + q}{q} v'(c_H) - \frac{R - A_L - q}{q} v'(c_L) \right) + \frac{1 - \alpha}{2} \left( v'(c_H) \frac{dc_H}{dq} \frac{dq_H}{dk} + v'(c_L) \frac{dc_L}{dq} \frac{dq_L}{dk} \right) = 0 \]  

(5)

Recall how the equilibrium price of capital (1) depends on individual firms’ level of capital:

\[ q = a + Rn + \sum_{i \in H} \frac{(A_H - R) k_i}{2N} - \sum_{j \in L} \frac{(R - A_L) k_j}{2N} = a + Rn - (R - A) k \quad \text{for} \quad k_i = k_j = k \]

While the relationship between the capital price and the aggregate level of capital is negative, the effect on the capital price as a high or low type differs. A high type firm has a positive effect on the price since its cash surplus, which is used for purchases, increases with its initial investment; a low type firm has a negative effect on the price since its cash shortfall, which forces sales, also increases with its initial investment:

\[ \frac{dq_H}{dk_i} = \frac{A_H - R}{2N} > 0 \quad \text{and} \quad \frac{dq_L}{dk_i} = -\frac{R - A_L}{2N} < 0 \]

Combining the effects on consumption, $dc_H/dq < 0$ and $dc_L/dq > 0$, with the price impacts, we therefore have

\[ \frac{dc_H}{dq} \frac{dq_H}{dk_i} < 0 \quad \text{and} \quad \frac{dc_L}{dq} \frac{dq_L}{dk_i} < 0. \]

(6)

That is, both as a buyer and as a seller, the extra term in a Cournot firm’s first-order condition is negative, biasing downward investment at $t = 0$. 

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Comparison to the WE allocation. The Walrasian first-order condition (3) and the Cournot first-order condition (5) differ only in the price-effect term. From (6), we know that the extra term is negative, so the Cournot equilibrium will always have less capital than the Walrasian equilibrium. In this leverage trade-off model, internalizing the price impact therefore does correct the pecuniary externality, $k_{\text{CE}} < k_{\text{WE}}$. The question is how much.

Comparison to SP allocation. Cournot firms will hold even less capital than the social planner if, at the social planner allocation, the extra term in the Cournot first-order condition (5) is smaller than the extra term in the social planner first-order condition (4). Substituting in for the derivatives in (4) and (5), and simplifying, we obtain a simple condition for when the Cournot term is smaller than the social planner term.

Proposition 2 (Over-correction in Cournot equilibrium). The pecuniary externality leads to inefficiently low investment in the Cournot equilibrium, $k_{\text{CE}} < k_{\text{SP}}$, if and only if

$$\frac{2N (R - A)}{2N (R - A) - R + A_H} < \frac{v'(c_H) x_H}{v'(c_L) z_L}. \quad (7)$$

While the right-hand side of condition (7) is positive, the left-hand side is negative for small $N$ and large $A_H - A_L$, holding $R$ and $A$ constant with $R - A$ not too large. This yields the following comparative statics

Corollary 1. Cournot behavior is more likely to over-correct the pecuniary externality in the leverage trade-off model if

- the degree of idiosyncratic productivity risk is larger (high $A_H - A_L$);
- the number of Cournot agents is smaller (low $N$).

Figure 1 illustrates the potential for Cournot to not only mitigates the inefficiently high investment of the Walrasian equilibrium but to overcorrect it. The figure compares the levels of investment in capital in the Walrasian and Cournot equilibria to the efficient level.\(^7\) As the degree of productivity risk increases, the efficient level of investment declines and is always lower than the level of investment in the Walrasian equilibrium. The Cournot equilibrium corrects this inefficiency as long as productivity risk is sufficiently low. Once idiosyncratic risk is sufficiently high, the Cournot equilibrium over-corrects the

\(^7\)For graphical clarity we use $N = 1$ (two Cournot firms) as a baseline and also show the case with $N = 3$ (six Cournot firms), but choosing higher $N$ would not qualitatively change the results so long as $N$ is not too large (see the discussion of empirical plausibility in Appendix C). The figure varies the degree of idiosyncratic risk by varying $A_H - A_L$ on the horizontal axis while keeping average productivity $A$ constant.
over-investment of the Walrasian equilibrium, leading to inefficiently low investment. Naturally, the region of over-correction is larger the smaller the number of Cournot agents is.

The intuition is clear: A higher price always hurts buyers and benefits sellers. For the social planner choosing aggregate capital, less capital increases the price and there is a trade off between the effects on buyers and sellers. In contrast, Cournot firms reason as follows: holding more capital will push down the price when they are a seller, but that is when they want the capital price to be higher; holding more capital will push up the capital price when they are a buyer, but that is when they want the capital price to be lower. Thus, whether they will be a buyer or a seller, strategically it makes sense to hold less capital. The social planner takes into account that less aggregate capital ex ante is not actually better for buyers of capital because, in the aggregate, less capital will push up the price.

Of course, for sufficiently low idiosyncratic risk $A_H - A_L$, condition (7) for under-investment compared to the social planner reverses and the Cournot equilibrium leads to investment higher than efficient but lower than in the Walrasian equilibrium. In particular, this is what happens with Cournot in the standard model of Lorenzoni (2008), which our model nests in the case of no idiosyncratic risk ($A_H = A_L < R$). Empirically, whether Cournot over-corrects the inefficiency mainly depends on the degree of idiosyncratic risk. In Appendix C, we argue that the parameter values necessary for the surprising Cournot effects are not implausible. We let various moments from data determine likely values for
parameters within this stylized model and find that, in reality, internalizing price impact likely over-corrects the externality.

In sum, while Cournot does mitigate the pecuniary externality as in the standard formulation of the model, for sufficiently high idiosyncratic risk, Cournot will over-correct, leading to under-investment relative to the social planner.

3 Cournot in a liquidity trade-off model

We now consider a standard model of fire sales due to liquidity shocks, in a setting based on Diamond and Dybvig (1983) with interim trade à la Allen and Gale (2004), potentially at fire-sale prices. In this model, the key choice is an ex-ante portfolio allocation between a liquid low-return asset and an illiquid high-return asset. Ex post, the liquid asset is preferred if hit by a liquidity shock, while the illiquid asset is preferred otherwise; the ex-ante choice has to trade off the two. The model is therefore representative of intermediaries that engage in maturity and liquidity transformation, which we refer to as “banks.” The canonical result in this type of model is that a pecuniary externality leads to inefficiently low liquidity holdings in the Walrasian equilibrium; the (constrained) social planner internalizes the externality and chooses higher liquidity holdings which leads to higher fire-sale prices and increases welfare (Allen and Gale, 2004). We show that internalizing the pecuniary externality through Cournot behavior can exacerbate the standard inefficiency instead of mitigating it, leading to even lower liquidity holdings than in the Walrasian equilibrium. This is the polar opposite of the result in the leverage trade-off model of Section 3 where Cournot always mitigates but can over-correct the inefficiency. This model-specific nature of our results highlights how incomplete the naive conclusion that Cournot behavior would correct pecuniary externalities is when going beyond a standard formulation of these models.

Technology and preferences. There are three periods $t = 0, 1, 2$, and $2N$ banks that start with one unit of endowment at $t = 0$ and have two investment opportunities: (i) liquid assets, which, for each unit invested at $t = 0$, deliver 1 at $t = 1$ or $t = 2$; (ii) illiquid assets, which, for each unit invested at $t = 0$, deliver $R > 1$ at $t = 2$ but nothing before. Denote by $\ell_i$ the fraction of bank $i$’s funds invested in liquid assets (hence, $1 - \ell_i$ is invested in illiquid assets).

In the spirit of Diamond and Dybvig (1983), banks can be subject to “liquidity shocks” in which case they only value early consumption at $t = 1$; otherwise they discount utility
from consumption at $t = 2$ by $\beta \leq 1$:

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with liquidity shock,} \\ u(c_1) + \beta u(c_2) & \text{without liquidity shock.} \end{cases}$$

We will suppose throughout our analysis that banks’ utility $u$ has relative risk aversion of at least 1. Together with $\beta R > 1$, our assumptions on preferences imply a standard demand for liquid claims, and therefore a role for banks in providing liquidity insurance. We could instead assume that banks may have a high or low fraction of depositors withdraw, rather than a binary zero-one type of shock. This would have quantitative implications for our results without affecting the qualitative results. See Appendix A for a micro-foundation of such banks pooling resources from many households with correlated liquidity needs.

**Uncertainty.** Analogous to the leverage trade-off model in Section 3, there are two aggregate states at $t = 1$, a good state and a bad state, with probabilities $\alpha$ and $1 - \alpha$, respectively. In the good state, no liquidity shocks occur and no bank is forced to liquidate early. In the bad state, half the banks, randomly selected, receive liquidity shocks. These banks sell their illiquid assets to the other half that did not receive liquidity shocks, at an endogenous price $p$.

**Asset market.** Also analogous to the leverage trade-off model, we assume that banks behave strategically in the initial period $t = 0$ but are price takers at the interim date $t = 1$. We show in Appendix B that allowing for strategic behavior at $t = 1$ would again not affect our results. We assume that the asset price at $t = 1$ is determined by cash-in-the-market pricing, $p = \min\{p_{\text{CITM}}, R\}$, where $p_{\text{CITM}}$ is such that the total value of assets being sold equals the total cash available to buy assets (Allen and Gale, 1994):

$$\sum_{i \in \text{sell}} (1 - \ell_i) \times p_{\text{CITM}} = \sum_{j \in \text{buy}} \ell_j$$

(8)

Note that the market clearing condition (8) with cash-in-the-market pricing differs from the market clearing condition (1) in the the leverage trade-off model of Section 2 which has additional demand from households. We show in Appendix D that adding such outside buyers to the liquidity trade-off model does not materially affect our results.\(^8\)

\(^8\)In the leverage trade-off model, $L$ types have a cash shortfall forcing asset sales, i.e. supply a fixed dollar amount, and $H$ types have a cash surplus to buy assets, i.e. demand a fixed dollar amount); without a
**Table 2:** Summary of aggregate states and investor consumption.

<table>
<thead>
<tr>
<th>Aggregate state</th>
<th>Prob.</th>
<th>Liquidity shock</th>
<th>Consumption</th>
<th>Asset price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good state</td>
<td>$\alpha$</td>
<td>Nobody hit</td>
<td>$\tau_i = \ell_i + (1 - \ell_i) R$</td>
<td>$\overline{p} = R$</td>
</tr>
<tr>
<td>Bad state</td>
<td>$1 - \alpha$</td>
<td>Hit (prob. 1/2)</td>
<td>$c_{iL} = \ell_i + (1 - \ell_i) p$</td>
<td>$p &lt; R$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Not hit (prob. 1/2)</td>
<td>$c_{jH} = \ell_j R \overline{p} + (1 - \ell_j) R$</td>
<td></td>
</tr>
</tbody>
</table>

**Payoffs.** In the good state, all banks consume at $t = 2$ and bank $i$’s consumption $\tau_i$ is given by

$$\tau_i = \ell_i + (1 - \ell_i) R.$$ 

In the bad state, a bank $i$ receiving a liquidity shock sells its illiquid assets and and consumes at $t = 1$ an amount $c_{iL}$ ($L$ for low) given by

$$c_{iL} = \ell_i + (1 - \ell_i) p.$$ 

A bank $j$ that does not receive a liquidity shock uses its liquid assets to buy illiquid assets and consumes at $t = 2$ an amount $c_{jH}$ ($H$ for high) given by

$$c_{jH} = \ell_j \frac{R}{p} + (1 - \ell_j) R.$$ 

It is clear that $p \leq R$ in equilibrium since $H$ types would not be willing to pay more than $R$ for illiquid assets. We suppose that the only buyers of illiquid assets are other banks and liquidity shocks therefore lead to an asset price strictly below $R$ in the bad aggregate state.\(^9\) Table 2 summarizes the states. Accordingly, the expected utility of banks is given by

$$U(\ell_i) = \alpha \beta u(\overline{c}_i) + (1 - \alpha) \left( \frac{1}{2} u(c_{iL}) + \frac{1}{2} \beta u(c_{iH}) \right)$$

$$= \alpha \beta u(\ell_i + (1 - \ell_i) R)$$

$$+ (1 - \alpha) \left( \frac{1}{2} u(\ell_i + (1 - \ell_i) p) + \frac{1}{2} \beta u(\ell_j \frac{R}{p} + (1 - \ell_j) R) \right).$$

\(^9\)The model easily generalizes to additional buyers of illiquid assets as long as they are second-best users or have limited resources as is standard in the fire-sale literature (Shleifer and Vishny, 2011).
3.1 Walrasian equilibrium

As in the leverage trade-off model, we first consider the allocation arising in the Walrasian equilibrium (WE), to which we then compare the social planner (SP) allocation and the allocation arising in the Cournot equilibrium (CE). In the Walrasian equilibrium, all banks act as price takers with respect to the $t = 1$ price of illiquid assets when choosing their portfolio at $t = 0$ to maximize their expected utility from 9. Taking $p$ as exogenous, a bank’s first order condition in the Walrasian equilibrium is

$$
\alpha \beta (R - 1) u'(\tau) = \left(1 - \alpha\right) \left(\frac{1}{2} (1 - p) u'(c_L) + \frac{1}{2} \beta \left(\frac{1}{p} - 1\right) Ru'(c_H)\right) = \frac{1 - \alpha}{2} (1 - p) \left(u'(c_L) + \beta \frac{R}{p} u'(c_H)\right).
$$

(10)

The left-hand side (LHS) is the cost of holding extra liquidity in the good aggregate state, where no one receives a liquidity shock and holding more illiquid assets instead of liquid assets yields a net return $R - 1 > 0$. The RHS is the benefit of extra liquidity in the bad state; since the LHS is positive, it must be that the liquidation price satisfies $p < 1$. Holding extra liquidity in the bad state then is good both as a seller of assets (i.e. if hit by a liquidity shock) since it requires fewer sales at net cost $1 - p > 0$ and as a buyer of assets since it allows more asset purchases with net return $\frac{1}{p} - 1 > 0$. Note the contrast to the leverage trade-off model in Section 2 where, in the Walrasian equilibrium, holding more capital benefits a buyer since it allows more purchases but hurts a seller since it requires more sales.

If there is no aggregate risk ($\alpha = 0$) so that only the bad state can occur, then the first-order condition 10 implies $p = 1$ in equilibrium. If $p < 1$, then assets are traded below cost so no-one wants to invest in them; sellers (state $L$) would rather hold liquidity and buyers (state $H$) would rather buy assets cheaply. Vice versa for $p > 1$. Equilibrium in the case of no aggregate risk is pinned down by the no-arbitrage condition, $p = 1$, which leads to $c_L = 1$ and $c_H = R$. The resulting wedge in marginal utilities, $u'(c_L) > \beta Ru'(c_H)$, represents the standard insufficient liquidity risk sharing of Diamond and Dybvig (1983). If there is aggregate risk ($\alpha > 0$), then the wedge in marginal utilities is $u'(c_L) > \beta \frac{R}{p} u'(c_H)$ which maintains the insufficient risk sharing.\(^{10}\)

\(^{10}\)Making use of the proof in Diamond and Dybvig (1983), we have $2\ell u'(2\ell) > 2 (1 - \ell) \beta Ru'(2 (1 - \ell) R)$. Since $c_L = 2\ell$, $c_H = 2 (1 - \ell) R$ and $p = \ell / (1 - \ell)$ in equilibrium, this implies $u'(c_L) > \beta \frac{R}{p} u'(c_H)$. 

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3.2 Social planner

We now consider the allocation chosen by a social planner who maximizes bank utility while being constrained to a choice of liquidity holdings at $t = 0$, just like the banks themselves. We replicate the standard (and intuitive) result that the Walrasian equilibrium provides inefficiently low liquidity.

The social planner chooses a single level of liquidity holding for all banks to maximize their expected utility from (9) but takes into account the effect on the equilibrium asset price (8) which, setting $\ell_i = \ell_j = \ell$ for all $i$ and $j$, simplifies to $p = \ell / (1 - \ell)$. Compared to the Walrasian first-order condition (10), the social planner’s first-order condition has an additional term on the RHS which considers how liquidity holdings will affect the asset price:

$$
\alpha \beta (R - 1) u'(\bar{c}) = \frac{1 - \alpha}{2} (1 - p) \left( \frac{\beta}{p} u'(c_H) \right) + \frac{1 - \alpha}{2} (1 - \ell) \left( u'(c_L) - \frac{\beta}{p} u'(c_H) \right) \frac{dp}{d\ell}.
$$

(11)

The social planner considers that more liquidity increases the price by $dp/d\ell = 1/(1 - \ell)^2$, which benefits sellers who gain $u'(c_L)$, and hurts buyers who lose $\beta R u'/(c_H)$. Since $u'(c_L) > \beta R u'(c_H)$, the social planner chooses higher liquidity than the Walrasian equilibrium.

**Proposition 3** (Standard inefficiency of Walrasian equilibrium). *The pecuniary externality leads to inefficiently low liquidity holdings in the Walrasian equilibrium, $\ell^{WE} < \ell^{SP}$.*

The intuition for the standard constrained inefficiency of the Walrasian equilibrium is that the market incompleteness prevents full insurance against the liquidity risk. The constrained social planner, by changing the price, can perform a reallocation that is outside the asset span and thereby increase welfare (Geanakoplos and Polemarchakis, 1986).

In the case without aggregate risk ($\alpha = 0$), the social planner’s first order condition yields the standard optimal risk sharing condition, $u'(c_L) = \beta Ru'(c_H)$, of Diamond and Dybvig (1983). Without aggregate risk, our setup with trading at $t = 1$ essentially corresponds to the Jacklin (1987) model, and our result that liquidity under-provision can be corrected by increasing the asset price is found also in Farhi, Golosov, and Tsyvinski (2009) and Geanakoplos and Walsh (2018).
3.3 Cournot equilibrium

In the Cournot equilibrium (CE), banks take into account the effect the of their own liquidity choice at $t=0$ on the equilibrium price at $t=1$, i.e., they maximize their expected utility from (9) subject to (8), also resulting in an additional term in the first order condition:

\[
\begin{align*}
\text{cost same as WE} & = \frac{1 - \alpha}{2} (1 - p) \left( u'(c_L) + \beta \frac{R}{p} u'(c_H) \right), \\
\text{benefit for given } p \text{ same as WE} & = \frac{1 - \alpha}{2} (1 - \ell_i) \left( \frac{dp_L}{d\ell_i} u'(c_L) - \frac{dp_H}{d\ell_i} \beta \frac{R}{p} u'(c_H) \right).
\end{align*}
\]

(12)

Extra CE price effect

Similar to the leverage trade-off model, a Cournot bank distinguishes between the price $p_L$ it faces as a seller and the price $p_H$ it faces as a buyer as well as the effect extra liquidity holdings have on the two. However, in contrast to the leverage trade-off model where the $t=0$ choice has opposite effects on the two prices (more capital increases the buyer price and decreases the seller price), here the $t=0$ choice has the same effect on the two prices (more liquidity increases the buyer price and the seller price). Specifically, from the equilibrium condition (8), the asset price is

\[
p = \frac{\sum_{i \in H} \ell_i}{\sum_{j \in L} (1 - \ell_j)}.
\]

As a buyer, the bank affects the numerator while as a seller it affects the denominator; with $2N$ banks and taking as given other banks’ (symmetric) equilibrium choice $\ell$, we have

\[
\frac{dp_H}{d\ell_i} = \frac{1}{N} \frac{1}{1 - \ell} > 0 \quad \text{and} \quad \frac{dp_L}{d\ell_i} = \frac{1}{N} \frac{\ell}{(1 - \ell)^2} > 0.
\]

(13)

Compared to the social planner’s price impact, $dp/d\ell = 1/(1 - \ell)^2$, the Cournot bank’s price impacts are uniformly lower, biasing downward the Cournot liquidity choice. We now consider how the allocation in the Cournot equilibrium compares to the constrained efficient allocation chosen by the social planner and the allocation in the Walrasian equilibrium.

Comparison to SP allocation. Cournot leads to inefficiently low liquidity if, at the social planner allocation, the price-effect term of the Cournot first-order condition (12) is less than the price-effect term of the social planner first-order condition (11). Substituting in
for the price effects, the condition becomes
\[
\frac{1}{N} \left( \ell u'(c_L) - (1 - \ell) \frac{R}{p} u'(c_H) \right) < u'(c_L) - \frac{R}{p} u'(c_H).
\] (14)

In the natural case where the good aggregate state lowers the efficient level, i.e. for \( p < 1 \) at the social planner allocation, we have \( u'(c_L) > \frac{R}{p} u'(c_H) \) and \( \ell < 1/2 \) so condition (14) is satisfied and, as expected, Cournot liquidity is inefficiently low, \( \ell_{CE} < \ell_{SP} \).\(^{11}\)

**Comparison to WE allocation.** To assess whether internalizing the price impact attenuates or exacerbates the inefficiently low liquidity holdings of the Walrasian equilibrium, the key comparison is between the allocations in the Cournot equilibrium and in the Walrasian equilibrium. Cournot yields less liquidity than the Walrasian equilibrium and therefore exacerbates the inefficiency if, at the Walrasian allocation, the price-effect term of the Cournot first-order condition (12) is negative.

**Proposition 4 (Exacerbation in Cournot equilibrium).** The pecuniary externality leads to even lower liquidity in the Cournot equilibrium than in the Walrasian equilibrium, \( \ell_{CE} < \ell_{WE} \), if and only if
\[
\frac{dp_L}{d\ell_i} \times u'(c_L) - \frac{dp_H}{d\ell_i} \times \frac{R}{p} u'(c_H) < 0.
\] (15)

We know that, at the Walrasian allocation, the marginal benefit of additional liquidity exceeds the marginal cost, \( u'(c_L) > \frac{R}{p} u'(c_H) \). But in the Cournot first-order condition (12), the price impacts act as weights on the benefit and the cost. Condition (15) is therefore satisfied if the seller price impact \( dp_L/d\ell_i \) is sufficiently low relative to the buyer price impact \( dp_H/d\ell_i \). From (13) we have that the seller price impact relative to the buyer price impact depends on the level of the asset price \( p \):
\[
\frac{dp_L/d\ell_i}{dp_H/d\ell_i} = p
\]

This implies that if liquidity \( \ell \) (and thus price \( p \)) is sufficiently low in the Walrasian equilibrium, then Cournot yields even less liquidity than the Walrasian equilibrium, exacerbating the inefficiency. For log utility, the “sufficiently low” condition simplifies to the fairly weak condition \( p < \beta \) and, more generally this case of sufficiently low \( p \) arises, e.g. if the bad state is not too likely.

\(^{11}\)The social planner will find it optimal to implement \( p < 1 \) as long as the good state is sufficiently likely and/or the illiquid asset sufficiently productive (high \( \alpha \) and/or \( R \)).
Corollary 2. Cournot behavior is more likely to exacerbate the pecuniary externality in the liquidity trade-off model if

- the likelihood of the bad aggregate state is smaller (high $\alpha$);
- the number of Cournot agents is smaller (low $N$).

Figure 2 illustrates the potential for Cournot to exacerbate the inefficiently low liquidity holdings of the Walrasian equilibrium. The figure compares the levels of liquidity provision in the Walrasian and Cournot equilibria to the efficient level. As the good state without liquidity shocks becomes more likely ($\alpha$ increases), the efficient level of liquidity declines but is always higher than the one provided by the Walrasian equilibrium. The Cournot equilibrium corrects this inefficiency only if the good state is sufficiently unlikely (liquidity risk is sufficiently high). Once the good state is sufficiently likely and liquidity risk therefore sufficiently low, the Cournot equilibrium exacerbates the under-provision of liquidity in the Walrasian equilibrium. The right panel shows that, for high $\alpha$, the Cournot level of liquidity is substantially below the Walrasian level. Naturally, the region of exacerbation is larger the smaller the number of Cournot agents is.

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12For graphical clarity we use $N = 1$ (2 banks) as a baseline and include $N = 3$ (6 banks), but choosing higher $N$ would not qualitatively change the results so long as $N$ is not too large (e.g., the results are similar with 10 banks with our preferred calibrations; see the discussion of empirical plausibility in Appendix C).
Figure 3: Effects of liquidity on Cournot price. The figure shows the price impacts for a seller, $\frac{dp_H}{d\ell_i} = \frac{1}{N} \frac{1}{1-\ell}$, for a buyer, $\frac{dp_H}{d\ell_i} = \frac{1}{N} \frac{\ell}{(1-\ell)^2}$, and for the social planner, $\frac{dp_H}{d\ell} = \frac{1}{(1-\ell)^2}$, for $N = 1$ (two banks).

For intuition, note that in the limit $\alpha \to 1$, liquidity has little ex-ante value and endogenously $\ell \to 0$ resulting in $p \to 0$. The seller price impact $\frac{dp_L}{d\ell_i}$, weighting the benefit of liquidity in condition (15), goes to zero while the buyer price impact $\frac{dp_H}{d\ell_i}$, weighting the cost of liquidity, does not, as illustrated in Figure 3. The Cournot equilibrium then holds very little liquidity because more liquidity would have a negligible price benefit when agents receive liquidity shocks and sell assets but a non-zero cost when agents do not receive liquidity shocks and instead buy assets.$^{13}$

To understand this difference in the limit behavior of buyer and seller price impact, consider the equilibrium condition (8) determining the price $p$. Additional liquidity of buyers enters directly in the form of more cash while additional liquidity of sellers enters indirectly in the form of more assets with a factor $p$. The marginal effect of cash on the equilibrium condition is therefore always 1 but the marginal effect of additional assets is low if the price $p$ is low.

Empirically, whether Cournot exacerbates the inefficiency mainly depends on the severity of the fire sale in the bad state. In Appendix C, we argue that the parameter values necessary for the surprising Cournot effects are not implausible. Internalizing price impact in the liquidity trade-off model can either mitigate or exacerbate the pecuniary externality, depending on parameter values. We let various moments from data determine likely val-

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$^{13}$This requires that $\frac{dp_L}{d\ell_i} u'(c_L)$ goes to zero as long as the marginal utility does not increase too quickly, which holds as long as if risk aversion is not too high or marginal utilities are bounded.
ues for parameters within this stylized model and find that, in reality, internalizing price impact likely exacerbates the externality.

In sum, with aggregate risk, Cournot can provide even less liquidity than the Walrasian equilibrium, in violation of the hypothesis that internalizing the pecuniary externality should lead to an allocation closer to the social planner’s.

4 Conclusion

In light of increasing concentration in both real and financial markets, we have considered the effects of market power in standard macro-finance models of fire sales where pecuniary externalities lead to constrained inefficiency. We show that market power can not only mitigate but over-correct the inefficiently high borrowing in a canonical model of leverage choice with productivity shocks, representative of firms with real investment. In contrast, we show that internalizing the price impact can exacerbate the inefficiently low liquidity holdings in a canonical model of portfolio allocation with liquidity shocks, representative of financial intermediaries engaged in liquidity transformation.

In terms of policy implications, our results highlight that intervention has to be tailored to both the type of activity and the competitiveness of a given sector. For the real sector, where market power over-corrects the tendency toward inefficient credit booms, our results imply a need for stronger investment stimulus as the sector becomes more concentrated, e.g. through an expansion of the favorable tax treatment of debt financing. For the financial sector, where market power exacerbates the tendency to hold insufficient liquidity, our results imply a need for stronger liquidity regulation as the sector becomes more concentrated, e.g. through a tightening of the Basel III Liquidity Coverage Ratio. Finally, our results speak to antitrust policy, highlighting additional welfare-relevant effects of market power. For example, in the debate whether concentration enhances financial stability (Bordo et al., 2015), our results show how important stringent liquidity regulation is for achieving stability benefits from concentration.
References


Appendix

A A single model of banks and real investment

In the main text, we present two separate models, one with a choice of investment level funded with debt and one with a choice of portfolio allocation between liquid and illiquid assets. In this appendix, we show how the two models can be viewed as representing two parts of an economy with financial intermediation. The liquidity trade-off model then represents the main decision of banks in allocating households savings between liquid assets and illiquid loans to firms; the leverage trade-off model then represents the main decision of equity constrained firms borrowing from banks to invest in productive assets.

The economy consists of households, banks, and firms, active in a real sector with productivity shocks and a financial sector with liquidity shocks. Households have funds to invest but are inefficient at operating capital and face uncertain consumption needs, while firms are efficient at operating capital but have small initial endowments. Banks, which are mutually owned by households, sit squarely between the two agents, taking deposits from households and providing loans to firms. There are three periods, \( t = 0, 1', 1'' \). Liquidity shocks determining banks’ solvency occur at \( t = 1' \), while productivity shocks determining firms’ output occur at \( t = 1'' \). The shocks can lead to fire sales of financial assets and real assets in the financial and real sector, respectively. Period \( t = 2 \) functions as “the future” or a continuation value for production in the economy.

At \( t = 0 \) banks have access to liquid and illiquid investment technologies. With the liquid technology, one unit of capital invested at \( t = 0 \) produces one unit of consumption good (“output”) either at \( t = 1' \) or \( t = 1'' \). With the illiquid technology, one unit of capital invested at \( t = 0 \) produces \( R > 1 \) units of output at \( t = 1'' \), but nothing at \( t = 1' \). Illiquid investments can be traded at \( t = 1' \) at any endogenous price \( p \).

In each period, firms have access to a linear production technology using capital. Production at \( t = 1'' \) is risky: capital \( k \) invested at \( t = 0 \) produces \( Ak \) units of output at \( t = 1'' \), where \( A \) is uncertain with \( \mathbb{E}[A] > R \). To simplify, production at \( t = 2 \) is risk-free, with every unit of period-1 capital producing one unit of output at \( t = 2 \). Firms have small endowments of capital at \( t = 0 \), denoted by \( n \), and have utility function \( v(c) \) over consumption in period 2.

In each period, households have access to a production technology that takes capital \( k \) and yields \( F(k) = a \log(1 + k) \) units of consumption goods in the next period. We suppose that \( a < 1 \) so that households are never the efficient users of capital. Each period contains a new generation of households endowed with one unit of capital. Importantly, however,
households born at $t = 0$ are subject to liquidity shocks à la Diamond and Dybvig (1983): they will either consume at $t = 1''$ (late types), or they will receive a liquidity shock and be forced to consume at $t = 1'$ (early types). Households receive utility $u(c)$ over early consumption and $\beta u(c)$ over late consumption, with $\beta \leq 1$ and $\beta R > 1$. To simplify the analysis, we suppose that households born at $t = 1''$ are not subject to liquidity shocks.

Banks pool resources from many households in order to offer deposit contracts that provide liquidity in the sense of Diamond and Dybvig (1983). Banks serve a restricted economic area as in Allen and Gale (2004), able to take deposits only from a set of households with correlated liquidity needs (i.e., banks cannot serve the entire population of households and completely diversify away liquidity shocks). To simplify, we assume that the households in each area have perfectly correlated liquidity needs. As a result, a bank whose consumers are early types will be forced to liquidate its assets in the interim period. Thus, we can say that the bank is itself subject to liquidity shocks.

At $t = 0$, firms can borrow from banks by issuing non-contingent debt due at $t = 1''$. Firms cannot default, and therefore bank loans to firms are identical to investments in the illiquid technology. Thus, firms can borrow at a gross interest rate $R$. Firms cannot borrow new funds at $t = 1''$ but must repay debt using proceeds from production or from selling capital at an endogenous price $q$.

The economy therefore features the following financial frictions. In the financial sector, banks and households cannot insure against liquidity risk and the price of illiquid assets is determined by cash-in-the-market pricing (i.e., bank capital is slow moving and so demand for assets must come from banks who do not receive liquidity shocks). In the real sector, firms cannot insure against productivity shocks (i.e., they are restricted to borrow using non-contingent debt) and firms are subject to borrowing constraints at $t = 1''$ (they cannot borrow to repay/roll over debts).

We assume that, relative to the firm sector, household endowments are sufficiently large and liquidity demands sufficiently small so that in equilibrium banks’ demands for illiquid investments exceed firms’ demands for borrowing. As a result, the flow of funds in the economy in equilibrium can be described as follows. At $t = 0$, households deposit all capital with banks. Banks allocate a fraction $\ell$ of capital to liquid investments and a fraction $1 - \ell$ to illiquid investments, a portion of which are loans to firms at an interest rate $R$. Firms borrow $d$ units from banks, allowing them to invest $k = n + d$ in capital for risky projects. (Our relative size assumption means that in equilibrium $d < 1 - \ell$.) At $t = 1''$, firms repay debts, perhaps by selling capital to households at price $q$ in order to do so, and remaining capital is invested in projects to produce at $t = 2$.

Given these assumptions, we can solve for equilibrium in this economy by considering
the financial and real sectors separately. First, we can consider the equilibrium provision of liquidity by banks at $t = 0$ and analyze how internalizing price impacts in the market for illiquid investments affects the price $p$ and the level of liquidity $\ell$. Second, we can consider the equilibrium borrowing decision of firms at $t = 0$ and analyze how internalizing price impacts in the market for capital affects the price $q$, the level of investment $k$, and borrowing $d$. Because all bank loans are risk-free, outcomes in the real sector (firm production and fire sales in capital) do not affect behavior or outcomes in the financial sector (provision of liquidity and fire sales in illiquid assets), and vice versa. As a result, we can also analyze pecuniary externalities in financial and real markets separately, and a social planner attempting to correct each externality can consider them separately without considering interactions between real and financial markets. The results therefore correspond to those in the main text.

B Strategic interim behavior

In the main analysis, we suppose that agents strategically choose portfolios at $t = 0$ (understanding that their portfolios will affect future prices), but in later periods agents act as price takers. In this section, we extend the previous analysis to allow agents to also act strategically when assets trade.

To incorporate strategic behavior in the interim period, we suppose that buyers choose a value of funds $f$ with which they purchase assets and sellers choose a quantity of assets $z$ to sell. The price is determined given the funds supplied to purchase assets and the quantity of assets supplied, as in the canonical strategic market game of Shapley and Shubik (1977) which converges to Walras in the limit (Dubey and Geanakoplos, 2003).

B.1 Strategic interim behavior in the leverage trade-off model

Consider the leverage trade-off model and now suppose that firms act strategically in the market for capital at $t = 1$. We show that low-productivity firms still find it optimal to sell the least amount of capital necessary to repay their debt. Taking into account their effect on price means that the amount they sell is the solution to a fixed point condition but their sales are still an increasing function of the capital they hold. We also show that, under mild conditions, high-productivity firms still find it optimal to use all their funds, just as in the non-strategic case. Allowing for strategic behavior at $t = 1$ therefore does not change the fact that additional investment in capital at $t = 0$ drives up the price paid as a buyer and down the price received as a seller. The potential for overcorrection of the
externality is therefore unchanged.

Firms with low productivity shocks choose an amount $z$ of capital to sell. Firms with high productivity shocks choose an amount $f$ of funds to purchase capital. Market clearing with $N$ low types, $N$ high types and $2N$ households requires

$$\sum_{i \in L} z_i = \sum_{j \in H} f_j + 2N \left( \frac{a}{q} - 1 \right),$$

which implies a price of capital given by

$$q(z, f) = \frac{2Na + \sum_{j \in H} f_j}{2N + \sum_{i \in L} z_i},$$

(16)

**Sellers.** Seller $i$ chooses $z_i$, taking as given other sellers’ choices $z_{-i}$ and buyers choices $f$, to solve the problem

$$\max_{z_i} \{ k_i - z_i + A_L k_i + qz_i - R_{di} \}$$

s.t.  
$qz_i \geq R_{di} - A_L k_i$
$z_i \leq k_i$
$q = q(z_i, z_{-i}, f)$

For the seller constraint $qz_i \geq R_{di} - A_L k_i$ to be binding, i.e. for them not wanting to sell more than necessary to repay debt, we need the price elasticity with respect to $z_i$ to satisfy:

$$-\frac{\partial q}{\partial z_i} \frac{z_i}{q} > 1 - \frac{1}{q}$$

(17)

For the case $q < 1$ that we are interested in, this condition is satisfied by a positive price elasticity which we naturally have from the price function (16):

$$-\frac{\partial q}{\partial z_i} \frac{z_i}{q} = \frac{z_i}{2N + \sum_{j \in L} z_j}$$

A seller acting strategically at $t = 1$ therefore finds it optimal to sell the least amount of capital possible to repay their debt. Since they take into account their effect on the price, their optimal sales are given by a fixed point condition

$$q(z_i, z_{-i}, f) z_i = R_{di} - A_L k_i.$$
Solving for $z_i$ and substituting in $d_i = k_i - n$, we obtain

$$z_i = \left( \frac{2N + \sum_{j \in L \setminus i} z_j}{2Na + \sum_{j \in H} f_j} \right) \left( (R - A_L) k_i - Rn \right)$$

which is increasing in $k_i$. Since the optimal level of sales with strategic behavior at $t = 1$ is increasing in the level of capital chosen at $t = 0$, the comparative statics underlying the results in the main text remain unchanged.

**Buyers.** Buyer $i$ chooses $f_i$, taking as given other buyers’ choices $f_{-i}$ and sellers’ choices $z$, to solve the problem

$$\max_{f_i} \left\{ k_i + \frac{f_i}{q} + A_H k_i - f_i - Rd_i \right\}$$

s.t. $f_i \leq A_H k_i - Rd_i$

$$q = q(z, f_i, f_{-i})$$

For the buyer constraint $f_i \leq A_H k_i - Rd_i$ to be binding, i.e. for them to use all their funds, we the price elasticity with respect to $f_i$ to satisfy:

$$\frac{\partial q}{\partial f_i} \frac{f_i}{q} < 1 - q \quad (18)$$

From the price function (16) we have an elasticity with respect to $f_i$ given by

$$\frac{\partial q}{\partial f_i} \frac{f_i}{q} = \frac{f_i}{2Na + \sum_{j \in H} f_j}.$$ 

Substituting in this elasticity, using (16), and the equilibrium conditions $f_i = f$ and $z_i = z$ for all $i$, condition (18) becomes

$$q < \frac{2Na + (N - 1)f}{2Na + Nf}. \quad (19)$$

Given that we are interested in the case $q < 1$, this is a weak condition that holds for sufficiently low $q$. A buyer acting strategically at $t = 1$ then finds it optimal to use all their funds to buy capital, exactly as in the case without strategic interaction at $t = 1$.

Figure 4 illustrates that condition (19) is satisfied for almost all parameter combinations shown in Figure 1 in the main text. The figure shows the difference between the equilib-
Figure 4: Condition for strategic interim behavior in leverage trade-off model. The figure shows the difference between the equilibrium price in the leverage trade-off model from the main text and the threshold from condition (19) such that non-strategic interim behavior is w.l.o.g. for different levels of idiosyncratic productivity risk, $A_H - A_L$. With relative risk aversion 1, $\alpha = 0.85$, $a = 0.93$, $n = 1$, $N \in \{1, 2, 3\}$, $\mathbb{E}[A] = 1.05$, $R = 1.02$, and $A = 0.99$.

### B.2 Strategic interim behavior in the liquidity trade-off model

We now consider the liquidity trade-off model and suppose that banks act strategically in the asset market at $t = 1$. We show that for $N > 1$, banks with liquidity shocks still find it optimal to sell all their assets. We also show that, under mild conditions, banks without liquidity shocks still find it optimal to use all their funds, just as in the non-strategic case. In sum, allowing for strategic behavior at $t = 1$ has no effect on the choices at $t = 1$ and therefore no effect on the optimization at $t = 0$. We derive the conditions in the more general setting with outside liquidity $N\phi \geq 0$ (see Appendix D).

Banks with liquidity shocks choose an amount $z$ of assets to sell. Banks without liquidity shocks choose an amount $f$ of funds to purchase assets. Market clearing implies an asset price given by

$$p(z, f) = \frac{N\phi + \sum_{j \in H} f_j}{\sum_{i \in L} z_i}.$$  

(20)
**Sellers.** Seller $i$ chooses $z_i$, taking as given other sellers’ choices $z_{-i}$ and buyers choices $f$, to solve the problem

$$\max_{z_i} u(\ell_i + pz_i)$$

s.t. $z_i \leq 1 - \ell_i$

$$p = p(z_i, z_{-i}, f)$$

For the seller constraint $z_i \leq 1 - \ell_i$ to be binding, i.e. for them to sell all their assets, we need the price elasticity with respect to $z_i$ to satisfy:

$$-\frac{\partial p}{\partial z_i} \frac{z_i}{p} < 1$$

This is satisfied by the price function (20) for $N > 1$:

$$-\frac{\partial p}{\partial z_i} \frac{z_i}{p} = \frac{z_i}{\sum_{j \in L} z_j}$$

A seller acting strategically at $t = 1$ therefore finds it optimal to sell all their assets, exactly as in the case without strategic interaction at $t = 1$.

**Buyers.** Buyer $i$ chooses $f_i$, taking as given other buyers’ choices $f_{-i}$ and sellers’ choices $z$, to solve the problem

$$\max_{f_i} u\left(\ell_i - f_i + R \frac{f_i}{p} + R (1 - \ell_i)\right)$$

s.t. $f_i \leq \ell_i$

$$p = p(z, f_i, f_{-i})$$

For the buyer constraint $f_i \leq \ell_i$ to be binding, i.e. for them to use all their funds, we need the price elasticity with respect to $f_i$ to satisfy:

$$\frac{\partial p}{\partial f_i} \frac{f_i}{p} < 1 - \frac{p}{R}$$

(21)

From the price function (20) we have an elasticity with respect to $f_i$ given by

$$\frac{\partial p}{\partial f_i} \frac{f_i}{p} = \frac{f_i}{N\phi + \sum_{j \in H} f_j}$$
Figure 5: Condition for strategic interim behavior in liquidity trade-off model. The figure shows the difference between the equilibrium price in the liquidity trade-off model from the main text and the threshold from condition (22) such that non-strategic interim behavior is w.l.o.g. for different values of the probability of the good state, $\alpha$. With log utility, $\beta = 0.96$, $R = 1.03/\beta$, and $N = 2$ (see Appendix C).

Substituting in this elasticity, using (20), and the equilibrium conditions $f_i = f$ and $z_i = z$ for all $i$, condition (21) becomes

$$p < \frac{N\phi + (N - 1)f}{N\phi + Nf} R$$  \hspace{1cm} (22)

Given that we are interested in the case $p < 1$ and have $R > 1$, this is condition holds for sufficiently low $p$ (i.e. high $\alpha$) and/or large $R$. A buyer acting strategically at $t = 1$ then finds it optimal to use all their funds to buy assets, exactly as in the case without strategic interaction at $t = 1$. Figure 5 illustrates that condition (22) is satisfied for the relevant case of high $\alpha$ where Cournot leads to severe underprovision of liquidity (see Figure 2 in the main text). The figure shows the difference between the equilibrium price and the threshold from condition (19) which is negative if the condition is satisfied. Note that for more outside liquidity (higher $\phi$), the region where strategic interim behavior is irrelevant increases considerable.

C  Empirical plausibility

The models presented in this paper are simple and the interesting results depend on parameters. In the leverage model, whether Cournot over-corrects the inefficiency mainly
depends on the degree of idiosyncratic risk. In the liquidity model, whether Cournot exacerbates the inefficiency mainly depends on the severity of the fire sale in the bad state. In this section, we argue that the parameter values necessary for the surprising Cournot effects are not implausible. Given the calibrations of our simple models, we can also compare welfare across the allocations of Cournot, Walras and the social planner. However, given the very stylized nature of the models, the quantitative welfare effects should be taken with a grain of salt.

### C.1 Empirical plausibility in the leverage trade-off model

The most important variable that determines the region we are in is the level of idiosyncratic risk facing firms in the bad aggregate state. There is substantial evidence that productivity dispersion is counter-cyclical (see Kehrig, 2015). Bloom et al. (2018) find that the standard deviation of micro-productivity shocks in recessions is 20.9\% which implies $A_H - A_L = 0.418$ in our model.\(^\text{14}\) Apart from the level of risk aversion, our results are not sensitive to the remaining variables, which we set in relatively standard ways. We set $\alpha = 0.85$, which corresponds to the frequency of expansions post-WWII. We set the real rate to 2\% and the expected return on capital to 5\% so that capital earns 3\% excess return in expectation. We let $a = 0.93$, so that the second-best user of capital has a 7\% productivity loss and we set $A = 0.99$, corresponding to an aggregate shock 5\% below average. Table 3 contains the parameters we use.

Table 3: Leverage Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>0.85</td>
</tr>
<tr>
<td>$a$</td>
<td>0.93</td>
</tr>
<tr>
<td>$n$</td>
<td>1</td>
</tr>
<tr>
<td>$R$</td>
<td>1.02</td>
</tr>
<tr>
<td>$\mathbb{E}[A]$</td>
<td>1.05</td>
</tr>
<tr>
<td>$\bar{A}$</td>
<td>0.99</td>
</tr>
<tr>
<td>$A_H - A_L$</td>
<td>0.418</td>
</tr>
</tbody>
</table>

Figure 6 plots capital holdings relative to the efficient level and consumption equivalent losses for several values of risk aversion $\sigma$ and varying the market size $N$. Regardless of the level of risk aversion, the Cournot equilibrium with $N = 1$ has capital investment that is about 10\% below the efficient level. In other words, the level of idiosyncratic risk is

\(^{14}\text{Bloom et al. (2018) find that the unconditional standard deviation of micro-productivity shocks is 5.1\% and that it is 4.1 times higher in recessions.}\)
high enough that Cournot over-corrects the externality. Importantly, the fire-sale discount is on the order of 80%, implying substantial efficiency losses from capital being allocated to second-best users (households). A critical element of our model is that some first-best users (firms) are positioned to buy capital cheaply during downturns, which may seem at odds with the intuition in Shleifer and Vishny (1992), where fire sales occur because first-best users must sell to second-best (inefficient) users of capital. Our results show that, indeed, the primary force driving fire sales is the reallocation to households, which pushes down the price of capital substantially. Opportunistic buying by lucky firms is important for our mechanism without violating the intuition of Shleifer and Vishny (1992).

It is thus empirically plausible that Cournot can over-correct the pecuniary externality, leading to inefficiently low levels of real investment. The levels of idiosyncratic risk present in the data are well above the level of idiosyncratic risk required for Cournot to over-correct in our model. Nonetheless, there are several caveats that could push against our results. First, a high level of competition (high $N$) would bring the level of capital closer to the Walrasian level, thus weakening the over-correction. Second, in the model, debt is the only vehicle available for firms to borrow, implying that firms retain all their idiosyncratic risk. If in reality firms can shed some of this productivity risk, then bad shocks need not lead to forced sales (and good shocks need not lead to higher levels of cash).

The welfare implications of the over-correction depend on the level of risk aversion for firms. Welfare decreases as the level of capital moves away from the efficient level, whether due to an over-correction or due to an under-correction of the externality. If risk aversion
is relatively high ($\sigma > 1$), then the over-correction from Cournot is preferred over the under-correction from the Walrasian equilibrium. With risk aversion $\sigma = 2$ (a plausible estimate for risk aversion for households), welfare losses are 52% of consumption in the Walrasian equilibrium and 6% in the Cournot equilibrium. With $\sigma = 1$ the consumption equivalent losses are 20% and 6.8% respectively. In these cases, the over-correction from Cournot is not so severe and thus welfare is higher with a low level of competition. The policy implications in this case would be to allow industry concentration but to provide incentives for investment. One could easily argue that firms should be modeled as less risk averse than the typical household. For low levels of risk aversion, the welfare results change substantially. With $\sigma = 0.5$, the welfare loss in the Walrasian equilibrium is 2.8% while the welfare loss from Cournot is 8%; with $\sigma = 0.25$, the welfare losses are 0.16% and 9.1%, respectively. In this case, the Cournot over-correction is very costly in terms of welfare losses and the policy implications are quite different because industry concentration is quite bad for welfare.

C.2 Empirical plausibility in the liquidity trade-off model

Internalizing price impact in the liquidity trade-off model can either mitigate or exacerbate the pecuniary externality, depending on parameter values. We find that, in reality, internalizing price impact likely exacerbates the externality.

Before considering the full exercise, consider the following back-of-the-envelope exercise. The risk aversion of banks is probably low; in the model, the lowest we can set risk aversion to is 1 (log utility). The impatience parameter $\beta$ determines how much banks discount illiquid relative to liquid claims. Estimates of liquidity premia are typically on the order of basis points (20bps in Gertler and Kiyotaki, 2015) and so $\beta$ should be close to 1. From our analytical results, Cournot will exacerbate the externality with log utility whenever the Walrasian fire-sale price is below $\beta$. Thus, if fire sales are a meaningful discount of fair value (more than 10% seems very conservative) and fair value is not too much greater than 1, then internalizing price impact will exacerbate the externality.

We consider two strategies to calibrate our parameters: target liquidity holdings to be 13% of banks assets;\(^{15}\) or target the fire sale to a 35% discount relative fair value. Table 4 contains the parameters used for each calibration, which we discuss in detail below. Figure 7 plots Cournot liquidity holdings relative to the Walrasian level and consumption equivalent losses for each calibration varying the market size $N$.

\(^{15}\)This corresponds to the ratio of bank liquid reserves to bank assets in the U.S. (IMF International Financial Statistics).
Table 4: Liquidity Model Parameters.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>0.92</td>
<td>0.99</td>
</tr>
<tr>
<td>$\beta^R$</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.98</td>
<td>0.97</td>
<td>0.91</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.01</td>
<td>1.05</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 7: Quantitative effects in the liquidity trade-off model. The figure shows the effect of market size $N$ on the liquidity provision of the Cournot equilibrium relative to the Walrasian equilibrium (left panel), and on the welfare of the Cournot equilibrium relative to the social planner allocation in terms of consumption equivalent (right panel) under the calibrations in Table 4.
We show two parameterizations to hit 13% liquidity. In the first, we let \( \beta = 0.96 \), which corresponds to a standard annual discount rate; we suppose that \( \beta R = 1.03 \) so that illiquid assets earn about 3% excess returns; we suppose that \( \alpha = 0.98 \), so that financial crises occur 2% of the time (see Gertler and Kiyotaki, 2015, for similar estimates). With relative risk aversion \( \sigma = 1.01 \), the model then delivers 13% liquidity holdings in the Walrasian equilibrium. At this calibration, the efficient level of liquidity is 3.6% higher than the Walrasian level, but a Cournot equilibrium with \( N = 1 \) holds 43% less liquidity than the Walrasian equilibrium (i.e., banks hold 7.4% liquid assets), exacerbating the externality. In terms of welfare losses, the Walrasian equilibrium has welfare that is negligibly below the efficient level, while welfare in the Cournot equilibrium corresponds to a 0.16% loss in terms of consumption equivalent compared to the efficient outcome.

While this parameterization is entirely plausible, the probability of crises is somewhat low. Instead we now let \( \alpha = 0.97 \) (following Gertler and Kiyotaki, 2015), which on its own would significantly increase liquidity holdings in equilibrium. To hit our liquidity target, we set \( \beta = 0.92 \), and get \( \sigma = 1.05 \). For this calibration, the efficient level of liquidity is 9.8% higher than the Walrasian level, but a Cournot equilibrium with \( N = 1 \) holds 38.5% less liquidity than the Walrasian equilibrium, exacerbating the inefficiency. In terms of welfare, the loss in the Walrasian equilibrium is 0.008% in terms of consumption equivalent, while the loss in the Cournot equilibrium is 0.26%. In either case, the parameters are well within the range of parameters for which Cournot competition exacerbates the externality.

In the model, the level of liquidity directly determines the fire sale price in the bad state. Liquidity holdings of 13% imply a fire sale price of \( p = 0.15 \). It is fair to wonder if the right variable to target is liquidity holdings and not the level of fire sales directly, since the externality is after all determined by the fire sale in the asset price. We now target \( p = 0.65 \times R \), which corresponds to a 35% discount over fair value for financial assets. One could reach this number, e.g. by considering the history of prices for ABX during the financial crisis and comparing trough levels to what prices ultimately returned to. It is more difficult to get the model to provide liquidity holdings high enough so that the fire sale price is this high. To do so, we set \( \beta = 0.99 \) and \( \alpha = 0.91 \), implying a very high likelihood of financial crises (higher than we believe to be empirically plausible). Maintaining \( \beta R = 1.03 \), the model requires \( \sigma = 1 \) in order to hit the target for fire sales. At this calibration, the efficient level of liquidity is 0.57% higher than the Walrasian level, but a Cournot equilibrium with \( N = 1 \) holds 19.3% less liquidity than the Walrasian equilibrium, exacerbating the inefficiency. In terms of welfare losses, the Walrasian equilibrium is negligibly below the efficient level, while welfare in the Cournot equilibrium corresponds to a 0.13% loss in terms of consumption equivalent.
In sum, all three calibrations are well within the range where Cournot exacerbates the externality, and the results for liquidity provision and welfare are meaningful. We do not take these results quantitatively seriously, but they do provide strong evidence at least for the direction of how the externality is affected (exacerbated, not mitigated). Furthermore, since we find that Cournot exacerbates the externality, the question “how much” depends on the competitiveness of the industry (i.e., on \( N \)). Less competition will lead to greater under-provision of liquidity. Thus, industry concentration is strictly bad for the fire sale externality, and policy should respond by providing greater incentives to hold liquid assets (disincentives to hold illiquid assets).

D Liquidity trade-off model with outside buyers

We now consider the case of outside buyers in the liquidity trade-off model of Section 3. Specifically, we assume that there are \( N \) outside buyers with \( \phi \geq 0 \) in cash to buy assets at \( t = 1 \). This collapses to the model in the main text for \( \phi = 0 \). With outside liquidity, the market clearing condition (8) becomes

\[
\sum_{i \in \text{sell}} (1 - \ell_i) \times p_{\text{CITM}} = N\phi + \sum_{j \in \text{buy}} \ell_j.
\]

The first-order condition (10) of Walrasian equilibrium remains unchanged and still implies \( p < 1 \) for \( \alpha > 0 \) and \( p = 1 \) for \( \alpha = 0 \).\(^{16}\) However, due to the additional outside liquidity \( \phi \), the equilibrium inside liquidity \( \ell \) will be lower. For example, in the case \( \alpha = 0 \), the equilibrium \( p = 1 \) implies that we can solve for \( \ell \) in closed form and it is decreasing in \( \phi \):

\[
\ell = \frac{1 - \phi}{2}.
\]

The social planner first-order condition (11) is affected by \( \phi \) through the price effect

\[
\frac{dp}{d\ell} = \frac{\phi + 1}{(1 - \ell)^2}.
\]

Combined with the effect of \( \phi \) on \( p \), the additional outside liquidity \( \phi \) will therefore also lower the efficient level of liquidity \( \ell \). Consider again the case \( \alpha = 0 \) where the social planner implements the standard risk sharing \( u'(c_L) = \beta Ru'(c_H) \) of Diamond and Dybvig (1983). Since outside liquidity substitutes for inside liquidity, the same risk sharing can be

\(^{16}\)Note that payoffs are in terms of the individual choice \( \ell_i \) and the equilibrium price so their formulas are as in the main text.
achieved with lower $\ell$.

Finally, we turn to the Cournot first-order condition (12). Notably, the outside liquidity $\phi$ appears only in the price impact as perceived by an $L$ type who sells assets, changing the price impacts in (13) as follows:

$$\frac{dp_L}{d\ell_i} = \frac{1}{N} \frac{\phi + \ell}{(1 - \ell)^2}$$

This price impact, which in the first-order condition weighs the benefit of holding extra liquidity, is increasing in $\phi$. The presence of outside liquidity therefore biases downward the inside liquidity $\ell$ in the Cournot equilibrium as well. The Cournot equilibrium can still lead to lower liquidity than the Walrasian equilibrium if the seller price impact is sufficiently low relative to the buyer price impact. We still have that the ratio of the two satisfies

$$\frac{dp_L/d\ell_i}{dp_H/d\ell_i} = \frac{\phi + \ell}{1 - \ell} = p.$$ 

Since $\phi > 0$ bounds the price (and therefore the ratio of price impacts) away from zero, higher outside liquidity attenuates the underprovision of liquidity in the Cournot equilibrium.