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Abstract

Loose financial conditions forecast high output growth and low output volatility up to six quarters into the future, generating time-varying downside risk to the output gap, which we measure by GDP-at-Risk (GaR). This finding is robust across countries, conditioning variables, and time periods. We study the implications for monetary policy in a reduced-form New Keynesian model with financial intermediaries that are subject to a Value at Risk (VaR) constraint. Optimal monetary policy depends on the magnitude of downside risk to GDP, as it impacts the consumption-savings decision via the Euler constraint, and financial conditions via the tightness of the VaR constraint. The optimal monetary policy rule exhibits a pronounced response to shifts in financial conditions for most countries in our sample. Welfare gains from taking financial conditions into account are shown to be sizable.

Key words: monetary policy, financial conditions, financial stability

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1 Introduction

Financial conditions play an important role in monetary policy. Central banks commonly adjust the stance of policy, and forward guidance as a function of shocks to financial conditions. The asset purchase programs that were undertaken by major central banks after the financial crisis aimed at influencing financial conditions in risky asset markets, such as longer term sovereign debt markets via the term premium, mortgage markets via mortgage spreads, and even credit markets in some jurisdictions.

In this article, we provide a novel rationale for the importance of financial conditions in the conduct of monetary policy, by showing that financial conditions are highly significant forecasting variables for the conditional distribution of the output gap. We thus extend recent work by Adrian et al. (2016) to a multi country setting, focusing on the GDP gap (difference between GDP and its potential), instead of GDP growth. We document that loose financial conditions forecast a high GDP gap and low GDP volatility up to six quarters into the future. This finding is robust to variations in indicators of financial conditions, countries, and time sample.

We summarize the downside risk to GDP using the notion of GDP-at-Risk (GaR), which was recently proposed by the IMF in its October 2017 *Global Financial Stability Report* (GFSR (2017)). GaR(τ) is the value-at-risk (VaR) of the GDP gap τ quarters into the future. GaR is shown to vary primarily as a function of financial conditions, while economic conditions are not a significant forecaster of downside risk to the output gap. We argue that financial conditions should be a key variable for the conduct of monetary policy because they determine GaR significantly, even if financial conditions do not enter the central bank's objective (or loss function).

In order to study the quantitative importance of GaR for monetary policy making, we calibrate an optimal monetary policy rule in a reduced form macro-financial model. The model features a standard New Keynesian (NK) setup with a Phillips curve that is determined by producers with staggered price setting that give rise to a Philips curve. Relative to the standard NK model of Woodford (2001) and Galí (2015), we introduce a financial intermediation sector that is subject to a VaR constraint, as in Adrian and Duarte (2016). The price of risk varies as a function of the tightness of the VaR constraint of intermediaries, shifting households consumption Euler equation (the IS curve). Importantly, the state variables that impact the second moment of the Euler equation also impact the first moment of the Euler equation. Hence monetary policy moves both first and second moments.

We derive the optimal monetary policy rule in this reduced form setting, and show that it depends not only on the output gap and inflation, but also on financial conditions. We calibrate the optimal monetary policy rule across countries, and find that optimal monetary policy deviates significantly from classic Taylor rules. This is because financial conditions carry important information about the evolution of the variance of output gaps. In other words, financial conditions help policymakers better take into account the distribution of output gaps, including downside risks.

The welfare gains from using the augmented Taylor rule as opposed to the classic Taylor rule are sizable. In the augmented Taylor rule, monetary policy is allowed to respond to financial conditions, whereas it is constrained not to do so when following a classic Taylor rule. Results are robust to the choice of country and time sample, whether or not including the global financial crisis. Welfare gains are approximately equal between advanced and emerging market economies, while the tradeoff between the mean and variance of output appears somewhat more attenuated in emerging markets. In all cases, though, optimal monetary policy responds to financial conditions.

Our findings contribute to the recent debate about the role of financial stability in monetary policy. To date, the debate essentially focussed on whether monetary policy should pursue an additional mandate—that of minimizing risks of costly financial crises. The literature, summarized in Smets et al. (2014) and expanded upon in Fund (2015) and **?**, focuses on the costs and benefits of increasing policy rates more than warranted to satisfy inflation and output objectives, so as to diminish risks of occasional crises. In general, under plausible calibrations, it seems like costs of doing so are greater than benefits. In addition, macroprudential policy appears better suited to directly target the imperfections that undermine fiancial stability. Monetary policy should remain focused on its output and price stability mandate. That is precisely the assumption taken in this paper. However, we suggest that to do so optimally, monetary policy should take into account financial conditions, as a useful variable to forecast the mean and variance of output.

Our findings are also closely related to the recent literature on the role of financial intermediation in monetary policy. Papers such as Curdia and Woodford (2010), Gertler and Karadi (2011), and Gambacorta and Signoretti (2014)—surveyed in more details later—consider welfare gains from monetary policy responding to credit spreads. These papers generally find that it is preferable to do so following financial sector shocks, but not necessarily in response to other shocks, such as productivity shocks. This paper studies instead a non-linear model better suited to emphasize second moments, namely the variance and risk of GDP. Doing so is warranted by the capacity of financial conditions to mostly forecast downside risk to GDP.

The remainder of the paper is organized as follows. We start with a description of the stylized features of the data in Section 2. We then present the NK vulnerability model (NKV), a NK model augmented with financial vulnerability in Section 3. In Section 4, we calibrate the model and present welfare calculations of the costs and benefits of including financial conditions in the optimal monetary policy rule. Section 6 places our findings within the recent macro-financial monetary policy literatures. Section 7 concludes. The analytical details are given in the Appendix.

2 Financial Conditions and GDP-at-Risk

We start by investigating the conditional distribution of the GDP gap as a function of financial conditions. We model the mean and variance of the output gap as functions of financial conditions for a sample of advanced and emerging market economies (AEs and EMEs). We run panel regressions to gauge average behavior across countries.

Our goal in this initial analysis is to quantify the tradeoff between the mean and variance of the output gap. This tradeoff is a key ingredient in monetary policy decisions, as output and inflation stabilization crucially depends on the conditional mean and variance of the output gap. The analysis also derives the distribution of the output gap conditional on financial conditions, and offers a measure of vulnerability as discussed earlier.

To preview our results, we find a clear negative relationship between the mean and variance of the output gap. When financial conditions become tighter (higher credit spreads or price of risk), the output gap falls and variance grows. This emphasizes a key tradeoff for policymakers. Results are robust to the sample and period of study, though there is heterogeneity across countries. If anything, the tradeoff has become slighter more recently, and is somewhat lesser in EMEs relative to AEs.

All our data is quarterly. We obtain consumer price indices (CPIs) and real GDP from the *International Financial Statistics* (IFS). We compute the output gap by filtering the GDP data with a Hodrick-Prescott filter with coefficient $\lambda = 1600$. Inflation rate is the year-on-year percentage changes in the CPI. We use the financial conditions indices (FCIs) from the IMF's October 2017 Global Financial Stability Report (GFSR (2017)). Univariate FCIs offer a parsimonious way of summarizing the information contained in asset prices and credit aggregates from a broad set of domestic and global financial variables.[1] Higher value in FCIs correspond to tighter financial conditions or a higher price of risk. We conduct all of the estimation using the logarithm of the FCIs.[2] The panels are unbalanced, with data for AEs starting in 1973, and for EMEs starting in 1990. Country coverage is presented in Table (1).

2.1 Estimation

For the panel estimation, we apply a two-step procedure. We first estimate the relationship between the output gap and past measures of financial conditions. From this equation we extract the estimated variance of the output gap, which we regress on past financial conditions in a second step.

We first estimate

$$\Delta y_{i,t} = \alpha_0 + \alpha_1 s_{i,t-1} + \alpha_2 y_{i,t-1} + \alpha_3 \pi_{i,t-1} + \sigma_{i,t}\epsilon_{i,t}, \tag{1}$$

where $y_{i,t}$ is the output gap for country i and $\Delta y_{i,t}$ is its change between periods $t-1$ and t, $\pi_{i,t-1}$ is the lagged inflation rate, $s_{i,t-1}$ is the lagged FCI. The error term is given by the product of the conditional standard deviation $\sigma_{i,t}$ and a white noise term $\epsilon_{i,t}$.

In the second step, we take the estimated residuals $\sigma_{i,\hat{t}}\epsilon_{i,t}$ and, given that $\epsilon_{i,t}$ is white noise, we calculate $ln(\hat{\sigma}^2_{i,t})$. We interpret these log-squared residuals as the volatility of the output gap and regress them into the same variables used in Equation (1):

$$ln(\hat{\sigma}^2_{i,t}) = \beta_0 + \beta_1 s_{i,t-1} + \beta_2 y_{i,t-1} + \beta_3 \pi_{i,t-1} + \nu_{i,t} \tag{2}$$

Both panel regressions are estimated by OLS with fixed effects and bootstrapped standard errors. Results for regressions (1) and (2) are presented in Table (2).

Financial conditions are significant in explaining the first and second moments of the output gap, both for the AE and EME samples. This is in line with Adrian and Duarte (2016) and GFSR (2017). The output gap depends negatively on financial conditions (a higher FCI—higher spreads—induces a lower output gap), while the correlation is positive for the variance of the output gap. As a result, the conditional distribution of the output gap is skewed, even if shocks are normal. This is as in Adrian et al. (2016).

< Insert Table 2 here >

To show the negative relationship between the mean and variance of the output gap, we regress the fitted values of the dependent variables in equation (1) onto those of equation (2). Figure (1) shows the scatter plots and fitted OLS line.

< Insert Figure 1 here >

[1] See GFSR (2017) for more details on the underlying data and estimation methods.

[2] As FCIs are standardized, we first add a positive constant for the FCIs to have strictly positive domain and then standardize the resulting logarithm.

The negative relationship underscores the intratemporal tradeoff faced by policy makers. A closed output gap is consistent with positive variance; a result of the positive intercept. A tradeoff therefore arises given by the slope of the best-fit line. Reducing the output gap from a positive level, for instance, comes with an increase in variance. The steeper the line, the shallower the tradeoff. In the limiting case of a vertical line, the tradeoff disappears. Variance would be constant (homoskedastic) and the central bank would be able to adjust the output gap at no cost to its volatility.

Interestingly, this tradeoff seems to have diminished in recent years. The absolute value of both intercept and slope have increased over time; curves have become steeper. Figure (2) shows the scatter plot and fitted OLS regression lines for two subsamples for each set of countries. For AEs, we divide the sample between 1973Q1-1990Q4 and 1991Q1-2016Q4. For EMEs, we divide the sample between 1990Q1-1995Q4 and 1996Q1-2016Q4. If anything, the tradeoff between mean and variance of the output gap appears somewhat stronger for AEs than for EMEs.

< **Insert Figure 2 here** >

3 The Model

We use the model of Adrian and Duarte (2016), which is a microfounded non-linear New Keynesian model augmented by a financial intermediation sector with vulnerabilities but otherwise standard. Here, we briefly describe the model in words and then present the equations of a reduced-form version that Adrian and Duarte (2016) show is a good approximation to the full model.

There are four types of agents in the economy: a representative household, firms that produce consumption goods, banks that intermediate household savings and finance firms, and a central bank.

- Firms are exactly as in the standard New Keynesian model. They produce a continuum of differentiated goods in a monopolistically competitive way with sticky (Calvo-style) prices using a production technology that is linear in labor. There is no physical capital and productivity is constant.
- The representative household maximizes its utility of consumption and leisure subject to its budget constraint. Unlike the standard New Keynesian model, the household cannot directly finance the firms that produce consumption goods in the economy. Instead, it can only invest in the intermediary sector by trading a complete set of zero net supply Arrow-Debreu securities with intermediaries (which can replicate, among other payoffs, the payoff of riskless deposits).
- The intermediary sector invests the resources obtained from households and its previously accumulated net worth in a portfolio of assets. Intermediaries have the necessary information, expertise or relationships to finance the good producers directly. Therefore, intermediaries can hold stocks and bonds of the good producing firms in their portfolio. In addition, each intermediary can hold stocks and bonds of other intermediaries, and can trade a complete set of Arrow-Debreu securities with the household and with other intermediaries. There are two frictions in the intermediary sector. First, intermediaries are subject to exogenous preference shocks, which are the only shocks in the economy. These preference shocks can be interpreted as shocks that shift intermediaries' effective risk aversion and/or their beliefs. Second, when picking their optimal portfolio to maximize the expected net present value of dividends (shareholder value), intermediaries are subject to a value-at-risk constraint

that limits the amount of tail risk they can take[3].

- The dual-mandate central bank pursues its objective to minimize the present value of mean square deviations of inflation and the output gap from target.

Policy cannot achieve the first or second best equilibrium. The first best is the allocation that coincides with the one obtained in the decentralized equilibrium in which firm prices are fully flexible and households can finance firms without any frictions (without the need for an intermediary). The second best can be obtained when retaining sticky prices but removing the friction that households cannot invest into firms without an intermediary. For both the first and second best equilibria, all endogenous variables are constant, since the only shock in the economy is to the preferences of intermediaries and they are bypassed completely. When the three frictions we introduce—the inability of the household to finance firms directly, the preference shocks to intermediaries and the value-at-risk constraint—are present, the decentralized equilibrium always results in allocations with lower welfare than in the second best case.[4] This holds true independently of monetary policy. From the point of view of the central bank, the frictions are taken as given and cannot be eliminated by monetary policy. The best the central bank can hope for is to achieve a third best equilibrium.

The reduced-form version of the model is given by

$$
\begin{aligned}
dy_t &= \frac{1}{\gamma}\left(i_t - \pi_t - r\right)dt + d\left(rp_t\right), && (3)\\
d\pi_t &= \left(\beta\pi_t - \kappa y_t\right)dt, && (4)\\
i_t &= \psi_0 + \psi_\pi \pi_t + \psi_y y_t + \psi_v V_t, && (5)\\
V_t &= -\mathbb{E}_t\left[dy_t\right]\tau - \alpha \mathbb{V}_t\left[dy_t\right]\sqrt{\tau}, && (6)\\
d\left(rp_t\right) &= \xi\left(V_t - s_t\right)dZ_t, && (7)\\
ds_t &= -\rho_s\left(s_t - \overline{s}\right) + \sigma_s dZ_t. && (8)
\end{aligned}
$$

The core of the model consists of a traditional IS and Phillips curve, through the former is expanded to include the risk premium. Equation (3) is the dynamic IS equation, the linearized first-order condition[5] of the representative household. The constant $\gamma^{-1} > 0$ is the elasticity of intertemporal substitution[6] and the constant r is the natural rate of interest. The endogenous variables in the dynamic IS are the output gap, y_t, the nominal interest rate, i_t, inflation, π_t and the risk premium rp_t. Equation (4) is the New Keynesian Phillips Curve (NKPC), the firms' linearized first-order conditions when they maximize profits by picking the price of differentiated consumption goods under monopolistic competition while subject to consumers' demand and Calvo pricing. The constant $\beta > 0$ is the representative household's discount rate and $\kappa > 0$ is related to the amount of price stickiness in the economy. As $\kappa \to \infty$, prices become fully flexible, while as $\kappa \to 0$, prices become fixed.

[3] Adrian and Shin (2010) provide extensive motivation.

[4] If the decentralized economy with all three frictions were to replicate the first or second best, it would have to feature constant consumption for the household. The trading between the household and the intermediaries would have to be such that intermediaries insure the household fully against all shocks. If that were the case, intermediaries would bear all the risk in the economy. But bearing all the risk in the economy always implies that, eventually, intermediaries must violate their value-at-risk constraint, which shows that there are no equilibria with constant consumption for the household.

[5] For all stochastic processes, linearization means linearizing the drift and stochastic parts of the true non-linear process around the deterministic steady-state.

[6] The representative agent has CRRA utility, and hence γ is its coefficient of relative risk aversion.

Monetary policy is set by a standard Taylor rule, which can be complemented to respond to the output gap vulnerability. Equation (5) is the central bank's policy rule, where ψ_0, ψ_π, ψ_y and ψ_v are constants picked by the central bank. In addition to responding to inflation and the output gap, as is usual in traditional Taylor rules, the central bank can also respond to output gap vulnerability, V_t.

The central bank may want to respond to vulnerability because, as equation (7) shows, it is the key endogenous determinant of the risk premium rp_t that, in turn, is a direct input into the output gap dynamics in the IS equation. The parameter ξ in equation (7) is a reduced-form parameter that captures the strength of the frictions –preference shocks and value-at-risk constraint– in the intermediation sector. When $\xi = 0$, the risk premium vanishes and the model collapses to a standard deterministic New Keynesian model in continuous time identical to that studied in Werning (2011) and **?**. When $\xi \neq 0$, on the other hand, fluctuations in risk premia induce changes in the conditional mean and volatility of the output gap through equation (3).

Vulnerability is defined in equation (6) as the value-at-risk of the output gap projected at horizon $\tau > 0$ and level $\mathcal{N}(-\alpha)$, where $\mathcal{N}$ is the cumulative distribution function of a standard normal distribution. This means that V_t is the $\mathcal{N}(-\alpha)$ quantile of the projected distribution of $y_{t+\tau}$ conditional on time t information. For example, if $\tau = 1$ and $\alpha = \mathcal{N}^{-1}(0.05) = -1.96$, V_t is the $5th$ percentile of the one-year-ahead output gap distribution.

Output gap vulnerability is a consequence of intermediaries' value-at-risk constraint combined with the trading between the household and intermediaries of a complete set of Arrow-Debreu securities. The value-at-risk constraint creates vulnerability in the intermediaries' net worth; the trading between the household and intermediaries equalizes their marginal utilities, transmitting the vulnerability from intermediaries to households. Of course, without any risk in the economy, the value-at-risk constraint would never bind, so it is crucial to have some uncertainty for vulnerability to arise.

Uncertainty comes from exogenous shocks to vulnerability given by s_t, which also affect the risk premium as shown in equation (7). The process for s_t is a simple autoregressive process given by equation (8). The constant $\overline{s}$ is the long-run mean of s_t, $\sigma_s > 0$ is its instantaneous volatility and $\kappa > 0$ controls its rate of mean reversion.

This model exhibits a key amplification mechanism. Fluctuations in risk premia induce changes in the conditional mean and volatility of the ouput gap, as discussed earlier. Changes in the output gap, in turn, feed into vulnerability through equation (6). As vulnerability changes, so does risk premia, which again affects the output gap, vulnerability, and so on. The endogenous feedback between risk premia, vulnerability and the output gap is the result of the amplification mechanisms in the intermediation sector that arise because of financial fricitions.

As a result, when considering how to conduct monetary policy, the central bank must consider vulnerability not only because it is informative about the state of the economy but also because changes in policy endogenously change V_t, and changes in V_t feed back into inflation and the output gap. Note that even if the central bank set $\psi_v = 0$, it would still have an important impact on V_t through its influence on π_t and y_t.

4 Calibration

The reduced form model takes the form

$$\begin{aligned} dy_t &= \frac{1}{\gamma}\left(R_t - r + \gamma\hat{\eta}\xi\left(V_t - s_t - \frac{1}{2}\frac{\hat{\eta}}{\xi\gamma}\right)\right)dt + \xi\left(V_t - s_t\right)dZ_t && (9) \\ V_t &= -\mathbb{E}_t\left[dy_t\right]\tau - \alpha\mathbb{V}_t\left[dy_t\right]\sqrt{\tau} && (10) \\ ds_t &= -\rho_s\left(s_t - \overline{s}\right) + \sigma_s dZ_t && (11) \end{aligned}$$

where y_t is the output gap, R_t is the real risk-free rate, V_t is vulnerability (defined by equation (10)), s_t is an exogenous shock and Z_t is a standard Brownian motion. As for the constants, γ^{-1} is the elasticity of intertemporal substitution, r is the natural rate and $\hat{\eta}, \xi, \alpha, \tau, \rho_s, \overline{s}, \sigma_s$ are parameters related to vulnerability that have to be calibrated.

To calibrate, we assume we have available a time series for the conditional mean, $\mathbb{E}_t\left[dy_t\right]$, and conditional volatility, $\mathbb{V}_t\left[dy_t\right]$, of output gap growth. In the model, they are given by

$$\begin{aligned} \mathbb{E}_t\left[dy_t\right] &= \frac{1}{\gamma}\left(R_t - r + \gamma\hat{\eta}\xi\left(V_t - s_t - \frac{1}{2}\frac{\hat{\eta}}{\xi\gamma}\right)\right) && (12) \\ \mathbb{V}_t\left[dy_t\right] &= \xi\left(V_t - s_t\right) && (13) \end{aligned}$$

Using equations (9), (10), (12) and (13), we can derive the following equation:

$$\mathbb{E}_t\left[dy_t\right] = -\frac{1}{\tau}\overline{s} - \frac{1 + \alpha\sqrt{\tau}\xi}{\tau\xi}\mathbb{V}_t\left[dy_t\right] - \frac{1}{\tau}\left(s_t - \overline{s}\right) \qquad (14)$$

Now run a regression of $\mathbb{E}_t\left[dy_t\right]$ on $\mathbb{V}_t\left[dy_t\right]$

$$\mathbb{E}_t\left[dy_t\right] = A \times \mathbb{V}_t\left[dy_t\right] + B + \varepsilon_t \qquad (15)$$

and obtain OLS estimates for $\hat{A}$ and $\hat{B}$. Comparing (14) to (15), we identify

$$\begin{aligned} \hat{A} &= -\frac{1 + \alpha\sqrt{\tau}\xi}{\tau\xi} \\ \hat{B} &= -\frac{1}{\tau}\overline{s} \\ \hat{\varepsilon}_t &= -\frac{1}{\tau}\left(s_t - \overline{s}\right) \end{aligned}$$

We set by hand

$$\begin{aligned} \alpha &= -1.645 \\ \sqrt{\tau} &= 1 \end{aligned}$$

to have a V@R at a 1-year horizon ($\tau = 1$) at around the 5% level ($\alpha = -1.645$). Then we have that

$$\begin{aligned}
\overline{s} &= -\hat{B}\tau \\
\xi &= -\frac{1}{\hat{A}\tau + \alpha\sqrt{\tau}} \\
\rho_s &= -\log\left(\frac{Cov\left(\hat{\varepsilon}_{t+1}, \hat{\varepsilon}_t\right)}{Var\left(\hat{\varepsilon}_t\right)}\right) \\
\sigma_s &= \frac{Std\left(\hat{\varepsilon}_t\right)}{\sqrt{\Delta t}}
\end{aligned}$$

where we have adjusted σ_s by $1/\sqrt{\Delta t}$ so that σ_s represents an annual volatility (Δt is the frequency of the data used in regression (15); for example, if the data is quarterly, we have $\Delta t = 1/4$).

Table (3) shows the calibrated parameter values from estimating the conditional mean and conditional volatility obtained with the panel data regressions.

We calibrate κ by choosing $\beta = 0.01$, estimating the model's Phillips curve country-by-country and averaging the values.

5 Welfare

We investigate the welfare gains from responding to financial conditions. To do so, we consider two Taylor rules, a classic and an augmented rule. In the first case, central banks are constrained to respond only to changes in output gaps and inflation. However, under the augmented rule, central banks can also respond to changes in financial conditions. In all cases, we allow central banks to pick weights in the Taylor rule to minimize the net present value of their loss function. In the end, we compare welfare outcomes from both rules following shocks to financial conditions.

In all cases, it seems optimal to respond to financial conditions. In other words, central banks chose to increase interest rates when financial conditions loosen. This is captured by the negative coefficients on financial conditions in the augmented Taylor rules, reported in Table 4. Results are robust to the choice of country and time samples, despite some heterogeneity. Coefficients on output and inflation are rather stable across rules and samples; as expected, rates are increased to counter inflation rising above target and growth above potential.

< Insert Table 4 here >

Coefficients on financial conditions may appear small, however, this does not mean central banks put little emphasis on financial conditions in their optimal responses. A shock to financial conditions also affects the output gap and inflation. Thus, by optimally chosing high weights on these variables, the central bank is in practice already responding to financial conditions.[7]

The fact that optimal policy responds at all to financial conditions, over and above its decisive reaction to output and inflation, is a notable feature of our model. Financial conditions contain information about the variance of output, which also enters the central bank's loss function. Importantly, the strong relationship

[7] Our model simply cannot quantify how much bigger these weights become relative to a benchmark economy with other shocks requiring a more tepid response to output gap and inflation deviations.

between the mean and variance of output, documented earlier, holds conditional on financial variables. Thus, if the central bank does not respond to, or is oblivious to, the financial conditions shock, it will not be able to anticipate changes to the variance of output. From the standpoint of the data, this is equivalent to saying that the unconditional relationship between the mean and variance of output gaps is non-linear; thus knowing the realization of one, at any given time, is not especially informative for the level of the other.

Consequently, welfare gains from responding to financial conditions are significant. In most cases, they are of the order of 10 percent, though are smaller in consumption equivalent units. Again, results are robust to the choice of sample, as shown in Table 5. Interestingly, welfare gains have decreased in the more recent sample. This is consistent with the lower tradeoffs between mean and variance of output gaps in more recent years, as documented and discussed in section 2. On the whole, welfare gains are approximately equal among advanced and emerging market economies.

< Insert Figure Table 5 here >

In reality, welfare differences between the classic and augmented Taylor rules are likely to be greater than suggested by this stylized model. For instance, in the data, the effect of a financial conditions shock on output is not immediate. Thus, responding to changes in financial conditions allows monetary policy to be more forward looking, as opposed to waiting for the output gap to respond. Also, a richer model with capital and investment could produce a drop in inflation following a compression of financial conditions; lower credit spreads would decrease the cost of capital and thus marginal costs of production, as in Gourio et al. (2017). In such a model, the classic Taylor rule might respond by cutting rates, because of lower inflation. However, an augmented rule taking spreads into account might instead recommend a lower cut. In our model, a financial conditions shock is similar to a demand shock in that it increases output and inflation therfore requiring an unequivocal hike without a stark tradeoff between output and inflation stabilization.[8]

Finally, the model could be extended to have a stronger inter-temporal tradeoff. Currently, a shock to financial conditions that compresses spreads (looser conditions) is met with higher rates to stabilize output and inflation, and the cost of somewhat higher variance of output. But a rich literature has documented mean reversion in financial conditions (see, for instance, López-Salido et al. (2017) and others discussed in the literature survey below—section 6). In that case, a rise in output, and a drop in variance, stemming from looser financial conditions would require an additional hike in interest rates, not just to dampen current activity, but to guard against a reversal in future activity.

6 Related Literature

This paper highlights the role of financial conditions in the relation between policy interest rates and the mean and variance of output gaps. Two steps are relevant: one linking policy rates to financial conditions, and the other linking financial conditions to output. Both steps are underpinned by a rich literature—empirical and theoretical—emphasizing financial frictions.

The first step rests on the risk-taking capacity of intermediaries. As this capacity evolves over time, in part due to changes in policy rates, so does the compensation intermediaries require to hold risk—the risk

[8]The similarity is all the more striking when digging into the model; as explained earlier, the financial conditions shock is modelled as a shock to the preferences of bankers which translates to households whose marginal utilities are closely related through trade.

premium. In segmented markets, this determines credit spreads as well as asset prices. Risk-taking capacity is typically limited by intermediaries balance sheets—their size, value of collateral, net worth, value at risk, or other such measures—in what is referred to as a financial friction. Examples are Adrian et al. (2010), and Adrian et al. (2014), in which broker-dealers price assets according to their balance sheets; if these are strong and the marginal value of wealth is low, expected returns on risky assets can also afford to be low. This papers model shares these mechanisms, insofar as policy rates affect the value-at-risk constraint of intermediaries.

A growing empirical literature documents the link between policy rates and financial conditions. Gilchrist and Zakrajšek (2012) provide a hint: shocks to profits of broker dealers—presumably coming in part from monetary policy—affect their credit default swap (CDS) rates, and the excess bond premia (a measure of credit spreads). Gertler and Karadi (2015), as well as Boyarchenko et al. (2016), examine the issue directly, while carefully identifying monetary policy shocks using high frequency analysis. In both cases, higher policy rates increase credit spreads.

The second step relevant to this paper—the link between financial conditions and output—has also been explored. Examples of empirical papers are Philippon (2009), Gilchrist and Zakrajšek (2012), Krishnamurthy and Muir (2016), and López-Salido et al. (2017). A common result emerges despite the differences in methods and samples: higher credit spreads—or tighter financial conditions—coincide with a contraction in output. The forecasting power of spreads varies somewhat among papers. Krishnamurthy and Muir document that prior to financial crises spreads remain particularly tight despite strong credit growth. Similarly, López-Salido et al. (2017) suggest that spreads are mean reverting so that a period of tight spreads forecasts one of wider spreads and lower growth.

Some empirical papers extend the analysis from the mean to the variance of output. Notably, Adrian et al. (2016), emphasizes that deteriorating financial conditions (wider spreads) are associated with an increase in the conditional volatility of GDP growth, and a decrease in its conditional mean. Together, these results make for a left-skewed conditional distribution of GDP growth, and emphasize the importance of financial conditions to forecast downside risks—or vulnerabilities—to growth. These same findings emerge from this paper's model, and are corroborated by its empirical investigation.

The relationship between financial conditions and output finds root in an older theoretical literature. Seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke et al. (1999) emphasize the role of the financial sector in amplifying effects of monetary policy shocks. These models exhibit borrowing and lending (between households and firms; later models introduce heterogeneous households). But equity issuance is constrained due to agency costs, and debt issuance is subject to frictions that are either exogenous, or depend endogenously on collateral values or levels of debt. As a result, monetary policy shocks have a larger effect on output than in representative agent models that abstract from the financial sector.

The more recent theoretical literature has evolved in two ways: one explaining, and the other expanding upon, basic dynamics. A first strand has rationalized why agents would take on excessive debt or leverage if doing so undermines economic stability. The answer lies in overlooking the macro implications of individual actions—a phenomenon called externalities. See, for instance, Stein (2012), Dávila and Korinek (2016), Farhi and Werning (2016), as well as Korinek and Simsek (2016).

A second strand has emphasized non-linear effects of financial frictions on intermediaries balance sheets, as in He and Krishnamurthy (2014), and Brunnermeier and Sannikov (2014). Doing so allows for stronger amplification effects—closer to those observed in the data. The new models study global dynamics as they are not linearized around a steady state. And as constraints on intermediaries balance sheets bind only occasionally and following negative as opposed to positive shocks, the models allow for more realistic dynamics: asymmetric amplification of shocks, and periods of crisis following others of more normal growth. He and Krishnamurthy (2014) show that such dynamics help explain asset prices. Brunnermeier and Sannikov (2014) also has important implications for regulation; the model economy can remain in a crisis state for prolonged periods, if initial capital cushions are too slim. This papers underlying model shares these same characteristics, as needed to explicitly study output volatility.

While this newer class of models does not dwell on monetary policy—unlike this paper—somewhat older papers do. Curdia and Woodford (2010), spurred by the global financial crisis, considers whether monetary policy should respond to financial conditions—credit spreads in particular—following the recommendations of McCulley and Toloui (2008), and Taylor et al. (2008) (while Christiano et al. (2010) favors a response to aggregate credit growth). Gambacorta and Signoretti (2014) builds on Curdia and Woodford (2010) by adding further frictions, and Gertler and Karadi (2011) advances a model where central banks can complement the private sector's intermediation function, thereby carving out a role for asset purchases.

In these models, though, welfare gains are not clear-cut from responding to financial conditions (especially in Curdia and Woodford (2010)). Doing so following financial disturbances—a shock to credit spreads—is optimal; but not in response to other shocks. A productivity shock, for instance, will increase output and encourage lending, thereby raising credit spreads. But a cut in interest rates to stabilize spreads would lead to inefficient inflation. In the end, welfare gains from responding to financial conditions are shock dependent. This paper instead remains focussed on the effects of a single shock to financial conditions.

7 Conclusion

Economists and policy makers have debated for some time to what degree financial conditions should or should not enter monetary policy rules. Some have argued that monetary policy should take financial conditions only into account to the extent that they change the forecast of inflation or output (Bernanke et al. (1999) Bernanke and Gertler (2001), Svensson (2017)). We extend that logic by arguing that monetary policy makers should not just care about the conditional mean forecasts of inflation and output, but also about the downside risks of those quantities. A monetary policy maker that minimizes the expected present discounted value of squared output losses and squared inflation deviations naturally cares about the conditional variance of output and inflation, not only the conditional mean.

Empirically, we document that both the conditional mean and the conditional volatility of the output gap are significantly related to financial conditions. This finding builds on a long literature that has studied the forecasting power of term spreads, credit spreads, and market volatility for downside risks to output. Importantly, we find that the conditional means and conditional voliatilites of the output gap are negatively correlated: when financial conditions deteriorate, the conditional mean declines, and the conditional volatility increases. This negative correlation between the conditional mean and the conditional volatility of GDP as a function of financial conditions gives rise to a strongly negatively unconditional distribution of GDP. These findings are broadly present in advanced economies and emerging markets.

We calibrate the empirical relationship between financial conditions and the time varying moments of the output gap distribution to a reduced from New Keynsian model that features financial vulnerability. The micro foundation of that model is provided by Adrian and Duarte (2016). When we calculate the intertemporally optimal monetary policy rule, we find that monetary policy should condition on financial vulnerability in addition to the output gap and inflation. Welfare gains from doing so are significant. Intuitively, monetary policy that takes financial conditions into account mitigates GDP skewness.

Our results are very robust across time periods (pre and post financial crisis), types of countries. Welfare gains from responding to financial conditions are approximately equal across a sample of advanced economies and one of emerging market economies. The results suggest some reformulation of the policy approach to the relationship between financial stabiltiy and monetary policy. Clearly, monetary policy should take financial conditions into account, even if macroprudential policy is the appropriate tool to lean against the buildup of major financial imbalances. In fact, Peek et al. (2016) document that references to financial conditions are more and more common in monetary policy statements.

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Figures and Tables

Figures

Figure 1: **Estimated Mean and Volatility from Panel Estimation**

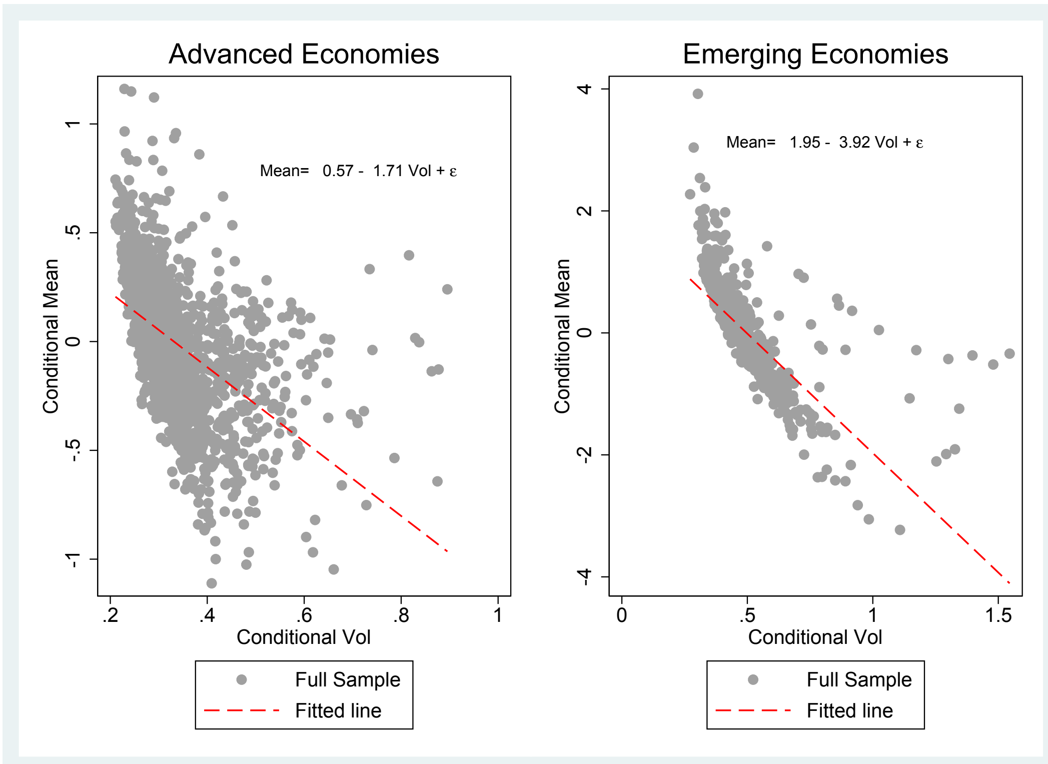
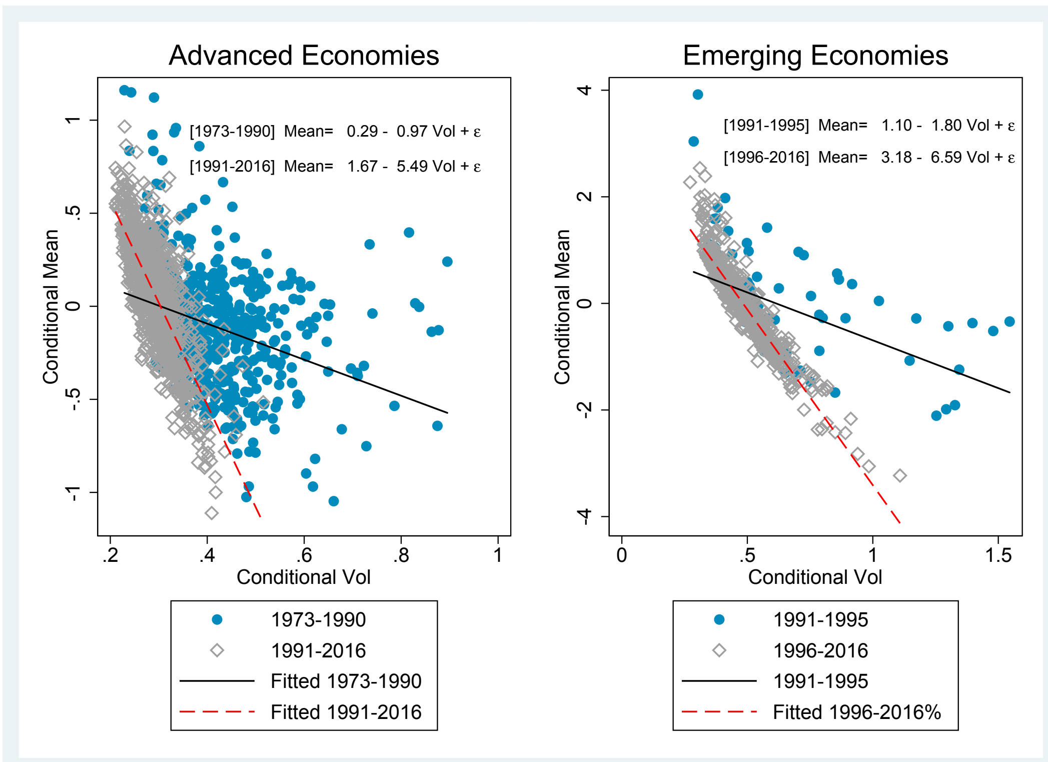


Figure 2: **Estimated Mean and Volatility from Panel Estimation on Sub-samples**



Tables

Table 1: **Country Coverage**

Advanced Economis (AE)	**Emerging Economies (EME)**
Australia (AUS)	Brazil (BRA)
Canada (CAN)	Chile (CHL)
France (FRA)	China (CHN)
Germany (DEU)	India (IND)
Italy (ITA)	Indonesia (IDN)
Japan (JPN)	Korea (KOR)
Sweden (SWE)	Mexico (MEX)
Sweden (SWE)	Russia (RUS)
United Kingdom (GBR)	South Africa (ZAF)
United States (USA)	Turkey (TUR)

Table 2: **Output Gap Conditional Mean and Volatility. Panel Estimation.**

	AE		**EME**	
	(1)	(2)	(1)	(2)
	Δy_t	$\ln \epsilon^2$	Δy_t	$\ln \epsilon^2$
s_{t-1}	-0.57***	0.65***	-1.28***	0.93**
y_{t-1}	-0.18***	0.05	-0.30***	0.14***
π_{t-1}	0.01	0.09***	-0.00	0.01
Constant	0.64***	-3.32***	0.41***	-1.84***
Observations	1,602	1,602	918	918
R-squared	0.13	0.04	0.18	0.03
Number of cty	10	10	10	10

*** p<0.01, ** p<0.05, * p<0.1

Table 3: **Calibration Values.** From Panel Estimates

	AE(1973-2016)	AE(1973-1990)	AE(1991-2016)	EME(1996-2016)
s	-0.57	-0.29	-1.67	-3.18
ξ	0.30	0.38	0.14	0.12
ρ_s	0.14	0.21	0.12	0.15
σ_s	0.27	0.29	0.19	0.25
κ	0.18	-0.04	0.20	-0.36

Table 4: **Monetary Policy Rule Coefficients**

	Optimal rule			Taylor rule		
	y_t	π_t	x_t	y_t	π_t	x_t
AE 1973-2016	-3.54	3.13	-0.14	-3.46	3.06	0.00
AE 1973-1990	-2.86	2.59	-0.64	-2.65	2.49	0.00
AE 1991-2016	-3.86	3.38	-0.01	-3.85	3.38	0.00
EME 1990-2016	-3.82	3.36	-0.03	-3.80	3.34	0.00
EME 1990-1995	-3.57	3.16	-0.50	-3.21	3.04	0.00
EME 1995-2016	-3.87	3.39	-0.01	-3.86	3.38	0.00

Table 5: **Welfare under Optimal and Classic Taylor Monetary Policy Rules**

	Optimal rule	Taylor rule	Difference in Welfare	Welfare Gain (in %)
AE 1973-2016	3.03	2.72	0.32	0.12
AE 1973-1990	3.53	2.58	0.95	0.37
AE 1991-2016	2.34	2.22	0.12	0.06
EME 1990-2016	5.64	5.21	0.43	0.08
EME 1990-1995	4.53	2.23	2.30	1.03
EME 1995-2016	5.15	4.73	0.42	0.09

A Appendix: Optimal Monetary Policy

The central bank solves

$$L\left(y_t,\pi_t,s_t\right)=\min_{\{i_s\}_{s=t}^{\infty}}\mathbb{E}_t\int_t^{\infty}e^{-s\beta}\left(y_s^2+\pi_s^2\right)ds$$

subject to equations (3)-(8). Using equations (25) and (26) from Appendix **??**, we can write the central bank's problem as

$$\begin{aligned}
L\left(y_t,\pi_t,s_t\right) &= \min_{\{V_s\}_{s=t}^{\infty}}\mathbb{E}_t\int_t^{\infty}e^{-s\beta}\left(y_s^2+\pi_s^2\right)ds \\
&\quad s.t. \\
dy_t &= -\xi\left(\frac{1+\alpha\xi\sqrt{\tau}}{\xi\tau}V_t-\frac{\alpha}{\sqrt{\tau}}s_t\right)dt+\xi\left(V_t-s_t\right)dZ_t \\
d\pi_t &= \left(\beta\pi_t-\kappa y_t\right)dt \\
ds_t &= -\kappa\left(s_t-\overline{s}\right)+\sigma_s dZ_t
\end{aligned}$$

The Hamilton-Jacobi-Bellman (HJB) equation is

$$\begin{aligned}
0 &= \min_V\left\{\xi\left(\sigma_s L_{ys}-ML_y\right)V+\frac{\xi^2}{2}\left(V-s\right)^2L_{yy}\right\} \\
&\quad +y^2+\pi^2-\beta L+\frac{\xi\alpha}{\sqrt{\tau}}L_y s-\kappa\left(s-\overline{s}\right)L_s+\left(\beta\pi-\kappa y\right)L_\pi+\frac{\sigma_s^2}{2}L_{ss}-\sigma_s\xi L_{ys}s
\end{aligned}$$

The FOC is

$$V=s+\frac{M}{\xi}\frac{L_y}{L_{yy}}-\frac{\sigma_s}{\xi}\frac{L_{ys}}{L_{yy}}$$

Plugging the optimal V into the HJB we get

$$\begin{aligned}
0 &= 3M\sigma_s\frac{L_{ys}L_y}{L_{yy}}-\frac{\sigma_s^2}{2}\frac{L_{ys}^2}{L_{yy}}-\frac{M^2}{2}\frac{L_y^2}{L_{yy}}+y^2+\pi^2-\beta L \\
&\quad +\xi\left(\frac{\alpha}{\sqrt{\tau}}-M\right)L_y s-\kappa\left(s-\overline{s}\right)L_s+\left(\beta\pi-\kappa y\right)L_\pi+\frac{\sigma_s^2}{2}L_{ss}
\end{aligned}$$

we look for a solution of the form

$$L\left(y,\pi,s\right)=c_0+c_1y+c_2y^2+c_3s+c_4s^2+c_5ys+c_6\pi+c_7\pi^2+c_8y\pi+c_9\pi s$$

Plugging into the HJB, using

$$\begin{aligned}
L_y &= c_1+2c_2y+c_5s+c_8\pi \\
L_{yy} &= 2c_2 \\
L_\pi &= c_6+2c_7\pi+c_8y+c_9s \\
L_s &= c_3+2c_4s+c_5y+c_9\pi \\
L_{ss} &= 2c_4 \\
L_{ys} &= c_5
\end{aligned}$$

and setting the coefficients in front of combinations of the state variables to zero, we get the following system of equations in $c_0, ..., c_9$:

$$\begin{aligned}
[y^2] &: \; 0 = \left(1 - \beta c_2 - \kappa c_8 - c_2 M^2\right) \\
[y\pi] &: \; 0 = \left(-c_8 M^2 - 2\kappa c_7\right) \\
[ys] &: \; 0 = \left(2c_2\xi\left(\frac{\alpha}{\sqrt{\tau}} - M\right) - \kappa c_9 - \beta c_5 - M^2 c_5\right) \\
[y] &: \; 0 = \left(3\sigma_s c_5 M - c_1 M^2 - \beta c_1 - \kappa c_6\right) \\
[\pi^2] &: \; 0 = \left(\beta c_7 - \frac{1}{4}\frac{M^2}{c_2}c_8^2 + 1\right) \\
[\pi s] &: \; 0 = \left(c_8\xi\left(\frac{\alpha}{\sqrt{\tau}} - M\right) - \frac{1}{2}\frac{M^2}{c_2}c_5 c_8\right) \\
[\pi] &: \; 0 = \left(\frac{3}{2}M\frac{\sigma_s}{c_2}c_5 c_8 - \frac{1}{2}M^2\frac{c_1}{c_2}c_8\right) \\
[s^2] &: \; 0 = \left(c_5\xi\left(\frac{\alpha}{\sqrt{\tau}} - M\right) - \beta c_4 - \frac{1}{4}\frac{M^2}{c_2}c_5^2\right) \\
[s] &: \; 0 = \left(c_1\xi\left(\frac{\alpha}{\sqrt{\tau}} - M\right) - \beta c_3 + \frac{3}{2}M\frac{\sigma_s}{c_2}c_5^2 - \frac{1}{2}M^2\frac{c_1}{c_2}c_5\right) \\
[const] &: \; 0 = \left(\sigma_s^2 c_4 - \beta c_0 - \frac{1}{4}\frac{\sigma_s^2}{c_2}c_5^2 - \frac{1}{4}M^2\frac{c_1^2}{c_2} + \frac{3}{2}M\sigma_s\frac{c_1}{c_2}c_5\right)
\end{aligned}$$

with solution

$$\begin{aligned}
c_8 &= \begin{cases} \frac{2\kappa\left(\beta + M^2\right)}{M^2\beta} & , \text{ if } M^2 = \beta \\ \frac{1}{M^2-\beta}\left(-\frac{\beta}{\kappa} \pm \kappa\sqrt{\beta^2 + 4\kappa^2\left(M^2 - \left(\frac{\beta}{M}\right)^2\right)}\right) & , \text{ if } M^2 \neq \beta \end{cases} \\
c_1 &= \frac{3\sigma_s}{M}\frac{2\xi}{M^2}\left(\frac{\alpha}{\sqrt{\tau}} - M\right)\left(\frac{1}{\beta + M^2} - \frac{\kappa}{\beta + M^2}c_8\right) \\
c_2 &= -\frac{1}{M^2 + \beta}\left(\kappa c_8 - 1\right) \\
c_3 &= -\frac{6\sigma_s\xi^2\left(\alpha - M\sqrt{\tau}\right)^2}{M^3\beta\tau\left(M^2 + \beta\right)}\left(\kappa c_8 - 1\right) \\
c_4 &= -\frac{\xi^2\left(\alpha - M\sqrt{\tau}\right)^2}{M^2\beta\tau\left(M^2 + \beta\right)}\left(\kappa c_8 - 1\right) \\
c_5 &= -\frac{2\xi\left(\alpha - M\sqrt{\tau}\right)}{M^2\left(M^2 + \beta\right)\sqrt{\tau}}\left(\kappa c_8 - 1\right) \\
c_6 &= \frac{6\beta\sigma_s\xi\left(\alpha - M\sqrt{\tau}\right)}{M^3\kappa\left(M^2 + \beta\right)\sqrt{\tau}}\left(\kappa c_8 - 1\right) \\
c_7 &= -\frac{M^2}{2\kappa}c_8 \\
c_9 &= \frac{2\xi\beta\left(\alpha - M\sqrt{\tau}\right)}{M^2\kappa\left(M^2 + \beta\right)\sqrt{\tau}}\left(\kappa c_8 - 1\right) \\
c_0 &= \frac{1}{\beta}\left(\sigma_s^2 c_4 - \frac{\sigma_s^2}{4}\frac{c_5^2}{c_2} - \frac{M^2}{4}\frac{c_1^2}{c_2} + \frac{3M\sigma_s}{2}\frac{c_1 c_5}{c_2}\right)
\end{aligned}$$

B Appendix: Derivation of Solution

Plugging equation (7) into equation (3) we can see that

$$\begin{aligned} \mathbb{E}_t\left[dy_t\right] &= \frac{1}{\gamma}\left(i_t-\pi_t-r\right) && (16)\\ \mathbb{V}_t\left[dy_t\right] &= \xi\left(V_t-s_t\right) && (17) \end{aligned}$$

so that equation (6) can be written as

$$V_t=-\frac{1}{\gamma}\left(i_t-\pi_t-r\right)\tau-\alpha\xi\left(V_t-s_t\right)\sqrt{\tau} \tag{18}$$

Solving equation (18) for i_t gives

$$i_t=\pi_t+r-\frac{\gamma\left(\alpha\xi\sqrt{\tau}+1\right)}{\tau}V_t+\frac{\gamma\alpha\xi}{\sqrt{\tau}}s_t \tag{19}$$

Using (19) in equation (3) gives

$$dy_t=-\xi\left(\frac{1+\alpha\xi\sqrt{\tau}}{\xi\tau}V_t-\frac{\alpha}{\sqrt{\tau}}s_t\right)dt+\xi\left(V_t-s_t\right)dZ_t \tag{20}$$

We can again identify

$$\begin{aligned} \mathbb{E}_t\left[dy_t\right] &= -\xi\left(\frac{1+\alpha\xi\sqrt{\tau}}{\xi\tau}V_t-\frac{\alpha}{\sqrt{\tau}}s_t\right) && (21)\\ \mathbb{V}_t\left[dy_t\right] &= \xi\left(V_t-s_t\right) && (22) \end{aligned}$$

Eliminating V_t from (21) and (22), we get

$$\mathbb{E}_t\left[dy_t\right]=M\times\mathbb{V}_t\left[dy_t\right]-\frac{1}{\tau}s_t \tag{23}$$

where I have defined

$$M\equiv-\frac{1+\alpha\xi\sqrt{\tau}}{\xi\tau}$$

Plugging (5) into (18) and solving for V_t gives

$$V_t=-\frac{1}{\psi_v-M\gamma\xi}\left(\psi_0-r+\left(\psi_\pi-1\right)\pi_t+\psi_y y_t\right)+\frac{\alpha\gamma\xi}{\sqrt{\tau}\left(\psi_v-M\gamma\xi\right)}s_t \tag{24}$$

Finally, plugging (24) into (20) and re-arranging, we get

$$\begin{aligned} dy_t &= \frac{\xi M}{\psi_v-\xi M\gamma}\left(r-\psi_0+\left(1-\psi_\pi\right)\pi_t-\psi_y y_t+\frac{\alpha}{\sqrt{\tau}}\frac{\psi_v}{M}s_t\right)dt && (25)\\ &\quad+\frac{\xi}{\psi_v-\xi M\gamma}\left(r-\psi_0+\left(1-\psi_\pi\right)\pi_t-\psi_y y_t+\left(M\gamma\xi+\frac{\alpha}{\sqrt{\tau}}\frac{\gamma}{\xi}-\psi_v\right)s_t\right)dZ_t && (26)\\ d\pi_t &= \left(\beta\pi_t-\kappa y_t\right)dt \end{aligned}$$

Equations (25) and (26) determine y_t and π_t as a function of the exogenous variables s_t and Z_t. Given y_t, π_t and the exogenous variables s_t and Z_t, equation (24) determines V_t. Once we have found V_t equation (5) determines i_t.