This paper presents preliminary findings and is being distributed to economists and other interested readers solely to stimulate discussion and elicit comments. The views expressed in this paper are those of the author and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System. Any errors or omissions are the responsibility of the author.
Firms’ Precautionary Savings and Employment during a Credit Crisis
Davide Melcangi
Federal Reserve Bank of New York Staff Reports, no. 904
November 2019
JEL classification: E44, G01, G32, L25

Abstract

Can the macroeconomic effects of credit supply shocks be large even when a small share of firms are credit-constrained? I use U.K. firm-level accounting data to discipline a heterogeneous-firm model in which the interaction between real and financial frictions induces precautionary cash holdings. In the data, firms increased their cash ratios during the last recession, and cash-intensive firms displayed higher employment growth. A tightening of firms’ credit conditions generates the same dynamics in the model. Unconstrained firms pre-emptively respond to credit supply shocks, and this precautionary channel crucially matters for the aggregate dynamics and the model fit with microeconomic data.

Key words: financial frictions, precautionary savings, employment, heterogeneous firms

Melcangi: Federal Reserve Bank of New York (email: davide.melcangi@ny.frb.org). This paper was part of the author’s doctoral dissertation at University College London. The author thanks Vincent Sterk and Marco Bassetto for invaluable advice. For helpful comments, he also thanks Kasper Kragh-Sorensen, Morten Ravn, Wei Cui, Mariacristina De Nardi, Wendy Carlin, Michela Altieri, and many seminar and conference participants. The views expressed in this paper are those of the author and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

To view the author’s disclosure statements, visit https://www.newyorkfed.org/research/staff_reports/sr904.html.
1 Introduction

The Great Recession has renewed interest in financial and labour markets, and the potential interconnections that may link them. One of the proposed narratives suggests that firm credit tightening is at the root of the increase in unemployment. However, the extent to which financial frictions affect firm’s decision-making, and hiring decisions in particular, is controversial. At the aggregate level, firms have large savings and generate internal funds substantially in excess of what they need to finance operations, as documented by Shourideh et al. (2016) for the US and the UK. Moreover, empirical proxies suggest that only a moderate fraction of firms is credit-constrained. These observations might lead to conclude that firm-level credit constraints play a limited role for the cyclical behaviour of aggregate employment.

In this paper, I show that the macroeconomic effects of a credit tightening can be large even in economies in which the share of credit-constrained firms is small. A persistent tightening of credit constraints affects not only the decisions of currently constrained firms, but also those of firms that are currently not credit-constrained but which face some probability of becoming constrained in the future. In the wake of a shock that tightens the credit constraints, these firms may cut investment in capital and hiring for precautionary reasons, as this allows them to build up larger cash holdings. More cash alleviates the impact of a credit tightening and reduces the probability of hitting the constraint.

The first contribution of this paper is to build a quantitative model to investigate this precautionary mechanism. I develop a partial equilibrium model with shocks to firms’ idiosyncratic productivity and aggregate credit uncertainty, where precautionary savings in cash arise endogenously from the interaction between real and financial frictions, and affect the transmission mechanism of credit supply shocks onto labour demand. Firms have to finance their wage bill in advance of production and can do so through accumulated cash holdings and an intraperiod loan. Such loans are collateralised with capital and subject to aggregate shocks. The structure of the model allows me to study simultaneously firms’ employment and portfolio decisions. Firms face a tradeoff; on the one hand, more cash reduces the probability of being credit-constrained. On the other hand, saving in cash may require cutting back on capital investment and hiring, which in turn reduces production. Firms have incentives to hoard cash because they face non-smooth labour and capital adjustment costs, and it is costly to issue equity.

The second contribution of this paper is to use balance sheet data from UK firms to motivate and discipline the model. I use the FAME (Financial Analysis Made Easy) dataset, a large panel of UK firms between 2004 and 2013. This is a much broader sample

---

1Campello et al. (2010) report that 20% of US CFOs identified their firms as very affected by financial constraints. Angelini and Generale (2008) use Italian matched data and estimate a share of 5%.
than other alternatives often used in the literature, as it mainly includes private firms, whereas US Compustat, for example, is limited to publicly listed firms. This feature makes FAME particularly suitable for the study of financial frictions, because private firms are often small and young. They may rely more heavily on external finance (Zetlin-Jones and Shourideh (2017)), and have a more limited access to credit (Spaliara (2009)). Finally, the data contain information on both the employment and the asset structure of the firms, used to directly discipline the model.

In the FAME data, I document two stylised facts. First, the average cash to assets ratio increases when aggregate employment falls. With a simple back of the envelope calculation, I show that the increase in aggregate cash between 2008 and 2009 would have been more than enough to keep the net job creation at pre-crisis levels, if used to hire workers at the average wage. Even if only a share of this excess cash was allocated to the wage bill, the 2009 increase in unemployment rate would have been a third of the one observed in reality. Moreover, the increase in cash ratios in 2009 is common to firms with different fundamentals. Second, I show that cash-intensive firms cut their workforces by less when aggregate employment falls.

The model is calibrated to the UK economy using both aggregate and firm-level moments. Most importantly, the real frictions on capital, labour and dividend payouts are disciplined using FAME firm-level data. Among others, the model matches the cross-sectional distribution of cash ratios. The calibrated model also performs well in approximating additional microeconomic features of the sample, not explicitly targeted. For instance, it correctly predicts that small and more labour intensive firms will hold relatively more cash.

I evaluate the model’s ability to explain macroeconomic and firm-level outcomes during the aftermath of the financial crisis, simulating an exogenous tightening of the credit conditions. I show that the precautionary channel allows the model to explain the joint evolution of three key variables: (i) the decline in aggregate employment, (ii) the increase in the average cash-to-assets ratio and (iii) the initial increase and subsequent decline in the cross-sectional correlation between the firm-level cash-to-asset ratio and employment growth rate. The predicted decline in aggregate employment is as large as in the data, despite the fact that in the model the share of credit-constrained firms never exceeds 20%. In fact, I show that unconstrained firms that act for precautionary reasons account for nearly half of the fall in aggregate employment upon impact, and are crucial for the subsequent decline.

2The finding is robust to the median and the aggregate cash ratio.
3The model displays aggregate uncertainty, which means that firms know the possibility of a credit shock and attach a conditional probability to this event. This is in sharp contrast with other papers evaluating the effects of an unexpected aggregate credit supply shock, as Buera et al. (2015).
Each of the real and financial frictions present in my model can be found in earlier literature. The financial friction is closely related to Jermann and Quadrini (2012). Non-smooth adjustment costs in labour and capital can be found, for example, in Bloom (2009). Costs that limit the speed at which firms can raise additional equity are often implemented in the corporate finance literature.\footnote{Falato et al. (2013) and Hennessy and Whited (2007) are some examples. Moreover, the cost is also used in the macroeconomic literature, as Jermann and Quadrini (2012).} I show that, for the precautionary channel to arise in full, these frictions need to be included simultaneously in the model. Indeed, they all play a complementary role: they make it costly for firms to quickly circumvent the effects of a binding credit constraint by either selling capital, firing workers or raising additional equity. I show quantitatively that removing one of the frictions substantially weakens the precautionary channel. In contrast with the data, these versions of the model predict small and short-lived aggregate employment dynamics. Without all real frictions, the model also predicts a rise in aggregate capital after a credit supply shock and an unrealistically large increase in cash ratios. All alternative models also fail to generate many microeconomic features of the data, as the right tail of the cash ratio distribution, or the fact that smaller and more labour-intensive firms are more cash-intensive.

This paper is organized as follows. After briefly reviewing the literature, in section 2 I document empirical stylised facts on cash ratio and employment, which motivate the model, introduced thereafter. Section 3.5 provides intuition for the key model mechanisms. The quantitative analysis starts with the description of the calibration strategy and the data used. I then turn to the steady state performance of the model, before investigating the aggregate effects of a credit tightening and its microeconomic drivers. In section 4.5 I use the model to shed light on the identification of financial constraints. My findings suggest that simple proxies, as those typically used in the empirical literature, do not identify well financial constraints even when we use a structural model calibrated to the data. Finally, I show in section 5 how versions of the benchmark model without some, or all, real frictions would fail to match key empirical predictions.

Related literature

This paper fits into the vast literature that incorporates firm-level financial frictions into macroeconomics models. Among seminal and influential contributions, Bernanke et al. (1999) propose a “financial accelerator” mechanism that amplifies and propagates shocks to the macroeconomy, while Cooley and Quadrini (2001) show that financial market imperfections, in presence of persistent shocks, affect the dynamics of firms. A more recent strand of literature has focused on the direct effect of shocks to these frictions on the real economy. Examples include Khan and Thomas (2013), who mainly focus on

The main focus of my paper, instead, is to study the effect of firm credit tightening on aggregate employment.\(^5\) In corporate finance, most of the literature focuses on the role played by financial constraints in distorting investment decisions, as surveyed by Strebulaev et al. (2012).\(^6\) In particular, labour is typically hired on the spot market and firms can always implement the static optimum: this implies that financial frictions have no direct and independent effect on employment decisions. Petrosky-Nadeau (2014) extends the baseline search-and-matching model of equilibrium unemployment with financial constraints to vacancy posting and shows that these frictions can generate persistence in the dynamics of labour market tightness. Jermann and Quadrini (2012) develop a real business cycle model with debt and equity financing, in which a representative firm finances working capital through a collateralised intra-period loan. In their setting, financial frictions show up as a labour wedge. Representative firm models as the latter, however, are not suitable to study economies with a small share of constrained firms and a large amount of savings, as typically observed in the data. Buera et al. (2015) use a model with heterogeneous entrepreneurial productivity and search frictions to argue that a credit crunch can translate onto a protracted increase in unemployment. While tracing a link between credit constraints and employment, these papers do not focus on firms’ precautionary savings, which is instead the central channel investigated here.

Finally, my work contributes to the literature studying differences in liquidity across firms’ assets. While firms’ cash holdings, and their determinants, have been extensively studied in corporate finance,\(^7\) they are often ignored in macroeconomic models.\(^8\) My paper brings together non-smooth capital and labour adjustment costs, as in Bloom (2009) and Bayer (2006), and portfolio decisions in face of financial frictions and aggregate shocks, to study the interaction between precautionary cash holdings and labour demand. Bacchetta et al. (2019) also combine the analysis of firms’ cash holdings and employment, in order to distinguish the effects of credit and liquidity shocks. Compared to their work, I focus on the role played by firms’ precautionary behaviour in amplifying the macroeconomic effects of credit tightening. The interaction between occasionally binding constraints and real frictions allows my model to generate a large and persistent drop

---

5Examples of empirical papers of this sort are Chodorow-Reich (2013) and Duygan-Bump et al. (2015).
6An exception is recent work by Michaels et al. (2018), who integrate costly external finance with labour and capital adjustment costs, to study the negative correlation between wages and leverage.
8Some recent examples to the contrary include Cui and Radde (2016) and Kiyotaki and Moore (2012). These works, however, do not explicitly focus on employment fluctuations.
in aggregate employment following a credit supply shock, even if the share of credit-constrained firms is small.

## 2 Stylised facts

This section documents empirical stylised facts on firms’ precautionary savings and employment in the UK, during the Great Recession. I show that, during the recent crisis, the average firm started hoarding cash while simultaneously cutting employment, and that cash-intensive firms reduced their workforces by less. I will use these findings to motivate a model in which firms have precautionary reasons to respond to changes in credit conditions.

The primary data source used in this paper is the FAME (Financial Analysis Made Easy) dataset. It comprises panel data observations for a large number of UK firms, for the period 2004-2013. The key advantage with respect to US Compustat consists in the ownership structure of the firms. Indeed, 93.7% of the FAME sample considered in this paper are non-publicly traded. This allows to have a much broader and representative sample, where the size and age distribution of firms is close to the UK universe of firms. The presence of young and small firms makes FAME particularly suitable for the analysis of financial frictions, since those firms are likely to rely more heavily on external finance and face more difficulties in accessing credit. The dataset contains firm-level information on both the asset and the employment structure of the firms. It also includes data on cash holdings, recorded in firm’s balance sheets as Bank deposits. Appendix A provides additional information on the data.

I document that, in the sample considered, the average cash ratio increases when aggregate employment falls. Figure 1 shows the first differences of average cash ratio and aggregate employment. In 2009, the year of the trough in aggregate employment of the UK dataset, firms hoarded a large amount of cash with respect to their assets.

The increase in cash ratio in 2009 is quantitatively sizeable and a simple back of the envelope calculation can show this. Suppose the cash ratio remained constant after 2008, and the cash in excess of this counterfactual cash was used to hire workers at the average

---

9Bank deposits are the British analogue of cash & equivalent in global accounting format. This limits the concern that the rise in cash is just driven by a substitution away from other cash securities. Appendix A explores this issue more in detail.

10The cash ratio is defined as the share of cash holdings over total assets. The results are very similar for aggregate cash ratio and median cash ratio across firms.

11Bacchetta et al. (2019) find that, in the US, deviations of aggregate cash ratio and employment are negatively correlated. The correlation for my data is -0.66.
Figure 1: Aggregate increase in cash ratio and aggregate net job creation in the UK

![Graph showing aggregate increase in cash ratio and net job creation in the UK from 2005 to 2013.]

Notes: Firm-level net job creation is the difference in number of employees for a given firm from one year to the other. The cash ratio is the sum of total Bank deposits over total assets. The dashed line shows the year-on-year differences in the cross-sectional average cash ratio. The sample considers firms that have weakly positive observations for employment, cash and total assets for all the years 2004 - 2013. Results hold for an unbalanced panel too. Firms with UK SIC code referring to "Financial and insurance activities" are excluded.

wage. In this scenario, 317,089 additional workers would have been hired, more than offsetting the fall in aggregate employment observed in FAME.

It may be argued that salaries are not the only expenses that a firm faces. In the FAME data I find that the wage bill accounts, on average, for 27% of operating and capital expenditures. Allocating only this share of additional cash to the wage bill, firms in FAME could have hired 85,249 additional workers in 2009, unwinding more than half of the negative job creation. Figure 2 shows how important this is in the aggregate. I scale up FAME additional net job creation to the UK economy and compute the counterfactual unemployment rate, shown by the dashed line. Under this scenario, the increase in unemployment rate in 2009 is less than a third of the one observed in reality, because part of the excess cash is used to hire workers. By doing so, the firms in the counterfactual scenario have a lower stock of cash in the following years, and thus unemployment rate increases faster between 2010 and 2011 than in reality.

The increase in cash ratio does not seem to be driven only by constrained firms. I classify the firms in 2005 by quartiles of size distribution. Size is a popular proxy for financial

---

12 The counterfactual cash is the aggregate cash required to keep the aggregate cash ratio constant, taking aggregate total assets as given. The average wage in 2008, in the FAME sample, is £31,377.

13 Operating expenditures are all the expenses before the EBITDA.

14 In 2008, 29.6 million workers were employed in the UK, five times the aggregate employment in my FAME sample. This factor remains constant over time, confirming that FAME and ONS data display similar employment dynamics.
Figure 2: Cash and unemployment: a back of the envelope calculation

Note: The dashed line is calculated as follows. At each period, the excess cash is computed as $\tilde{C}_t = (C_t - \hat{C}_t) - (C_{t-1} - \hat{C}_{t-1})$, where $C_t$ is the empirically observed aggregate cash and $\hat{C}_t$ the aggregate cash required to keep the cash ratio constant at 5.38%, its value in 2008, conditional on observed aggregate total assets. Every period, 27% of $\tilde{C}_t$ is used to hire additional workers at the cross-firm average wage, and the remaining excess cash is used for other purposes and gone. The additional workers are scaled up by a factor 4.99, which is the ratio between ONS and FAME aggregate employment, and added to the time series of unemployed workers (ONS). Finally, the unemployment rate is computed dividing the counterfactual unemployment by the ONS labour force.

constraint in the empirical literature.\textsuperscript{15} The largest firms experience the largest increase in cash ratio between 2009 and 2008, slightly above 12%. Small firms, by contrast, increase their cash ratios by 1% only.

Besides documenting the cyclical patterns of cash ratios, the data can shed some light on the role played by precautionary cash holdings in the transmission mechanism of credit shocks onto the labour market. Figure 3a shows, at different years, the cross-firm correlations between lagged cash ratio and employment growth.\textsuperscript{16} The correlation more than doubles in 2009, while then turning negative in 2011.\textsuperscript{17} I propose a possible explanation to this behaviour. Consider a credit supply shock that dries up external liquidity available to firms. Firms with higher cash ratios are better equipped to cope with the crisis, and thus they have to cut their workforces by less. During the credit tightening period, most firms hoard cash to partially counteract the scarce external funding, and smooth employment growth throughout the recession. A share of firms, however, is likely to be so disrupted by the credit supply shock that is not able to internally generate liquidity. This cash-scarce

\textsuperscript{15}Repeating the same analysis using age as a proxy delivers the same qualitative results.

\textsuperscript{16}Bacchetta et al. (2019) find a negative cross-firm correlation between deviations in cash ratio and employment among US quoted firms. In figure 3a, instead, I correlate the level of cash ratio with deviations in employment.

\textsuperscript{17}To get a sense of the magnitude of these fluctuations, I regress employment growth on lagged cash ratio at every year and then compute the marginal elasticities. A 1% increase in cash ratio in 2008 is associated with 0.63% higher employment growth in 2009. Similar results hold with a panel regression with interacted year dummies.
Figure 3: Cash ratio and employment growth

(a) Cross-firm correlations between lagged cash ratio and employment growth

(b) Employment growth rates by cash ratio

Note: UK FAME firm-level data, YoY changes. The sample considers firms that have weakly positive observations for employment, cash and total assets for all the years 2004 - 2013. Firms with UK SIC code referring to “Financial and insurance activities” are excluded. Employment growth for a firm $j$ at year $t$ is calculated as $\Delta n_{j,t} = \frac{a_{j,t}-a_{j,t-1}}{a_{j,t}+(1-a_{j,t-1})}$, with $\alpha = 0.5$; Moscarini and Postel-Vinay (2012) explain the advantages of this symmetric approach. Shaded grey bands indicate 95% confidence intervals. In figure 3b, firms are reclassified every year depending on whether their lagged cash ratio was above or below the sample average. Within group average employment growth rates are plotted.

A group of firms leads the recovery when credit conditions are restored, driving the cross-firm correlation negative. The large swings in employment growth, both in 2009 and in the recovery, are clearly shown in figure 3b. Employment growth at firms with cash ratio below the median, in 2009, was 2.6 percentage points lower than for firms with cash ratio above the median, compared to a pre-crisis gap of typically less than 1 percentage point. Schoefer (2015) finds a similar pattern for the US. Differently from his analysis, I reclassify firms every period, investigating the dynamic relationship between cash ratio and employment. This uncovers the differential behaviour in the recovery period. Indeed, in 2011, employment at more illiquid firms grew 1 percentage point more than in the rest of the sample.

I will show how the model will be able to generate similar dynamics. An increase in cash has a twofold effect: on one hand, it takes resources away from production, amplifying the negative effects of credit shocks. On the other hand, in the following periods, it helps the adjustment to tighter credit conditions and, coupled with labour adjustment costs, smooths the recovery.

There could be alternative potential explanations of the dynamic evolution of the cross-firm correlation. Firms that manage to grow even during the crisis could receive more revenues, which would translate in higher cash. As suggestive evidence against this explanation, I regress employment growth on lagged cash ratio at each year, including
cash flow as a control: the dynamic evolution of the coefficients on lagged cash ratio tracks the correlations shown in figure 3a. Similarly, this should account for the possibility of an unexpected negative productivity or demand shock that induces firms to lay off workers and generates more cash flow. If that was the case, we should see a drop in the correlation between cash ratio and employment growth in 2009, which does not happen instead. Finally, the results are robust to the inclusion of sector fixed effects.

3 The model

I consider a partial equilibrium model that investigates firms’ behaviour. The economy is populated by heterogeneous firms that are subject to idiosyncratic productivity shocks and aggregate credit shocks. Firms can invest in physical capital, used for production together with labour, or in liquid assets. They face a liquidity need originated by the payment of the wage bill, which can be covered either by external intra-period loans or by cash holdings. This assumption generates interactions between employment and portfolio choices. Short-term borrowing is collateralised by tangible assets, in the form of capital, and subject to persistent credit shocks, which restrict the amount of loans for a given level of collateral. Firms incur non-smooth capital and labour adjustment costs and can issue equity, at an increasing and convex cost. These elements give rise to firms’ precautionary behaviour, further exacerbated during tight credit periods. I will start presenting the main features of the model and the firm’s value function. Section 3.5 will shed further light on the key mechanisms of the model.

3.1 Technology

The economy is populated by a very large number of infinitely-lived heterogeneous firms that use capital $k$ and labour $n$ to produce a final good. I assume that each firm operates a diminishing returns to scale production function with capital and labour as the variable inputs. A firm produces output $y$ according to:

$$y_t = z_{jt}^\nu k_t^{\nu} n_t^{\omega}, \nu + \omega < 1$$

(1)

where $z_{jt}$ is a stochastic and persistent idiosyncratic productivity\(^\text{18}\) that follows a Markov chain: $z \in Z \equiv \{z_1, \ldots, z_N\}$, with $Pr(z_{t+1} = z_i | z_t = z_j) = \pi^z_{ji} \geq 0$ and $\sum_{i=1}^{N_z} \pi^z_{ji} = 1$.

\(^\text{18}\)Since the model is in partial equilibrium, the production function can be seen as a revenue function where $z$ combines productivity and demand terms into one index, as in Bloom (2009).
3.2 Working capital constraint

Firms need to pay their wage bill in advance of production. As in Jermann and Quadrini (2012), this stems from the cash-flow mismatch between the payments made at the beginning of the period and the realization of revenues.\(^{19}\) Corugedo et al. (2011) analyse UK firms working capital positions over the business cycle and find that firms have typically a funding gap between the payments of the costs of the inputs to production and the sales revenues, which typically come much later.

The timing goes as follows. At the beginning of the period, after the realization of the idiosyncratic and aggregate shocks, firms choose the stock of workers that will be productive in the same period. They have to pay the associated wage bill \(\bar{w}n_t\) out of accumulated cash \(m_t\), before the realization of revenues, which come at the end of the period. If the wage bill exceeds the accumulated cash, firms can obtain external funds at the beginning of the period and repay at the end of it. This form of intra-period loan entails no interest, as in Jermann and Quadrini (2012), and cannot be larger than a stochastic fraction \(\phi\) of collateral, that is, the liquidation value of capital. The following equation describes how, according to the collateral constraint, the financing funds need to be greater or equal than the financing needs:

\[
\phi_s \cdot \left(1 - \vartheta \right) \cdot \left(1 - \delta_k \right) k_t + m_t \geq \bar{w}n_t \tag{2}
\]

The ability to borrow intra-temporally is bounded by the limited enforceability of debt contracts. Since liquidity can be easily diverted, the only asset available for the liquidation is physical capital \(k_t\), as in Kiyotaki and Moore (1997). In particular, this will be the non-depreciated fraction of capital; moreover, lenders incorporate the fact that, in case of default, they will sell the seized capital at a lower, resale price \((1 - \vartheta)\). This form of partial irreversibility will be described in the following subsection.

The collateral fraction \(\phi \in \phi_1, \ldots, \phi_{N_\phi}\) is assumed to be common to all firms and will be referred to as credit tightness. It is assumed to follow a Markov chain, with \(Pr(\phi_{t+1} = \phi_m | \phi_t = \phi_s) = \pi_{sm}^\phi \geq 0\) and \(\sum_{m=1}^{N_\phi} \pi_{sm}^\phi = 1\). This variable can be interpreted in many ways. It could reflect the efficiency of the economy’s financial sector, as in Khan and Thomas (2013), or capture the variations over time in the degree of credit market tightness (Finocchiaro and Mendicino (2016)). Eisfeldt and Rampini (2006) provide some evidence about the cyclicality of \(\phi\). The quantitative analysis in section 4 will consider a drop in \(\phi\), resembling an exogenous reduction in the amount of available external funds.

Similarly to Svensson (1985), cash holdings decisions are made before the realization

\(^{19}\)Christiano et al. (2010) and Mendoza (2010) are other examples of models with similar constraints.
of the shocks, which gives rise to precautionary incentives to accumulate cash. The latter will be softened by the possibility to top up the wage bill payment through external financing. As will be explained later, the presence of real frictions act in the opposite direction, amplifying the incentives to behave pre-emptively. Finally, the wage is assumed to be fully rigid and common across firms.\footnote{Wage rigidity is often assumed in quantitative macroeconomic models, as Christiano et al. (2005), and search and matching literature, for instance Shimer (2005).}

### 3.3 Real frictions

Besides the working capital constraint, firms face three real frictions. As will be made clearer later on, the interaction between real and financial frictions implies that precautionary cash holdings arise endogenously in the model.

Firms face linear and symmetric hiring and firing costs, as in Bloom et al. (2018). The firm begins the period $t$ with a pre-determined employment stock $n_{t-1}$, a fraction $\delta_n$ of whom immediately separates. Firms choose the new stock of workers, pay the wage bill and use pre-determined capital and the newly available labour to produce. The labour adjustment costs can be summarized as follows:

$$AL(n_{t-1}, n_t) = \chi |n_t - (1 - \delta_n) n_{t-1}|$$

Consistent with the typical timing convention, capital $k_t$ is chosen at time $t - 1$ and predetermined at time $t$. It evolves according to $k_{t+1} = (1 - \delta_k)k_t + i_t$ where $i_t$ is investment and $\delta_k$ is the depreciation rate. As in Bloom (2009), capital is partially irreversible and its installation is subject to fixed costs. Firms buy capital at a unitary price, as in a neoclassical growth model, but, for each unit of used assets, only $(1 - \vartheta)$ fraction is useful for other buyers. $\vartheta$ represents the reallocation costs, the partial irreversibility of the capital stock due to capital specificity or adverse selection problems. Capital investment, net of partial irreversibility cost, is then given by:

$$ACP(k_t, k_{t+1}) = \begin{cases} k_{t+1} - (1 - \delta_k)k_t & \text{if } k_{t+1} \geq (1 - \delta_k)k_t \\ -(1 - \vartheta)[(1 - \delta_k)k_t - k_{t+1}] & \text{if } k_{t+1} < (1 - \delta_k)k_t \end{cases}$$

Moreover, when new capital is installed or gross investment is negative, a fixed fraction $\Theta$ of output is lost. Therefore we define fixed adjustment costs of capital as:

$$ACF(k_t, k_{t+1}, y_t) = \begin{cases} \Theta y_t & \text{if } k_{t+1} \neq (1 - \delta_k)k_t \\ 0 & \text{otherwise} \end{cases}$$
Finally, I assume that firms incur a quadratic cost\textsuperscript{21} when they deviate from a target level of dividends, given by: $\xi(d_t) = \kappa(d_t - \overline{d})^2$, as in Jermann and Quadrini (2012). This cost is in line with empirical evidence that underwriting fees display increasing marginal cost in the size of the offering (Altinkilic and Hansen (2000)). It is also a reduced form to capture the fact that managers are concerned with smoothing dividends over time. The seminal work by Lintner (1956), repeatedly confirmed by more recent studies, found that approximately 90% of firms smooth their dividend payments with respect to their earnings. For simplicity, the dividend target $\overline{d}$ is set to 0. This especially helps the interpretation of the cost in terms of equity issuance too.

3.4 The Firm’s value function

I denote $V(m_t, k_t, n_{t-1}, z_{j,t}; \phi_{s,t})$ the value function of a firm. The 5 state variables are given by (1) the firm’s cash stock $m_t$, (2) the firm’s capital stock $k_t$, (3) the firm’s stock of workers $n_{t-1}$, (4) the firm’s idiosyncratic productivity $z_{j,t}$ and (5) the aggregate credit tightness $\phi_{s,t}$. The dynamic programming problem of the firm consists of choosing dividends, labour, capital next period and cash next period to maximise the present discounted value of future dividends:

$$V(m_t, k_t, n_{t-1}, z_{j,t}; \phi_{s,t}) = \max_{d_t, m_{t+1}, k_{t+1}, l_t, n_{t+1}, y_t} \left\{ d_t - \xi(d_t) + \beta \sum_{m=1}^{N_\phi} \pi_{s,m} \sum_{i=1}^{N_z} \pi_{j,i} V(m_{t+1}, k_{t+1}, n_{t+1}, z_{i,t+1}; \phi_{m,t+1}) \right\}$$

subject to:

\begin{align*}
y_t - l_t &= AC^P(k_t, k_{t+1}) + AC^F(k_t, k_{t+1}, y_t) + AL(n_{t-1}, n_t) + m_{t+1} + d_t \quad (6) \\
l_t &\geq \bar{w}n_t - m_t \quad (7) \\
l_t &\leq \phi_{s,t}(1 - \vartheta)(1 - \delta_k)k_t \quad (8) \\
m_{t+1} &\geq 0 \quad (9) \\
\xi(d_t) &= \kappa d_t^2 \quad (10)
\end{align*}

and (1), (3), (4) and (5). I denote with $l_t$ the intra-period loan that the firm receives at the beginning of the period and repays at the end of it. In equilibrium, (7) is always bind-

\textsuperscript{21}The majority of macroeconomic models with heterogeneous firms restrictively assumes no equity issuance. Given the absence of inter-temporal debt, I allow all firms to issue equity, but at a cost.
ing, since firms borrow exactly the amount of wage bill that exceeds the accumulated cash. In contrast, (8) binds occasionally over time and across firms. All firms face the same aggregate credit tightness $\phi_{s,t}$. The wage $\bar{w}$ is exogenous, time-invariant, common across firms and taken as given by each firm. Let $k^*(m_t, k_t, n_{t-1}, z_{j,t}; \phi_{s,t})$, $m^*(m_t, k_t, n_{t-1}, z_{j,t}; \phi_{s,t})$, $n^*(m_t, k_t, n_{t-1}, z_{j,t}; \phi_{s,t})$ and $d^*(m_t, k_t, n_{t-1}, z_{j,t}; \phi_{s,t})$ represent the optimal choices of next-period capital and cash, labour and dividends respectively, made by the firm with current idiosyncratic productivity $z_{j,t}$ and under aggregate credit tightness $\phi_{s,t}$. I characterize these decision rules in section 3.5.

3.5 Firm’s behaviour

Before turning to the quantitative part of the paper, it is useful to shed some light over the main mechanisms generated by the model. I will start by showing the trade-off between capital and cash, and how this is affected by the real frictions. I will then turn to the hiring decision and finally show how the precautionary mechanism falls apart when each of the real frictions is removed.

Firms face a trade-off between very liquid but unproductive assets, denoted as cash, and productive but partly liquid and partly collateralizable assets, capital. This is shown in equations 11 and 12, which show the first order conditions of a firm with idiosyncratic productivity $z_{j,t}$ for capital and cash respectively:

$$
\beta E_t \left[ \lambda_{t+1} \left( \gamma_{t+1} + AC_{k}^p (k_{t+1}, k_{t+2}) - AC_{k}^F (k_{t+1}, k_{t+2}, y_{t+1}) \right) + \phi_{t+1} (1 - \vartheta) (1 - \delta_k) \mu_{t+1} \right] = \lambda_t AC_{k}^p (k_t, k_{t+1}) + \lambda_t AC_{k}^F (k_t, k_{t+1}, y_t) \tag{11}
$$

$$
\beta E_t [\lambda_{t+1} + \mu_{t+1}] = \lambda_t - \psi_t \tag{12}
$$

where $\lambda_t$, $\mu_t$ and $\psi_t$ are the Lagrange multipliers associated to (6), (8) and (9) respectively.\footnote{Complementary slackness conditions for $\lambda$, $\mu$ and $\psi$ have been omitted. For all the equations of this section, the expectation operator $E_t$ is used as a reduced form for $\sum_{m=1}^{N_0} \pi_m^0 \sum_{j=1}^{N_j} \pi_{j,t}$. The non-smoothness in labour and capital adjustment costs introduce kinks in the value function. As shown by Cui (2017), the differentiability of the value function can be proved using methods from Clausen and Strub (2012). In Appendix C I show how this works and deal with the envelope conditions in detail. In any case, the numerical solution presented in appendix B does not use the optimality of the first order conditions.} For illustrative purposes, I leave the derivatives of labour and capital adjustment
costs unspecified. The left hand side of equations (11) and (12) shows the marginal benefit of holding an additional unit of capital and cash respectively. For (11), this can be decomposed in three parts: (I) the expected marginal product of capital, [(II) + (III)] the expected marginal net benefit of an additional unit of capital brings tomorrow in terms of adjustment costs and, finally, (IV) the expected marginal benefit of holding capital as collateral. Capital is only partly collateralizable and hence its financing return \( \beta E_t \phi_{t+1} \mu_{t+1} \) is scaled down by a factor \((1 - \vartheta)(1 - \delta_k)\) smaller than 1. The portfolio allocation between capital and cash is forward-looking, since decisions taken this period affect the financing conditions in the following. In other words, employment growth can be sustained by different allocations of internal and external financing.

The real frictions play a crucial role for the endogenous accumulation of cash. Adjustment costs make capital less liquid and, in turn, shift the portfolio allocation towards cash. Partial irreversibility scales down the financing return by a factor \((1 - \vartheta)\), as shown in III. Moreover, both costs affect the marginal benefit and cost of holding capital. Intuitively, firms incorporate the fact that a negative shock may require to sell off capital, at a lower resale price. Moreover, this will trigger the fixed adjustment cost. Hence, they act pre-emptively and hoard more cash instead.

The dividend cost also adds to firms’ precautionary incentives. It implies that the shadow value of wealth can be different than 1, as shown in the FOC for dividends:

\[
1 - 2\kappa d_t = \lambda_t
\]  

On one hand, it limits the room for issuing equity in face of negative shocks, therefore inducing firms to accumulate cash instead. On the other hand, it also induces firms to retain cash instead of distributing it as dividends, after a positive productivity shock. Both effects go in the same direction, implying that cash is used as a tool to move resources from one period to the other. This feature has been documented empirically by Dittmar and Duchin (2011). In the quantitative part of the paper I will show that, in absence of inter-temporal substitution in the savings decision, relevant empirical moments of UK firm-level data would be missed by the model. Capital and labour optimal decisions are also affected by the dividend cost. Indeed, the Tobin’s Q for capital fluctuates around 1 even without capital adjustment costs. Moreover, firms cannot finance freely additional capital through equity issuance, and thus easily circumvent the financial constraint. This possibility could generate a counter-factual increase in investment during credit crunches.

23 Appendix C deals with this issue and shows how the FOCs can be rewritten in terms of marginal values of capital and labour that satisfy the envelope condition, without loss of generality. With a slight abuse of notation, \( AC^F_k \) refer to the total derivative of fixed adjustment costs with respect to \( k \), including its effect through \( y \).
as shown in section 5.

Finally, labour adjustment costs have two opposite effects, which show up in the optimal decision for labour:

\[
\lambda_t \omega \frac{y_t}{n_t} - \beta E_t \lambda_{t+1} AL_n (n_t, n_{t+1}) - \bar{w} \mu_t = \\
\lambda_t \left[ \bar{w} + AL_n (n_{t-1}, n_t) + AC^F_y (k_t, k_{t+1}, y_t) \frac{y_t}{n_t} \right]
\]

(14)

On one hand, hiring costs induce labour hoarding and make it less likely for booming firms to face a binding collateral constraint, because they increase the marginal cost of labour. On the other hand, firing costs imply that firms ”on the way down”, those that face negative shocks, may fear to be at the binding collateral constraint. Intuitively, the firing cost reduces the possibility of cutting labour. This affects the expected marginal benefit of having an additional worker next period, \( E_t \lambda_{t+1} AL_n (n_t, n_{t+1}) \). The dividend cost affects labour optimal choices through the budget constraint; in turn, currently unconstrained firms may have incentives to adjust labour to changes to credit conditions.

The precautionary mechanism shows up in full if all the real frictions are included simultaneously. To show this, I use a version of the model with no real frictions and then discuss the role played by each of them. Equations 15-17 show the optimal decisions for capital, cash and labour in this case:

\[
\beta E_t \left[ \nu \frac{y_t + 1}{k_t + 1} + 1 - \delta_k \right] + \beta E_t \phi_{t+1} (1 - \delta_k) \mu_{t+1} = 1
\]

(15)

\[
\beta E_t [1 + \mu_{t+1}] = 1 - \psi_t
\]

(16)

\[
\omega \frac{y_t}{n_t} = \bar{w} [1 + \mu_t]
\]

(17)

The timing assumption of the working capital constraint potentially gives rise to precautionary cash holdings by itself. Even in this version of the model, optimal capital and cash decisions still depend on the expectation of a binding financial constraint next period. Nevertheless, these expectations do not feed into the hiring decision through the budget constraint, because the absence of dividend costs implies that \( \lambda_t = 1 \). In other words, labour decisions of credit-unconstrained firms - for which \( \mu_t = 0 \) - are not affected by aggregate credit conditions when there are no real frictions. The dividend cost by itself is not enough to induce forward-looking hiring decisions: the combination of labour adjustment costs and dividend rigidity is required. Further introducing capital adjustment costs induces firms to tilt their portfolio allocation towards cash. Whether there will be firms, in a model without real frictions, for which the marginal benefit of holding cash is greater.
than the marginal return on capital is a quantitative question. I will show in section 5 that, for reasonable calibrations, the absence of real frictions implies that firms will strictly prefer to hold zero cash reserves. In this setting, firms hold cash only when $\phi$ approaches 0 and, effectively, the financial constraint resembles a cash-in-advance constraint.$^{24}$

4 Quantitative exploration

This section considers a quantitative version of the theoretical framework, in order to investigate the effect of an exogenous tightening of the collateral constraint. The model is calibrated to the UK economy, using aggregate and microeconomic moments. In the steady state, the model is able to match a set of additional moments not explicitly targeted. I then show the effects of a credit supply shock, in the form of a drop in $\phi$, both in terms of aggregate dynamics and microeconomic forces driving them.

4.1 Calibration

The model is parameterised so that the stationary equilibrium matches relevant aggregate and firm-level moments in the UK.$^{25}$ I set the time period to a quarter. Idiosyncratic productivity is assumed to follow a AR(1) in logs, which is then discretized using Tauchen and Hussey (1991) method to obtain $(\pi_{ij})_{i,j=1}^{N}$. Table 1 summarizes the calibrated parameter values, whereas Table 2 compares empirical and model-generated moments. Most of the empirical moments are obtained using UK firm-level balance sheets from the FAME dataset, and are averages of the pre-crisis period 2004-2006.

Six parameters are calibrated simultaneously using a simulated method of moments. The average cash to assets ratio is particularly informative of $\phi$, governing the tightness of the financial constraints. Since precautionary cash holdings are a key feature of the model, I do not only target first moments, but also the cross-sectional standard deviation of firm-level cash ratios. In section 5 I will show how versions of the model that do not entail the full precautionary mechanism are not able to generate the skewness and the right tail of the cash ratio distribution. The standard deviation of idiosyncratic productivity shocks is important in determining the dispersion of cash ratios. Indeed, higher dispersion of shocks magnifies the precautionary savings motive. The persistence of the productivity process, $\rho_z$, affects most of the targeted moments but it is mainly identified by the autocorrelation

$^{24}$The same result could also be obtained with strong assumptions as full depreciation of capital. This, however, would not allow to match the calibration targets outlined in sections 4.

$^{25}$The stationary equilibrium in the steady state is defined as follows: I solve the firm’s problem allowing for aggregate uncertainty and obtain the individual’s time-independent decision rules. Then I set $\phi = \phi_H$ and compute the invariant distribution of firms over $(m,k,n,z)$. 

16
# Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>0.76</td>
<td>Exponent on labour in production function</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.14</td>
<td>Exponent on capital in production function</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.60</td>
<td>Dividend rigidity cost</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.93</td>
<td>Quarterly persistence of idiosyncratic productivity</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.11</td>
<td>Quarterly standard deviation of innovations to idiosyncratic productivity</td>
</tr>
<tr>
<td>$\phi_H$</td>
<td>0.5</td>
<td>Steady state credit tightness</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
<td>Firm discount factor</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>1</td>
<td>Wage (normalisation)</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>0.0375</td>
<td>15% annual depreciation of capital stock (Riddick and Whited, 2009)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.072</td>
<td>Per worker hiring/firing cost in % of annual wage bill (Bloom 2009)</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>0.025</td>
<td>UK (ONS) average quarterly voluntary job separation rate 1996-2007</td>
</tr>
<tr>
<td>$\bar{\phi}$</td>
<td>0.34</td>
<td>Investment resale loss in % (Bloom, 2009)</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>0.06</td>
<td>Investment fixed cost in % of annual sales (Bloom 2009)</td>
</tr>
</tbody>
</table>

**Aggregate credit shock**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{HHH}^\phi$</td>
<td>0.978</td>
<td>Quarterly transition probability of remaining in high $\phi$</td>
</tr>
<tr>
<td>$\pi_{LH}^\phi$</td>
<td>0.212</td>
<td>Quarterly transition probability from low to high $\phi$</td>
</tr>
<tr>
<td>$\phi_L$</td>
<td>0.45</td>
<td>Tight credit conditions to match drop in short-term loans (UK FAME)</td>
</tr>
</tbody>
</table>
Table 2: Model fit

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate labour share</td>
<td>0.72</td>
<td>0.72</td>
<td>Bank of England</td>
</tr>
<tr>
<td>Aggregate fixed assets to sales ratio</td>
<td>0.57</td>
<td>0.60</td>
<td>FAME</td>
</tr>
<tr>
<td>Average cash-to-assets ratio</td>
<td>0.18</td>
<td>0.18</td>
<td>FAME</td>
</tr>
<tr>
<td>Cross-firm standard deviation of cash ratios</td>
<td>0.23</td>
<td>0.22</td>
<td>FAME</td>
</tr>
<tr>
<td>Autocorrelation of investment ratios</td>
<td>0.07</td>
<td>0.06</td>
<td>FAME</td>
</tr>
<tr>
<td>Cross-firm correlation between dividends and sales</td>
<td>0.74</td>
<td>0.73</td>
<td>FAME</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-targeted Moments</th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-firm correlation between cash ratio and capital-labour ratio</td>
<td>-0.11</td>
<td>-0.13</td>
<td>FAME</td>
</tr>
<tr>
<td>Scaled average volatility of dividends</td>
<td>0.73</td>
<td>0.44</td>
<td>FAME</td>
</tr>
<tr>
<td>25th percentile of cash ratio</td>
<td>0</td>
<td>0.02</td>
<td>FAME</td>
</tr>
<tr>
<td>75th percentile of cash ratio</td>
<td>0.26</td>
<td>0.26</td>
<td>FAME</td>
</tr>
<tr>
<td>Average cash ratio by quartile of size:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤25th percentile</td>
<td>0.25</td>
<td>0.31</td>
<td>FAME</td>
</tr>
<tr>
<td>&gt;25th &amp; ≤50th percentile</td>
<td>0.15</td>
<td>0.18</td>
<td>FAME</td>
</tr>
<tr>
<td>&gt;50th &amp; ≤75th percentile</td>
<td>0.16</td>
<td>0.14</td>
<td>FAME</td>
</tr>
<tr>
<td>&gt;75th percentile</td>
<td>0.17</td>
<td>0.11</td>
<td>FAME</td>
</tr>
</tbody>
</table>

Notes: Labour share is measured in the data as share of GDP, whole economy, excluding rents. Average over the period 1950-2006. FAME data are averages for the period 2004-2006. The panel is balanced, for consistency with the model that does not account for entry and exit. Investment ratio for a firm \( j \) at time \( t \) is defined as \( \frac{k_{jt} + \alpha k_{j,t-1}}{F_{jt} + (1-\alpha)F_{j,t-1}} \), with \( \alpha = 0.5 \), where \( k \) is fixed assets as recorded at balance sheet. Autocorrelation is one year. Size is measured in number of employees in the data and labour in the model. Empirical capital to labour is the ratio of fixed assets over number of employees. Capital to labour ratios are winsorized at the upper 99th percentile both in the model and in the data. The scaled average volatility of dividends is calculated in the model as follows. For each firm in a simulated panel with \( \phi = \phi_0 \), I calculate the standard deviation of annualized dividends over time. Then I take the cross-sectional average of these standard deviations, and scale it by the cross-sectional average level of dividends. In the data, I follow the same approach, before 2006. Extending the sample to 2013 increases the moment to 0.90. Similarly, including the credit crunch period in the model simulation increases the volatility to 0.79.

of investment ratios, as in Khan and Thomas (2013). The dividend cost is disciplined by matching the cross-firm correlation between dividends and sales. Firms’ capital intensity is directly tied to the fixed assets to sales ratio. Finally, the labour exponent of the production function is calibrated to match the aggregate labour share. The resulting decreasing returns to scale are close to the value calibrated by Khan and Thomas (2013).

The other parameters are externally set. Firms’ discount factor needs to be sufficiently low to ensure that at least some firms do not accumulate enough assets to completely save
themselves out of the constraint. The calibrated value is consistent with an annual steady state return from holding shares of 8.4%, slightly higher than in Jermann and Quadrini (2012). The discount factor is, however, larger than in Bacchetta et al. (2019), where all firms are constrained. The exogenous job separation rate $\delta_n$ is directly taken from ONS data on UK average quarterly voluntary job separation rate between 1996 and 2007. The wage is normalised to 1. Per worker hiring and firing costs are symmetric and equal to 1.8% of the annual wage bill. This estimate, as well as the capital adjustment costs, is taken from Bloom (2009). Ideally, these values should be estimated to the UK data as well. In Appendix D I explore the effects of using different estimates available in the literature, and show that the chosen values achieve a better fit of over-identifying empirical moments. Labour adjustment costs deserve special attention, given their important role in the model mechanisms. The OECD employment protection index for the UK is only mildly higher than in the US, suggesting that assuming the same firing costs may not be an unreasonable assumption. In Appendix D I explicitly focus on estimates from Nickell (1986), who consider linear labour adjustment costs 4 times larger than those employed in this calibration. With that calibration, the quantitative relevance of the precautionary channel is slightly larger, and the aggregate dynamics are even closer to the data. The benchmark calibration can be therefore seen as conservative, and the main results of this paper robust to stronger labour adjustment costs.

The model entails aggregate uncertainty with respect to the credit tightness $\phi$, whose stochastic process is discretized using a 2-states Markov chain. A credit shock consists of a fall in $\phi$. Agents form their expectations over future credit conditions according to the transition probability matrix. I define a credit shock as a 10% drop in $\phi$. This allows the model to match the HP-detrended reduction in aggregate short-term loans held by UK firms in FAME data, between 2007 and 2009. The issues related with the estimation of the credit shock process are particularly relevant for the UK, where data on aggregate financial conditions (i.e.: Bank of England Credit Conditions Survey) start in Q1 2007. In terms of timing, both survey and lending measures suggest that firms’ credit conditions start to deteriorate only at the beginning of 2009. I set the transition probabilities such that the credit tightening lasts on average 13 months and occurs every 10 years. The length of the credit tightening is in line with a various range of financial indicators in

---

26 It might also be argued that labour adjustment costs are asymmetric, and this might affect the precautionary mechanisms of this paper. The empirical evidence, however, is mixed. Abowd and Kramarz (2003) use French data to establish that hiring is cheaper than terminations. Pfann and Palm (1993) find that, in the UK, hiring costs are larger than firing costs for production workers, while the opposite is true for nonproduction workers.

the UK. The frequency may seem high, especially if we interpret it as a financial crisis. Nevertheless, the financial friction in my model restrains the ability to borrow within the period, resembling what is normally referred to as line of credit, as in Bacchetta et al. (2019). Shocks affecting the supply of this form of liquidity are likely to happen much more frequently than a full-blown financial crisis. Moreover, relatively frequent credit tightening episodes should dampen the disruptive effects of a credit supply shock, since firms have an additional incentive to save ahead of the crisis and therefore are better equipped to face episodes of credit crunch.

4.2 Steady state

The stationary distribution of firms in the stochastic steady state, with $\phi = \phi_H$, is in line with a number of UK empirical moments that are not explicitly targeted in the calibration. The lower panel of Table 2 shows some of them. The negative cross-firm correlation between cash ratio and capital to labour ratio informs us about important model dynamics. Consider a firm with a low capital to labour ratio: this implies that it needs to finance a large wage bill, especially relative to the available collateral. Hence, the firm has incentives to shift towards a more cash-intensive portfolio.

The dividend cost also reduces the volatility of dividends over time. Following a positive productivity shock, for instance, a firm would like to distribute more dividends to the shareholders. The dividend cost limits the amount of additional dividends the firm will pay out. With multiple dividend targets, this effect would be milder, because the target would adjust together with the productivity shock, making the cost of deviation smaller and leading to larger fluctuations in dividends. I show that, even with a single dividend target, the model overshoots the volatility of dividends over time compared to the data. I compute the standard deviation of dividends over time for each firm, then I take the average of these volatilities across firms and scale it by the cross-sectional average of dividends, to allow for comparability between the model and the data. In terms of equity issuance, the combination of the dividend cost parameters implies that, on average, 13% of firms issue equity every quarter, a proportion in line with other studies as Gilchrist and Zakrajsek (2014).

The model also does a very good job in matching the distribution of cash ratios, besides the first two moments. Moreover, smaller firms have relatively higher cash ratios. The negative correlation is present also in the model, although this is overly driven by very small firms. Intuitively, small firms have little collateral and are likely to face an increasing schedule in employment growth. Hence, they face a tradeoff between expanding

\[28\]

An alternative approach involves computing the coefficient of variation at the firm-level and then taking the cross-sectional average. This delivers similar results.
their capital stock together with labour or accumulating internal cash as a financing tool. While they are small, the low pledgeability of capital makes them to lean towards cash. As they grow, they get more easily access to external credit and thus start reducing their stock of cash.

### 4.3 The aggregate response to a credit shock

I simulate the aggregate dynamics of the model following a tightening of the collateral constraint. This consists in a 10% drop in $\phi$, as discussed in section 4.1, and can be interpreted as an increase in the haircut applied on collateral.

Figures 4a-4f show the aggregate dynamics of a credit tightening that lasts 5 quarters. A credit crunch has two main effects. First of all, it causes a substitution from capital to cash. During credit tightening periods, capital is less worth in terms of collateral. For this reason, firms switch from external financing to internally generated liquidity. Credit shocks enlarge capital inaction regions typical of non-smooth adjustment costs; this implies a very sluggish recovery in capital and, conversely, aggregate cash levels well above steady state conditions for many quarters after the end of the credit crunch.

The second effect is a sizeable fall in aggregate employment, which keeps falling throughout the credit tightening period. Less available credit implies that some firms are not able to finance the same wage bill as before, and thus they have to reduce their workforces. Aggregate employment slowly recovers as ordinary credit conditions are restored, although it takes many quarters to completely come back to pre-crisis levels.

These large aggregate effects materialize even though the share of constrained firms is small. As shown in Figure 4g, the share of firms facing a currently binding credit constraint increases during credit tightening periods, but never exceeds 20%. As a direct consequence of this, in the aggregate, the available financing funds are largely in excess of firms’ financing needs, as documented in the data by Zetlin-Jones and Shourideh (2017).

Figures 5a and 5b show that the effects of a credit shock in the model replicate fairly well key aggregate dynamics in the UK. This is especially true in the first year after the shock. The recovery of aggregate employment is faster in the model than in the data; this is probably due to the absence of demand effects that may propagate the effects of a credit shock. In contrast, while quantitatively sizeable, the increase in average cash ratio generated by the model is 40% of what observed in the data. I also show that a model without real frictions is unable to generate the empirically observed aggregate dynamics. The fall in aggregate employment is negligible. Absent virtually any precautionary mechanism,

---

29 An alternative approach consists of simulating many different economies where we allow the aggregate credit shock to evolve naturally after an initial one-period drop. The aggregate dynamics for this case are shown in Appendix E.
Figure 4: The effects of a credit supply shock

(a) Credit shock  
(b) Aggregate Employment  
(c) Aggregate Output  
(d) Aggregate Capital  
(e) Aggregate Cash  
(f) Average cash ratio  
(g) Share of constrained firms  
(h) $\text{Corr}(\frac{m_{i_t-1}}{m_{i_t-1}+k_{i_t-2}}, \%\Delta n_{i_t})$  
(i) Average employment growth by cash ratios

Note: 200,000 firms are simulated for 400 periods allowing for aggregate uncertainty but forcing $\phi = \phi_H$. $\phi$ falls to its low value thereafter, stays there for 5 quarters and reverts back. Simulations have been repeated 100 times. The IRFs show the mean response (across simulations). Shaded grey bands indicate 95% confidence intervals. Constrained firms are those for which equation (8) binds.
Figure 5: Model performance to the UK economy

Note: Model-generated data refer to the same simulation outlined in Figure 4. Quarterly simulated data have been annualised using standard accounting techniques, as explained in Appendix B. The credit shock hits the economy in the first quarter of 2009. In the "Non-precautionary model", $\chi$, $\vartheta$, $\Theta$, and $\kappa$ have been set to 0, while the remaining parameters are the same as in Table 1. UK data for aggregate employment refer to employment rate (aged 16-64) from the Office for National Statistics (ONS). Data for panel b are from the FAME dataset, where previous notes apply. To account for an upward trend in average cash ratios over the time period, the series is linearly detrended. The figure shows the difference, in percentage points, from 2008.

4.4 Micro-level effects of a credit shock

Besides correctly predicting the evolution of key aggregate variables, the model is able to replicate the dynamic evolution of the cross-firm correlation between lagged cash ratio and employment growth, as shown in Figure 4h. A different way to visualize this pattern is by looking at the differential behavior of employment growth rates for firms above and below the median cash ratio, see figure 4i. The model mechanisms help rationalise this result. Firms face a tradeoff when choosing their cash ratio. On one hand, they may need to reduce their hiring to finance an increase in cash, because cutting on dividends or capital investment is limited by the associated frictions. Moreover, reducing the capital stock exerts further negative pressure on labour through the production function. On the other hand, more cash alleviates the extent to which the credit constraint binds and helps sustaining employment growth. In normal times, these two effects generally offset each other, as shown in Figure 4h. As in the data, the steady state correlation is mildly positive.
because firms with lagged cash ratio above the median grow more in terms of employment. After a credit supply shock that dries up liquidity cash becomes an even more valuable source of financing. Cash-intensive firms are associated with higher employment growth throughout the crisis.

This behavior is reminiscent of what documented in the data, as shown in section 2. Quantitatively, the model does a good job in generating the increase in the cross-firm correlation. In other words, it opens a large enough gap in employment growth rates, for firms with different liquidity positions. Differently from the data, however, the model-generated correlation remains positive when ordinary credit conditions are restored, albeit it still drops. Employment growth at cash-scarce firms rebounds more than for the rest of the population, but only relative to their own steady state. In the data, instead, illiquid firms grow unconditionally more in the recovery.

The key mechanism that characterizes this paper is a precautionary behaviour in anticipation of future financial constraints. In order to evaluate its quantitative importance, I consider the simulation described in the previous section and classify the firms in four groups, depending on whether they faced a currently binding credit constraint in this period and/or in the previous. Firms that do not face a binding constraint are loosely labelled as unconstrained, although they will be clearly affected by financial frictions and more so the closer they are to the binding constraint. Figure 6 shows, for each period, the contribution of each group to the growth of aggregate employment. In normal times constrained firms are booming and hoard cash to sustain their employment growth. When a credit shock hits the economy, all the four groups are affected. Unconstrained firms react to changes in credit conditions by cutting employment more than in normal times. Upon impact, they account for nearly half of the total fall in aggregate employment. In the second quarter, firms that remain constrained are already growing more than in normal times. In other words, these firms drive the recovery even before ordinary credit conditions are restored. The persistent fall in aggregate employment is therefore led by unconstrained firms, as well as by firms becoming constrained because of idiosyncratic shocks. In particular, unconstrained firms account for more than half of the drop in employment in the second quarter, with firms that remained unconstrained since the beginning of the crisis being quantitatively important. Their employment growth starts increasing, relative to steady state, in quarter 3; firms becoming unconstrained, however, grow consistently less than in steady state for the entire crisis period, crucially contributing to the persistent fall in aggregate employment.

For each group, I compute the difference in employment growth between a certain quarter and the steady state (i.e.: quarter 0). Then I divide this object by aggregate employment growth. This allows me to compute contributions to growth that clean out for the fact that certain groups are associated with negative or positive employment growth rates in normal times.
Figure 6: The importance of unconstrained firms

Note: Each period $t$, firms are classified in 4 groups: "remaining constrained" if they faced a binding financial constraint in $t$ and $t-1$, "becoming unconstrained" if they moved from constrained to unconstrained, "becoming constrained" from unconstrained to constrained, "remaining unconstrained" if they remained unconstrained. Each bar denotes the contribution of the group to growth. The economy starts at the stationary distribution and experiences a credit tightening in quarter 1, lasting 5 quarters. Previous notes on the simulation apply.

4.5 The credit-constrained firms

The identification of credit constraints is still a debated topic in the empirical literature, and it is not clear whether it is possible to find observables that unambiguously help the identification of credit-constrained firms. The structural model outlined in this paper predicts that credit-constrained firms are generally productive, large and illiquid. The first two characteristics directly stem from the working capital constraint, which by construction is more binding for firms that have a large wage bill to finance. The relationship between cash ratio and the probability of being constrained is less clear ex ante. On one hand, higher cash implies that firms are less likely to face a binding financial constraint. On the other hand, this is normally associated with small firms, which do have little wage bill to finance but also little collateral to pledge.

As mentioned in the previous sections and shown in the quantitative analysis, being constrained is not a clear-cut concept in the model. For this reason, there are often non-linear and time-varying correlations between firm’s characteristics and the distance from the binding constraint.

Coming up with a simple proxy which identifies financial constraints in the model is difficult. To show this, I run the following experiment. I take different proxies for financial constraints normally used in the empirical literature and ask: if we were to classify firms
Table 3: The performance of financial constraint proxies in the model

<table>
<thead>
<tr>
<th>Proxy</th>
<th>% incorrectly classified as constrained</th>
<th>% incorrectly classified as unconstrained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (employees): below median</td>
<td>96.8%</td>
<td>30.3%</td>
</tr>
<tr>
<td>Size (total assets): below median</td>
<td>93.4%</td>
<td>26.7%</td>
</tr>
<tr>
<td>Leverage ratio: above median</td>
<td>75.1%</td>
<td>8.3%</td>
</tr>
<tr>
<td>No dividend payment</td>
<td>81.0%</td>
<td>16.2%</td>
</tr>
<tr>
<td>Size (employees): bottom tercile</td>
<td>99.9%</td>
<td>24.9%</td>
</tr>
<tr>
<td>Size (total assets): bottom tercile</td>
<td>96.0%</td>
<td>22.9%</td>
</tr>
<tr>
<td>Leverage ratio: upper tercile</td>
<td>73.7%</td>
<td>11.7%</td>
</tr>
<tr>
<td>All firms</td>
<td>83.3%</td>
<td>16.7%</td>
</tr>
</tbody>
</table>

Note: Stochastic steady state of the model, with steady state credit tightness \( \phi_H \). The first column lists some proxies for financial constraints used in the empirical literature. Size as a proxy for financial constraints has been used in a wide range of empirical papers and the cutoff is generally set to the median of the distribution. Fazzari et al. (1988) suggest that firms not paying dividends are financially constrained. The results reported in the second column identify the share of firms that would be constrained according to the proxy, but which are actually unconstrained. Conversely, in the third column, the share of firms that would be unconstrained in the proxy, but which are actually constrained in the model.

According to each of these proxies, how many would be correctly identified as financially constrained in the model? Table 3 shows the performance of these proxies. Only 3.2% of the firms that would be categorized as constrained according to their size, are actually facing a binding collateral constraint in the model, while 96.8% are not. In other words, categorizing firms by size would identify financial constraints more poorly than just looking at all firms. Given the precautionary mechanism featured in the model, however, the fact that large firms are generally more credit-constrained does not necessarily mean that they will be more affected by a credit supply shock.

Fazzari et al. (1988) suggest that firms not paying dividends are financially constrained. This proxy performs slightly better in the model, although dividend smoothing may be associated to non-financial factors. Leverage ratio is a better proxy for financial constraints in the model. Finally, not only simple proxies of financial constraints perform poorly in the model, but even multi-dimensional proxies cannot identify credit-constrained firms. A linear regression of a constraint dummy on all the firms’ state variables delivers a \( R^2 \) of 41%.\(^{31}\) This illustrative example suggests that the identification of financial constraints is hard when non-linearities and non-convexities matter.

\(^{31}\)Considering a probit, instead of a linear probability model, delivers similar results. McFadden’s pseudo-\( R^2 \) does not exactly mean the proportion of variance of the dependent variable explained by the regressors, as in OLS. Hence, interpretation should be taken with caution.
5 The importance of the precautionary mechanism

Previous sections have already shown how the precautionary channel allows the benchmark model to generate predictions in line with the data. In this final section, I formalize this statement by considering three variations of the benchmark model shown in section 3.

I start by shutting down all the real frictions at the same time. In this version of the model, firms generally find it optimal to invest in capital and set their cash reserves to 0. The average cash ratio is matched only if $\phi_H$ is driven to 0.15. At such low value, external credit is so limited that some firms prefer to resort to internally generated liquidity.32 Then I consider two models that add capital adjustment costs. One with only the partial irreversibility of capital, and another that combines it with a fixed cost. Even these models are not able to match the empirically observed average cash ratio. In fact, $\phi_H$ needs to be driven further down to 0.13 when capital is only partially irreversible.

Figure 7 shows the same quantitative exercise of section 4.3, where a credit tightening shock hits the economy for five quarters, for the benchmark model and four variations. When the economy does not face any real friction, the fall in aggregate employment is very mild, as shown by the dash-dotted blue line. Nobody holds cash in the steady state. When credit conditions tightens, the average cash ratio shoots up by more than 400 basis points, an increase at odds with the data, as shown before. When $\phi_H$ is recalibrated to a sufficiently low value, this version of the model generates a large drop in employment upon impact. This lasts, however, only one quarter. The increase in average cash ratio is half as large as the non-recalibrated model, but still much larger than observed in the data.

When no firm holds cash, moreover, capital counterfactually increases in response to a credit tightening. Two counteracting forces are at work: on one hand, absent any equity issuance cost, firms can respond to changes in credit conditions by issuing equity to fund capital investment, thus circumventing the financial friction. This is because a larger capital stock can be pledged as collateral, relaxing the credit constraint. On the other hand, constrained firms reduce labour upon impact, and this exerts a negative pressure on capital through the production function. Which effect dominates is a quantitative question: the findings shown in figure 7 suggests that this crucially hinges on the stationary distribution of cash ratios, which a model without real frictions typically misses.

---

32Extreme calibrations of the exponents of production function and the discount factor can generate the empirically observed aggregate cash ratio but miss other relevant targets as the sales to tangibles ratio and the aggregate labour share. An alternative approach departs from the assumption that cash earns no interest; in this setting, cash is held via a risk-free bond as in Riddick and Whited (2009).
Figure 7: Impulse response functions to a credit supply shock - alternative versions of the model

Note: The economy starts at the stationary distribution and experiences a credit tightening in quarter 1, lasting 5 quarters. Previous notes on the simulation apply. $\chi$ and $\kappa$ have been set to 0 in all versions of the model except the benchmark. Except for the dash-dotted blue model, all the other variants have been recalibrated such that the economy matches the empirical average cash ratio in steady state. The recalibrated $\phi_H$ is 0.15 in the economy with no real frictions, and in the one with all capital adjustment costs, and 0.13 in the economy with only partial irreversibility of capital.
Figure 8: Model with only capital adjustment costs

Adding capital adjustment costs contributes to a more reasonable response of average cash ratio. Capital also falls persistently, with a milder fall when there are fixed costs of adjustment. Neither model, however, generates a large prolonged fall in aggregate employment that we see in the data, and which is produced by the benchmark model. Without labour and dividend costs, firms can adjust rapidly to the change in credit conditions, mainly because labour is not a state variable. Most of the aggregate dynamics are driven by constrained firms, as shown in figure 8 for the model with only capital adjustment costs. In particular, constrained firms do not only disproportionately account for the response of aggregate employment upon impact.\footnote{With capital adjustment costs, precautionary motives are present, albeit weak. Unconstrained firms account for 20\% of the fall in aggregate employment in the first quarter.} As shown for the benchmark model in figure 6, their employment growth rates also quickly overshoots steady state counterparts already in the second quarter of the crisis. Differently from the benchmark model, however, unconstrained firms here do not offset this recovery. The absence of a prolonged effect on unconstrained firms’ employment growth rates lies behind the short-lived aggregate dynamics of employment in the alternative models.

The outperformance of the benchmark model is not driven by the specific parameterisation of labour adjustment costs. In appendix D, I show that alternative calibrations of the adjustment costs imply a worse fit of microeconomic over-identified moments. Increasing...
The bite of labour market frictions make the precautionary channel slightly stronger, and generate aggregate dynamics that are even closer to the data – in particular, a stronger fall in capital investment and a larger increase in average cash ratios. Hence, the benchmark calibration can be seen as conservative along this respect.

The alternative versions of the model also generate microeconomic predictions that are at odds with the data, as shown in Table 4. First of all, they are not able to generate the empirically observed right tail of the cash ratio distribution. This is true in spite of the fact that we recalibrate $\phi$ to match the empirically observed average cash ratio. The economy without real frictions counterfactually pushes up the cash ratios at the low end of the distribution, while leaving its symmetry basically unaffected. Adding capital adjustment costs increases the skewness of the cash ratio distribution, but much less than what observed in the data and what generated by the benchmark model.

Alternative models also perform poorly when looking at the relationship between cash ratio and other firm states. For example, they wrongly predict that large firms will hold relatively more cash. The model without real frictions is not able to generate the empirical finding that capital intensive firms are also more illiquid. Adding capital adjustment costs correctly reverts the sign, but strongly over-predicts this correlation. Finally, $\kappa$ affects the correlation between dividends and sales, although the direction of this effect strongly interacts with other frictions in the model. A monotonic pattern emerges when looking at the scaled volatility of dividends, $\frac{E\sigma(d_{it})}{E d_{it}}$. Without real frictions, dividends strongly fluctuate in response to idiosyncratic productivity shocks. Adding capital adjustment costs makes dividends more sticky. The dividend friction considered in this paper brings volatility...
further down, although it still overshoots what observed in the data.

6 Conclusions

The quantitative role of firm-level financial frictions and credit supply shocks in affecting the real economy has been recently object of debate in the literature. In this paper I argue that a precautionary mechanism, which induces firms to respond to changes in credit conditions in anticipation of future idiosyncratic shocks, plays a quantitatively important role. This paper entails two main contributions. Using UK firm-level data, I document that firms accumulated cash during the last recession and cash-intensive firms decreased their workforces by less. Motivated by these facts, I build a heterogeneous-firm model where precautionary cash holdings arise endogenously from the interaction between real and financial frictions. The model is disciplined by UK data and its ability to replicate empirical facts relies on the precautionary channel. Credit tightening implies a sizeable fall in aggregate employment and a substitution from capital to cash. As a key result, these aggregate dynamics are driven by firms not facing a currently binding constraint, who behave pre-emptively in anticipation of future shocks.
References


Appendix

A The data

The primary data source used in this paper is FAME dataset, gathered by Bureau van Dijk. It contains information on over 9 million companies in UK and Ireland, 2 million of which are in detailed format, over the period 2004-2013.\(^{34}\) I restrict the dataset to UK only. A standard company report includes a balance sheet, profit and loss account, turnover, employees and industry codes.\(^{35}\) In contrast to other datasets as US Compustat, 93.7% of the firms contained in the FAME sample are non-publicly traded.\(^{36}\) This implies that there is a large number of small and medium-sized companies.\(^{37}\) Since the model does not feature life cycle, I restrict the sample to a balanced panel; firms that have weakly positive observations for employment, cash and total assets are kept in the sample. Following the standard procedure employed in similar studies, I exclude from the sample firms with UK SIC code referring to "Financial and insurance activities". The final sample consists of 17,762 firms each year. Although the reporting requirements slightly bias the sample towards large firms, the size distribution is much closer to the UK universe than a dataset with only publicly quoted firms. For instance, the median firm in 2006 had 77 employees. The sample is representative also in terms of aggregate dynamics. The evolution of aggregate employment, for instance, closely resembles the one for Non-financial corporations published by the UK Office for National Statistics. Cash is recorded in firm’s balance sheets as Bank Deposits, which is the British format for cash & equivalent. Hence, this definition should already account for a potential substitution among cash securities. Moreover, the average share of short-term investments to total assets stays constant between 2009 and 2010 and even rises in the following year. This further excludes that the increase in cash is driven by a reduction in other liquid assets. Net job creation is defined as the difference in number of employees for a given firm from one year to the other. I define employment growth for a firm \( j \) at year \( t \) as \( \Delta n_{jt} = \frac{n_{jt} - n_{j,t-1}}{\alpha n_{jt} + (1-\alpha)n_{j,t-1}} \), with \( \alpha = 0.5 \). Moscarini and Postel-Vinay (2012) explain the advantages of this symmetric approach, which bounds employment growth between -2 and 2. Finally, I define investment ratio

---

\(^{34}\) A maximum of 10 years data history can be downloaded at once. Companies are registered at Companies House in the UK.

\(^{35}\) Some firms report also the Cash Flow statement. Moreover, the data includes detailed ownership and subsidiary information.

\(^{36}\) This share is comparable to recent studies that use the FAME database, as Brav (2009) and Michaely and Roberts (2011).

\(^{37}\) Unlike in the US, UK firms have to disclose their accounts even when not traded on the stock market. Following the UK Companies Act 1985, large firms have to report detailed accounts, whereas medium-size companies do not have to disclose turnover details and small firms are required to submit only an abridged balance sheet.
using the same strategy as for employment growth. Investment ratio for a firm \( j \) at time \( t \) is defined as \( \frac{k_{jt} - k_{jt-1}}{ak_{jt}+(1-\alpha)k_{jt-1}} \), with \( \alpha = 0.5 \), where \( k \) is fixed assets as recorded at balance sheet.

\section*{B Numerical Method}

The firm's problem is solved with value function iteration. The AR(1) process for the the log of idiosyncratic productivity is discretized using Tauchen and Hussey (1991) method over 7 grid points. In the spirit of Khan and Thomas (2013), I specify the value function over \( (n_{t-1}, m_k, k_t, z_t) \). Using \( m_k \) allows me to restrict the knot points to the feasible set.

The choice dimension is instead specified over a much finer grid. The choice grid for capital always comprises the inaction decision \( k_{t+1} = (1 - \delta_k)k_t \); this is quantitatively important given the capital adjustment costs. The choice grid for labour exploits the features of the financial constraint and thus has a \( (N_k, N_m, N_\phi) \) dimension which depends on the dimension of the state grid for capital, cash and \( \phi \). This also allows to account for a binding financial constraint exactly. Stochastic shocks are discretized as explained in section 4.1. Having defined the value function, I iterate on the Bellman equation until convergence. At each round of iteration, the value function is interpolated using linear interpolation techniques, to accommodate the discrepancy in the number of grid points between states and choices. Linear interpolation has the advantage of preserving the shape of the policy functions and the kinks arising from the constraints that characterize the model.

For the scope of the calibration and the quantitative results, as set out in section 4, the model is simulated over a large number of firms. The transition back from fine choices to coarser states is implemented using a nearest neighbour approach; the simulation keeps track of sequential inaction choices and adjusts the policy functions accordingly.

Six parameters are calibrated simultaneously so that the stochastic steady of the model matches six empirical moments. I solve the dynamic program allowing for aggregate uncertainty, I then fix the policy functions to the steady state aggregate credit tightness \( \phi_H \) and find the stationary distribution over \( (m, k, n, z) \). I compute the model moments and compare to the data moments. The simulation results shown in the paper refer to a credit tightening shock lasting 5 periods. I simulate the economy for 200,000 firms over 400 quarters, allowing for aggregate uncertainty but fixing \( \phi = \phi_H \). I then consider a credit shock such that \( \phi \) drops to its low value, stays there for 5 quarters and rebounds back to its steady state. I repeat this simulation for 100 economies, different only with respect to the realisations of the idiosyncratic productivity shock. For the aggregate results, I take the average of aggregate quantities across the economies. An alternative approach lets the
aggregate credit shock evolve naturally after a one-period shock. I show in Appendix E the impulse response functions for this case.

The time period in the model is a quarter, and the results shown in the paper follow this frequency. Little information on the frequency of the decision making at firm level is known (Bloom (2009)). I decide to strike a balance between monthly frequency of board meetings in public firms and the annual balance sheet data. When required, model-generated quarterly data is converted into annual figures using standard accounting techniques. Flow figures from the Income Statement are added across the quarters of the year, stock figures from the Balance sheet are taken from the year end values. As reported in FAME company reports, the number of employees is the average over the accounting year.

C The firm’s problem

As mentioned in section 3.5, non-smooth labour and capital adjustment costs raise potential concerns with respect to the differentiability of the value function. As shown by Cui (2017), the value function $V(m_t, k_t, n_{t-1}, z_t; \phi_t)$ is differentiable at $k_t > 0$ and satisfies the envelope condition.\footnote{The differentiability of $V(m_t, k_t, n_{t-1}, z_t; \phi_t)$ when $k_{t+1} \neq (1 - \delta_k)k_t$ and $n_t \neq (1 - \delta_n) n_{t-1}$ is standard, as proved by Benveniste and Scheinkman (1979). The differentiability at $k_{t+1} = (1 - \delta_k)k_t$ and $n_t = (1 - \delta_n) n_{t-1}$ can be proved using methods from Clausen and Strub (2012), as shown by Cui (2014). The intuition is that the value function is super-differentiable, but also sub-differentiable, given the potential downward kink stemming from the adjustment costs. Being both super-differentiable and sub-differentiable implies the differentiability of the value function.}

The first order conditions that pin down the optimal decisions for dividends, labour, capital and cash respectively, of a firm with idiosyncratic productivity $z_{j,t}$, are shown below. Equation (7) is always binding, which allows to combine it with (8) in (2). With a slight abuse of notation, $V'$ is a compact form for $V(m_{t+1}, k_{t+1}, n_t, z_{t+1}; \phi_{t+1})$.

\begin{align*}
1 - 2\kappa d_t &= \lambda_t \quad (18) \\
\lambda_t \omega_{nt} + \beta \frac{\partial EV'}{\partial n_t} &= \rho_q + \lambda_t \left[\rho + AL_{nt}(n_{t-1}, n_t) + AC^F_{yt}(k_t, k_{t+1}, y_t) \omega_{nt} \right] \quad (19) \\
\beta \frac{\partial EV'}{\partial k_{t+1}} &= \lambda_t \left[AC^F_{k_{t+1}}(k_t, k_{t+1}) + AC^F_{k_{t+1}}(k_t, k_{t+1}, y_t) \right] \quad (20) \\
\beta \frac{\partial EV'}{\partial m_{t+1}} &= \lambda_t - \psi_t \quad (21)
\end{align*}
And the envelope conditions for labour, capital and cash are:

\[ V_{n_{t-1}}(m_t, k_t, n_{t-1}, z_{j,t}; \phi_t) = -\lambda_t AL_{n_{t-1}}(n_{t-1}, n_t) \]  

(22)

\[ V_{k_t}(m_t, k_t, n_{t-1}, z_{j,t}; \phi_t) = \lambda_t \left[ \frac{V}{k_t} - AC^F_k(k_t, k_{t+1}) - AC^F_k(k_t, k_{t+1}, y_t) \right] + \phi_t (1 - \vartheta)(1 - \delta) \mu_t \]  

(23)

\[ V_{m_t}(m_t, k_t, n_{t-1}, z_{j,t}; \phi_t) = \lambda_t + \mu_t \]  

(24)

Combining equations (18-24) gives the first order conditions (11-14) shown in section 3.5. Following Cui (2014), it is possible to further decompose the derivatives with respect to labour and capital adjustment costs. For instance, let \( q(m_t, k_t, n_{t-1}, z_{j,t}; \phi_t) \) be the marginal value of capital that satisfies the envelope condition, which we shall refer to as \( q_t \) thereafter. Then, equation (23) can be rewritten as:

\[ V_{k_t}(m_t, k_t, n_{t-1}, z_{j,t}; \phi_t) = \lambda_t \left[ \frac{V}{k_t} - AC^F_k(k_t, k_{t+1}, y_t) + 1 \right] + q_t (1 - \delta) \mu_t + \phi_t (1 - \vartheta)(1 - \delta) \mu_t \]  

(25)

Intuitively, \( q_t \) is the marginal reward of adjusting capital. When it reaches 1, a firm buys capital. The lower bound of \( q_t \) is instead \( 1 - \vartheta \); selling capital is associated to this marginal reward to decrease capital. When the firm is inactive in its capital investment decision, \( q_t \) is less than 1 and greater than \( 1 - \vartheta \). Inside the inaction region, \( q_t \) is the option value of remaining inactive.\(^{39}\)

D Alternative parameterisations of the adjustment costs

In the calibration exercise outlined in section 4.1, labour and capital adjustment costs are not calibrated to the UK economy, but rather inherited from Bloom (2009). This may pose some concerns about the quantitative performance of the model, since labour and capital adjustment costs are important drivers of the model mechanisms. In this section, I evaluate the model performance using different estimates available in the literature. Table 5 shows how models with different labour and capital adjustment costs would imply a worse fit of the over-identifying moment restrictions.

Without fixed adjustment costs of capital, the model predicts an excessively negative correlation between cash ratios and capital to labour ratios, more so with stronger partial irreversibility. When capital is only mildly irreversible, however, this correlation becomes

\(^{39}\)As in the main text, \( AC^F_k \) is the total derivative of the fixed adjustment cost with respect to capital, incorporating the indirect effect via the production function. Similarly, the intuition about \( q \) disregards this channel.
Table 5: Model performance with alternative adjustment costs

<table>
<thead>
<tr>
<th>χ</th>
<th>φ</th>
<th>Θ</th>
<th>corr($\frac{m}{m+k}$, $\frac{k}{n}$)</th>
<th>corr($\frac{m}{m+k}$, n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.072</td>
<td>34%</td>
<td>0.06</td>
<td>-0.11</td>
<td>-0.07</td>
</tr>
<tr>
<td>0.072</td>
<td>34%</td>
<td>0</td>
<td>-0.44</td>
<td>0.36</td>
</tr>
<tr>
<td>0.32</td>
<td>34%</td>
<td>0.06</td>
<td>-0.10</td>
<td>-0.15</td>
</tr>
<tr>
<td>0.072</td>
<td>2.5%</td>
<td>0</td>
<td>0.37</td>
<td>0.36</td>
</tr>
<tr>
<td>0.072</td>
<td>50%</td>
<td>0</td>
<td>-0.62</td>
<td>-0.10</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td>-0.13</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

Note: corr($\frac{m}{m+k}$, $\frac{k}{n}$) is the cross-firm correlation between cash ratio and capital to labour ratio. corr($\frac{m}{m+k}$, n) is the cross-firm correlation between cash ratio and employment. Labour (χ) and Capital (θ, Θ) adjustment costs taken from other estimates often used in the literature. The first row refers to the benchmark calibration used in this paper. In the second row from the top, I get rid of fixed adjustment costs of capital. Nickell (1986) estimates partial irreversibility of labour at 8% of annual wage. In the third row, I use his parameterisation for labour adjustment costs and add it to the benchmark specification of capital adjustment costs. Cooper and Haltiwanger (2006) estimate partial irreversibility of capital at 2.5%, whereas Ramey and Shapiro (2001) between 40 and 80%. I consider these two estimates in the last two rows, before showing the data. Moments reported in the table are calculated from the stationary distribution of the stochastic steady state of the model, when credit conditions are at $\phi_H$.

Nickell (1986) is an especially interesting comparison, given the quantitative importance of labour adjustment costs for the precautionary channel. The table shows that both moments are not affected much by larger labour adjustment costs. To further explore the reliance of the main results to the parameterisation of the adjustment costs, I recalibrate the model in order to correctly match the targeted moments. In particular, I increase ν to 0.15, κ to 0.68 and decrease $\sigma_z$ to 0.105. Large labour adjustment costs make the distribution of cash ratio bi-modal, with a non-negligible portion of firms holding mostly cash. This implies it is difficult to simultaneously match the cash ratio distribution and the correlation between dividends and sales. The latter is biased upwards with respect to the data, at 0.81. In spite of much stronger labour market frictions, the main results shown in section 4 and 5 hold. Aggregate variables follow similar dynamics to those shown in figure 4. The fall in employment is 0.5 percentage points smaller at the trough, but more persistent. In contrast, aggregate capital falls as much as 2.4%, more than twice than in the baseline model. Average cash ratio also increases more, up to 95 basis points. The cross-firm correlation between lagged cash ratios and employment growth also increases during the credit crunch and then falls. Differently from the baseline model, the steady
state correlation is closer to 0, and the dynamic fluctuations are less pronounced. Finally, tighter labour market frictions further reduce the share of currently constrained firms to a maximum of 13%. The contribution of unconstrained firms to the upon impact fall in aggregate employment is 45%, just a couple of percentage points higher than in the benchmark model, and the evolution in the following periods is similar. Overall, stronger labour adjustment costs leave the main results of this paper unaffected. If any, the implied aggregate dynamics are closer to the data. The benchmark calibration can be therefore seen as conservative.

E Additional results

The simulation results shown in this paper all refer to a credit supply shock that lasts 5 quarters. Figure 9 shows an alternative approach. 100 Economies are simulated over 200,000 firms for 400 quarters, allowing for aggregate uncertainty in the solution but fixing $\phi = \phi_H$ in the simulation. I impose a credit shock in the same quarter for all economies. $\phi$ is allowed to evolve naturally in each economy from this quarter onwards. Then I compute the average and median aggregate levels across economies and plot the percent deviation from the pre-shock quarter.

In figure 10 I show the performance of the model against the data for additional aggregate dynamics. In the first year, the model generates a large fall in aggregate output. This is smaller than the fall in employment, relative to the data, because investment falls one third of the empirically observed drop. When all the real frictions are lifted, investment increases, at odds with the data.

As in the data, firms entering the credit crunch with a low cash ratio, cut employment the most, see figure 11. The model does a strikingly good job at matching the size of this effect relative to the data. When ordinary credit conditions are restored, all firms start the recovery within one year in the model. In the data, instead, very liquid firms grow less than the rest of the population, generating the negative correlation shown in figure 3a.
Figure 9: Aggregate responses to a credit supply shock

Note: Simulations over 200,000 firms for 400 quarters, allowing for aggregate uncertainty in the solution but fixing $\phi = \phi_H$ in the simulation. Simulations have been repeated for 100 economies. I impose a credit shock in quarter 1, allowing $\phi$ to evolve naturally thereafter. Then I compute the average and median aggregate levels across economies and plot the percent deviation from quarter 0.
Figure 10: Model performance to the UK economy

Note: Model-generated data refer to the same simulation outlined in Figure 4. Quarterly simulated data have been annualised using standard accounting techniques, as explained in Appendix B. The credit shock hits the economy in the first quarter of 2009. In the "Non-precautionary model", $\chi$, $\vartheta$, and $\kappa$ have been set to 0, while the remaining parameters are the same as in Table 1. Aggregate output is UK GDP in chained volume, from the ONS. GDP has been converted in logs, de-trended with HP filter over the period 1950-2016, and then plotted as percentage point deviations from the de-trended value in 2008. UK aggregate investment is Business Investment (ONS).

Figure 11: Employment growth by cash ratios: model and data

Note: Same notes as figure 10 and 3b.