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Abstract

We offer a parsimonious model of the reserve demand to study the trade-offs associated with various monetary policy implementation frameworks. Our model considers a reserve demand function that encompasses banks' preferences for reserves in the post 2007-2009 financial crisis world and incorporates shocks to the demand for and the supply of reserves. We find that the best policy implementation outcomes are realized when reserves are somewhere in between scarce and abundant. This outcome is consistent with the Federal Open Market Committee's 2019 announcement to implement monetary policy in a regime with an ample supply of reserves.

Key words: federal funds market, monetary policy implementation, ample reserves

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1 Introduction

A monetary policy implementation framework describes the targets and tools a central bank uses to transmit its desired stance of monetary policy to financial markets and the real economy. An implementation framework specifies, e.g., a target interest rate, reserve requirements, the rate of remuneration on reserves, the discount rate, tools—such as open market operations—used to adjust the quantity of reserves, parameters associated with standing repo and reverse repo facilities, the issuance of central bank bills and so on. Over time and across jurisdictions central banks have chosen a wide variety of monetary implementation schemes. Although monetary economics typically abstracts from implementation issues by assuming that the central bank achieves its policy stance by choosing *the* policy interest rate, such an assumption is not at all innocuous. In practice, the way in which monetary policy is implemented can constrain the choice of feasible policy stances and, conversely, the choice of a policy stance has implications for how the policy should be implemented. This paper attempts to shed some light on what constitutes an optimal implementation framework.

The 2007-2009 financial crisis and its aftermath highlight some of the interactions between a policy stance and implementation strategies. A conventional policy response at the outset of the financial crisis would call for negative nominal policy rates. However, it is challenging to implement non-negligible negative nominal policy rates in an economy with physical currency. As a result, many central banks adopted new policy tools, such as forward guidance and large-scale purchases of long-dated assets—quantitative easing—to provide additional stimulus (Bernanke, 2020). While large-scale asset purchases were effective at easing financial conditions, they also dramatically increased the amount of reserves supplied to the banking system. The pre-crisis tools for controlling overnight interest rates—such as open market operations via overnight repo and reverse-repo transactions—became ineffective since open market operations that result in small changes in the supply of reserves cannot affect overnight rates as reserves are no longer scarce. In response to the impotence of traditional tools, central banks introduced new and additional ones, such as payment of interest on reserves and overnight repurchase and reverse repurchase facilities, to better control the policy rate (Ihrig et al., 2015).

The framework for monetary policy implementation continues to evolve to this day. In the months prior to the coronavirus pandemic, central banks around the world were unwinding their responses to the financial crisis and "normalizing" their policy stances. However, the disruptions in money markets in September 2019 and those associated with the Covid-19 pandemic temporarily stalled the normalization process and led policymakers to once again reassess their implementation frameworks. As economic conditions improved, central banks resumed reducing the size of their balance sheets, bringing on board lessons learned since September 2019 that will likely shape the narrative for monetary policy implementation frameworks into the foreseeable future.

In this paper, we develop a simple model of the banking system's demand for reserves to better understand the choices policy makers face when selecting a monetary policy implementation framework. Throughout we focus on the experience of the Federal Reserve to create an intuitive link between theory and practice. Our model builds on the seminal work of Poole (1968): a model where banks hold reserves to meet reserve requirements, borrow or lend them in an interbank market to adjust end-of-day reserves, and face a late-period payment shock that can drain reserves after the interbank market closes and force banks to borrow from the central bank discount window.[1] We generalize the Poole (1968) model along several dimensions. First, we consider a reserve demand function that captures banks' preferences for reserves in the post-crisis world beyond required reserves. Second, we include shocks to banks' reserve demand functions that reflect the increased uncertainty associated with estimating the banking system's demand for reserves in the post-crisis period. And finally, we introduce shocks to supply of reserves to incorporate uncertainty that arises from factors outside the Federal Reserve's control, such as changes in the balances that the Treasury Department holds at the Federal Reserve or in the balances held at the overnight reverse repo facility, both of which have become rather pronounced and important in the post-crisis period.

Our model generates a downward sloping reserve demand curve with three main regions—a region of "high" aggregate reserves, a region of "low" aggregate reserves and a smooth transition between the two—that is consistent with the demand for reserves estimated by Afonso et al. (2023b). Through the lens of our model, we define the amount of reserves that a central bank supplies to the banking system as abundant, scarce or ample. Reserves are *abundant* when the equilibrium is characterized by no interest rate volatility. Such an

[1]Many models of monetary policy implementation use the Poole model as their starting point. See, for example, Ennis and Keister (2008), Keister et al. (2008), and references therein.

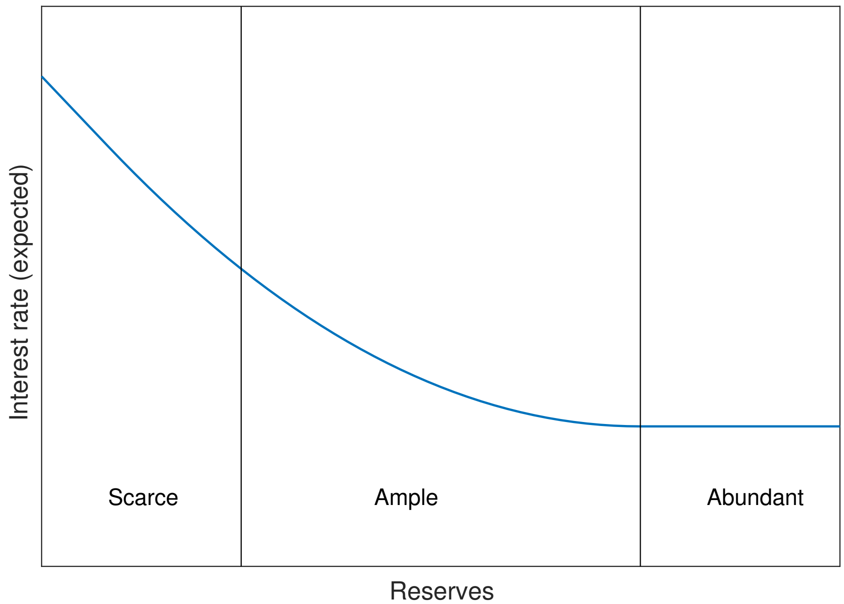

Figure 1: Reserve Demand

equilibrium occurs only if the central bank supplies large quantities of reserves. In Figure 1, the demand curve is flat when aggregate reserves are abundant since banks are able to meet their internal and external requirements for any (ex post) reserve demand shock or supply shock realization. When reserves are abundant, the value of trading reserves is simply the interest paid on overnight reserve balances. In contrast, reserves are *scarce* when the equilibrium displays high-rate volatility, even when the central bank undertakes open market operations in an attempt to stabilize rates. Intuitively, this equilibrium occurs when reserve supply is small and the equilibrium rate is on the downward-sloping part of the demand curve, as illustrated in Figure 1. In the region of scarce reserves, the marginal value of reserves increases as aggregate reserves decline and exceeds the interest paid on overnight reserves. Finally, in between scarcity and abundance, we define reserves to be *ample*. In this region, reserve supply and demand shocks result in a positive but suppressed equilibrium interest rate volatility.[2]

We use our model to study the choice of a central bank's monetary policy implementation framework and explore trade-offs between interest rate control, financial stability, and active management of reserve balances. In practice, policy makers have preferences over outcomes

[2]These definitions—scarce, abundant, and ample—are our own, and are intended to facilitate the discussion of central banks' implementation choices from within the perspective of our model.

and operations and, because of this, they face trade-offs when choosing their implementation framework. For example, policy makers may prefer low, rather than high, volatility in their policy rate. They may also prefer smaller, less frequent open market operations in response to reserve supply and demand shocks to larger or frequent operations. A central bank could achieve a low volatility policy rate, without relying on open market operations, by supplying a very large amount of reserves to the banking sector. But, if policy makers also prefer smaller to larger balance sheets, then a large balance sheet—that stabilizes the policy rate—may not necessarily constitute an optimal framework from the central bank's perspective. We capture these policy-maker preferences as a linear combination of costs associated with: (i) volatility in the policy rate; (ii) the size of the central bank's balance sheet; and (iii) the expected size of open market operations. The central bank's implementation framework specifies the initial quantity of aggregate reserves—small, moderate, or large—and the size and frequency of subsequent open market operations. The *optimal* implementation framework is the one that minimizes a linear combination of these costs.

Since the 2007-2009 financial crisis, the magnitudes of the shocks to the reserve supply have increased substantially and a new set of drivers of reserve demand have also emerged, both of which decrease the predictability of reserve supply and demand.[3] We show that in the post-financial crisis world the optimal monetary policy framework has aggregate reserve supply in the intermediate region, between scarcity and abundance. These findings are consistent with the Federal Reserve's plans to implement monetary policy over the longer run in an environment of ample reserves.[4] An important implication of this framework is that high frequency, active adjustment of the reserve supply is not needed to implement policy, although occasional adjustments may arise.[5]

[3]Post-crisis liquidity regulations, such as the Liquidity Coverage Ratio (LCR), living wills, stress testing, as well as banks' responses to the regulation via internal liquidity management strategies and targets, have transformed the demand for reserves. In recent years, changes in reserve supply due to factors outside of the control of the central bank—mainly, balances in the account of the U.S. Treasury or at the overnight reserve repo facility—have increased too, making reserve supply also more uncertain.

[4]In January 2019, the Federal Open Market Committee (FOMC) announced its intention to maintain an "ample supply of reserves" and to use administered interest rates, such as the rate paid on reserves, as its primary tools to ensure rate control. Our model predictions are also consistent with the Federal Reserve's longstanding plan to operate with a balance sheet that is no larger than necessary for efficient and effective policy implementation, see "Policy Normalization Principles and Plans," September 2014, available at https://www.federalreserve.gov/monetarypolicy/policy-normalization-discussions-communications-history.htm.

[5]See "Statement Regarding Monetary Policy Implementation and Balance Sheet Normalization." https://www.federalreserve.gov/newsevents/pressreleases/monetary20190130c.htm.

The next section provides a brief overview of the banking system's demand for reserves in the pre- and post-financial crisis periods, as well as a discussion of reserve supply in the post-crisis period. Section 3 introduces our model of a monetary policy implementation framework and provides insights into a central bank's choice of the optimal framework. Section 4 then focuses on how effective these regimes have been in practice and discusses potential financial stability implications of regimes with high reserves. Section 5 concludes.

2 Reserve Demand and Supply

To motivate our model and analysis, we first discuss the impact that post-crisis liquidity regulations have had on the banking sector's demand for reserves and how these regulations, along with other considerations, make it more challenging for a central bank to estimate total reserve demand. We then document changes in the supply of reserves for the pre- and post-crisis periods in the U.S. and show that reserve supply volatility has substantially increased in the post-financial crisis period.

2.1 Reserve Demand

Prior to the 2007-2009 financial crisis, the Federal Reserve and clearing banks provided intraday liquidity at generous terms so that banks could almost costlessly smooth out their daily payment flow obligations. As a result, banks demanded reserves mainly to satisfy their end-of-day reserve requirements. Since the primary driver for pre-crisis reserve demand was banks' reserve requirements, the Federal Reserve was able to estimate banks' total demand for reserves with a high degree of precision.

Post-crisis regulations have directly and indirectly affected banks' liquidity risk management in ways that have resulted in new and more uncertain sources of demand for reserves. For example, the liquidity coverage ratio (LCR) requires banks to hold a sufficient amount of high-quality liquid assets (HQLA) to meet net cash outflows over a thirty-day stress period. HQLA include central bank reserves and government securities, as well as some other safe and liquid assets. The LCR regulation implies that banks' demand for the sum of reserves *and* government securities will be higher than the pre-crisis period. Banks are, however, free to allocate their HQLA holdings between government securities and reserves as they see fit: the LCR does not, *per se*, specify any requirements about reserve holdings *vis-à-vis* government securities holdings. Since the LCR allows banks to choose different mixes of reserves and other types of HQLA to satisfy the requirements, the Federal Reserve may be unable to estimate banks' total demand for reserves with a high degree of precision, even though banks' demand for HQLA can be. For example, a bank may prefer to hold more government securities than reserves if the yields on government securities are relatively high but may suddenly reverse this preference if the bank believes that it might be challenging to quickly convert government securities into cash in the face of outflows.

In addition to the increase in demand for reserves associated with regulatory requirements such as the LCR, resolution plans under the Dodd-Frank Act, buffers of highly liquid assets under Regulation YY, and banks' internal liquidity stress tests also increase banks' demand for reserves. Importantly, the impact on the demand for reserves depends on banks' own risk assessments and their willingness to bear these risks. Owing to these new and more complex sources of demand for bank reserves, a central bank's ability to accurately predict reserve demand has been reduced in the post-financial crisis period.

In Section 3, we model the increased uncertainty regarding banks' reserve demand in the post-crisis period as an increase in the magnitude of shocks to reserve demand and as a decrease in the central bank's ability to predict the reserve demand. The central bank in our model determines the optimal implementation regime taking this increased uncertainty as an exogenous change. This approach is motivated by the observation that regulatory changes, such as the liquidity coverage ratio (LCR), were introduced for financial stability purposes, largely independent of monetary policy implementation considerations.

2.2 Reserve Supply

Traditional models typically assume that the reserve supply is under the complete control of the central bank. In practice, however, the level of reserves can change due to factors that are outside of the control of the central bank, the so-called autonomous factors. Two important examples of these factors in the U.S. are the balances at the overnight reverse repo (ON RRP) facility and the balances that the Treasury Department holds at the Federal Reserve. In this section, we show that variations in reserve supply due to autonomous factors have become much larger in the U.S. in recent years. This development poses challenges for monetary policy implementation when reserves are scarce and small changes in reserves affect the policy rate.

In the absence of offsetting open market operations, reserves available to banks change on a daily basis. Reserves may change for two reasons: First, the size of the central bank balance sheet may change as the result of, e.g., large-scale asset purchases. Second, the composition of the central bank liabilities may change when, e.g., bank reserves are converted into physical currency, which is also a liability of the central bank, or when currency is returned to the central bank and the returning bank's account at the central bank is credited (with reserves). Another example of changes in reserves occurs on tax payment dates when funds from a bank's account at the central bank are used to pay taxes, which reduces reserves in the banking system and increases the balance of the Treasury General Account (TGA) at the central bank. Prior to the 2007-2009 financial crisis, these exogenous, day-to-day changes in the supply of reserves were small and mostly predictable. Since then, the volatility of these changes has increased significantly.

The first 5 panels in Figure 2 illustrate the weekly volatility of selected autonomous factors from 2003 to 2022. Notice that the volatility of these factors have significantly increased since 2009. This, in turn, means that the volatility of the supply of reserves, the bottom-right panel in Figure 2, has also increased substantially. The volatility of the reserve supply was close to zero prior to the 2007-2009 financial crisis, partly because the Federal Reserve actively offset movements in the supply of reserves. However, given the much smaller volatility of autonomous factors in the pre-crisis period, the reserve volatility would have been much lower than it has since 2009 even in the absence of the Federal Reserve's operations.

The Federal Reserve could, in principle, reduce the volatility of autonomous factors, at least in a limited way.[6] It is not, however, obvious that reducing volatility is desirable. For example, since 2015 the Treasury Department has tried to maintain a five-day liquidity buffer in its account at the Federal Reserve to limit the risk that it may be unable to access markets due to an operational outage or a cyber-attack.[7] While such a buffer contributes

[6]It would be difficult to regulate the withdrawal and deposit of physical currency. The Federal Reserve could set up rules on the use of accounts held by non-banks. But it would be difficult to force these accounts to substantially reduce their volatility without impairing their operational needs. In fact, free withdrawal and deposit is a primary advantage of holding cash or reserves.

[7]Treasury's May 6, 2015, quarterly refunding statement notes: "Based on our review, the TBAC's [Trea-

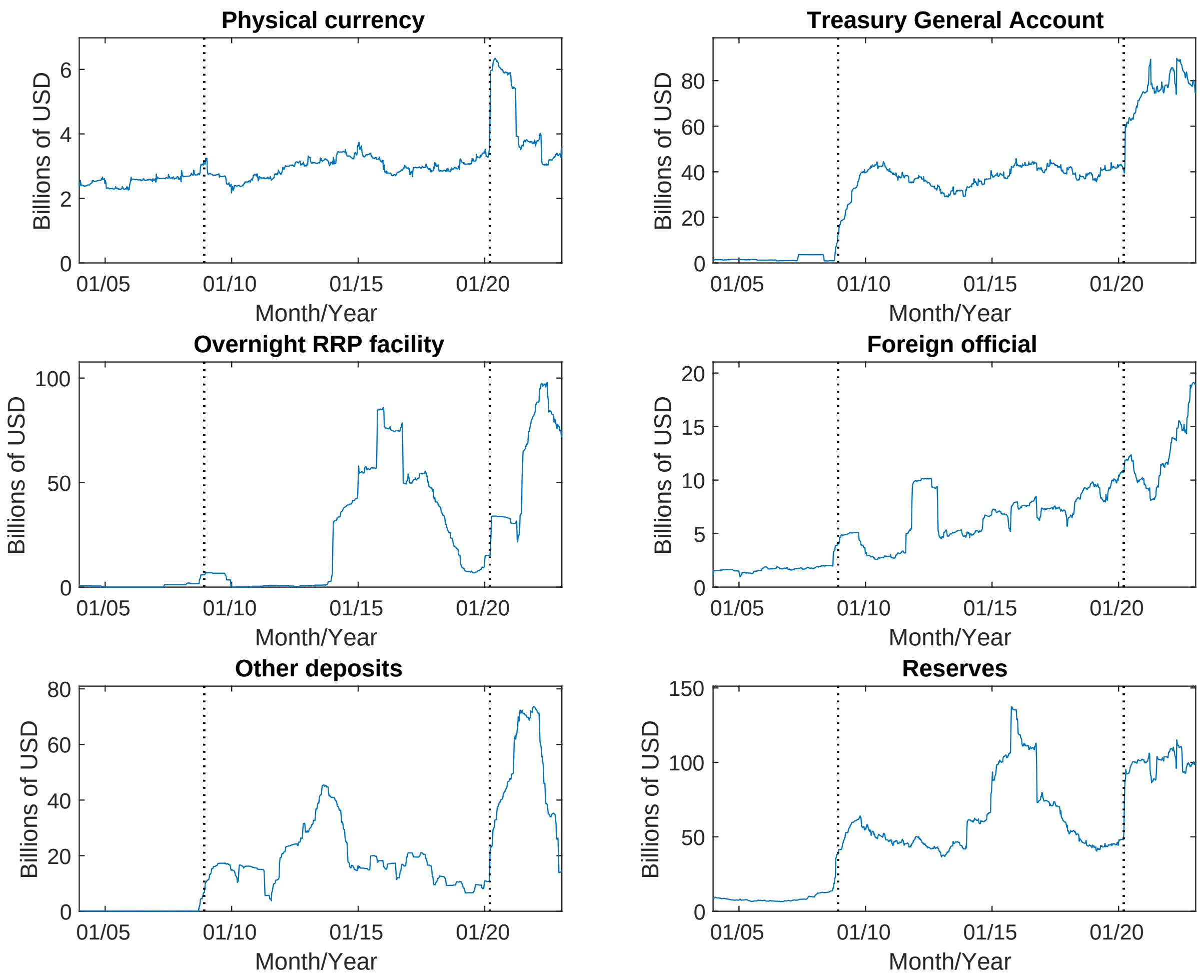

Figure 2: Volatility in Selected Autonomous Factors and Reserves in the United States

The volatility of autonomous factors and reserves generally increased since 2009, albeit not monotonically. Time period covered is 2003 to 2022. Volatility is calculated as the standard deviation of weekly differences over a 52-week trailing window, using publicly released weekly snapshots of Federal Reserve's liability (H.4.1 releases from the Federal Reserve). Vertical lines mark the beginning of asset purchases in response to the 2007-09 financial crisis in late 2008 and to the Covid-19 pandemic in early 2020. Other deposits are held by selected official and private entities.

to a larger and more volatile autonomous factor, it might not be possible or desirable to return to the pre-crisis balance. Alternatively, the Treasury Department could move its buffer into the banking sector, thereby reducing or eliminating this autonomous factor. But banks may not be interested in taking on large and volatile cash deposits since it is costly from both a regulatory and liquidity risk management perspective. Similarly, the volatility of the balances of various non-bank counterparties, such as deposits held by of foreign central banks, governments, and monetary authorities or by government-sponsored enterprises (see the mid-right and the bottom-left panels of Figure 2) might be reduced by establishing restrictions on account usage, but doing so would make these deposits less attractive to their holders. In addition, while in a frictionless world where reserves can be costlessly transferred to and from the ON RRP facility, the volatility of ON RRP balances could be considered irrelevant—endogenous to the supply of other Federal Reserve liabilities— in reality the volatility of the ON RRP balances remains a source of reserve supply shocks due to frictions in money markets.

As illustrated in the bottom-right chart on Figure 2, the weekly volatility of reserves was less than $10 billion prior to the financial crisis. Volatility increased sharply during the first round of large-scale asset purchases to around $60 billion in 2009. For several years after the crisis, reserve supply volatility remained near that level through different rounds of large-scale asset purchases. Between 2015 and 2017, a period between the end of asset purchases and the beginning of balance sheet normalization, reserve volatility was elevated and reached nearly $150 billion. During this period, volatility reflected changes in reserves due to exogenous factors as the Federal Reserve no longer conducted daily open market operations to fine-tune reserve supply nor changed the size of its balance sheet to conduct monetary policy.[8] Volatility fluctuated around $50 billion during balance sheet normalization over 2017-2019, and increased again as the Federal Reserve expanded the balance sheet in response to the Covid-19 pandemic.

sury Borrowing Advisory Committee's] recommendations, and an assessment of emerging threats, such as potential cyber-attacks, Treasury believes it is prudent to change its cash management policy starting this month. To help protect against a potential interruption in market access, Treasury will hold a level of cash generally sufficient to cover one week of outflows in the Treasury General Account, subject to a minimum balance of roughly $150 billion."

[8]Fine-tuning the reserve supply was a key element of the monetary policy implementation framework in the pre-crisis period. Therefore, pre-crisis figures do not reflect volatility in exogenous reserve supply since most of the autonomous changes were "reversed" through open market operations. Similarly, reserve-injecting operations in the aftermath of mid-September 2019 money market volatility tended to offset reductions in reserve supply.

3 A Model of a Monetary Policy Implementation Framework

We propose a model of interbank interest rate determination to study the trade-offs that policy makers face when choosing a monetary policy framework. Our model builds on the work of Poole (1968) and accommodates implementation frameworks with scarce, abundant, and somewhere in between (ample) reserves. As in Poole (1968), banks must hold a minimum level of reserves. In contrast to Poole (1968), where the minimum level is proportional to bank deposits, in our model, the minimum level of reserves captures banks' expanded demand for reserves in a post-financial crisis environment. More specifically, and as discussed in Section 2, changes in (i) liquidity regulation, (ii) supervision of banks, (iii) banks' risk management practices and (iv) the structure of the market for reserves have substantially transformed—and expanded—the banking system's demand for reserves relative to the pre-crisis period. These new and more complex sources of demand for reserves have reduced the central bank's ability to accurately predict demand for reserves. Our model captures this increased uncertainty, which is an important feature of the post-financial crisis period, by introducing shocks to the demand for reserves. These demand shocks decrease the central bank's ability to accurately predict aggregate reserve demand. Moreover, the volatility of factors outside the control of a central bank that affect the supply of reserves has also increased relative to the pre-financial crisis period, as illustrated in Figure 2. We model the increased uncertainty arising from these autonomous factors by introducing shocks to the supply of reserves.[9]

3.1 Agents

There are two types of agents: depository institutions, which we will refer to as "banks," and a central bank.

Banks. There are N banks in the banking system indexed by $i \in \{1, ..., N\}$. Banks are risk-neutral and maximize expected profits. Each bank has an initial level of desired reserves, $\bar{R}_i$, which is exogenous to the model. Each bank receives a demand shock, d_i, that changes

[9]Shocks in Poole (1968) are "late-period" shocks, which redistribute reserves between banks after the interbank market closes. As we discussed in Section 3.2, our model includes this late-period shock, and incorporates shocks in the demand for and supply of reserves.

the bank's desired reserves by d_i, where d_i can be either positive or negative. After the demand shock realizations, banks trade in a competitive interbank market for reserves, e.g., the federal (or fed) funds market in the U.S., to adjust their reserve holdings at rate r. After the interbank market closes, each bank receives a late (Poole) shock, u_i, which redistributes reserves among banks. If this shock causes reserves to fall below $\bar{R}_i + d_i$, then bank i borrows reserves from the central bank at a penalty rate, r_P, to get reserves back to its desired level $\bar{R}_i + d_i$. One can think of the penalty rate as the rate charged by the central bank on discount window loans or the minimum bid rate at the Standing Repo facility. Banks hold all of their reserves overnight at the central bank and earn the interest rate on these reserves equal to $r_{IOR} < r_P$.

Central bank. The central bank chooses an initial level of reserves to supply to the banking system, R. The supply of reserves is subject to a shock, s, which arises from factors outside the central bank's control. After the shock, the central bank either injects, $x > 0$, or drains, $x < 0$, reserves through an open market operation. The central bank lends to banks at a penalty rate, r_P, and remunerates reserve balances that banks hold at the central bank at the rate r_{IOR}.

3.2 Shocks

There are three important shocks at play: a supply shock, a demand shock, and a late (Poole) reserve redistribution shock.

Supply shock. The shock s to the supply of reserves captures that central banks do not perfectly control the supply of reserves. Changes in the balances at the ON RRP facility or at the account that the Treasury Department holds at the Federal Reserve are relevant examples of supply shocks that affect the level of reserves in the U.S. banking system.

Demand shock. The model incorporates a shock, d_i, to a bank's initial demand for reserves, where $d = \sum_i d_i$ denotes the aggregate shock to the banking system demand for reserves. This shock captures the ex ante uncertainty about a bank's demand for desired reserves, as well as the increased uncertainty associated with the post-financial crisis environment.

Late shock. Each bank receives a late shock, u_i. This shock represents increases or decreases in a bank's reserves due to, e.g., a payment that occurs late in the day after the interbank market closes. We can interpret the u_i shocks as a reshuffling of existing reserves among the N banks, where $\sum_i u_i = 0$. Late shocks will further have the indirect effect of increasing the supply of reserves in the system if banks borrow from the central bank at the end of the day.

3.3 Timing of Events

The model is static in the sense that events and actions occur over a finite number of periods, six to be precise. We think of the model as describing what happens over a typical day. The timing of events is as follows:

Time t = 1: The central bank: (i) chooses the level of reserves, R, to supply to the banking system; (ii) specifies the penalty rate, r_P, at which banks can borrow from the central bank; and (iii) sets the interest paid on reserves, r_{IOR}. Each bank has an initial (time $t = 1$) level of desired reserves, $\bar{R}_i$, where $\bar{R} = \sum_i \bar{R}_i$.

Time t = 2: The reserve supply shock, s, is revealed. Total reserves are now $R + s$.

Time t = 3: The central bank either injects $x \geq 0$ or drains $x \leq 0$ reserves. Total, and final, supply of reserves is $R + s + x$.

Time t = 4: Each bank receives a demand shock, d_i, to its initial level of desired reserves. Bank i's desired reserves are now given by $\bar{R}_i + d_i$. The banking system's demand for reserves is $\bar{R} + d$, where $d = \sum_i d_i$.

Time t = 5: A competitive interbank market opens, where banks can borrow and lend reserves at rate r. Reserves are redistributed in the interbank market, and then the market closes. Denote bank i's reserves at the end of period 5 by R_i.

Time t = 6: Each bank i receives a late shock, u_i, to its reserve holdings. If $R_i + u_i < \bar{R}_i + d_i$, then bank i must borrow the difference from the central bank at rate r_P. Banks hold their reserves at the central bank and earn interest r_{IOR}.

The central bank makes decisions about the level of reserves at $t = 1$, when it chooses the initial supply of reserves, and at $t = 3$, when it adjusts this initial level plus the reserve supply shock through an open market operation. When making these decisions, the central bank takes into account banks' expected borrowing and lending in the interbank market at $t = 5$.

Banks' behavior creates a demand for reserves which, along with the central bank's earlier supply decisions, results in an equilibrium interbank (fed funds) rate at $t = 5$. Intuitively, if the central bank cares only about rate control, and there are no costs associated with the size of its balance sheet or with open market operations, then the solution to the central bank's problem is relatively simple: the central bank provides an arbitrarily large supply of reserves—in the abundant region—so that the reserve supply intersects the reserve demand in the far right region where the demand curve is essentially flat in Figure 1. For a large enough level of reserves, the equilibrium interbank rate will (almost always) equal the rate on overnight reserves, r_{IOR}, regardless of the supply, s, and demand, d, shock realizations. In this scenario, the central bank is able to (almost) perfectly control its policy rate with an abundant reserve supply.[10] However, if there are costs associated with the size of the central bank's balance sheet, then having (almost) perfect control over the policy rate may not constitute an optimal implementation framework, as we shall see in Section 3.5.

Before describing and solving the central bank's policy implementation problem, we first characterize the individual bank's reserve holdings problem.

3.4 Reserve Demand

In this section, we first provide the intuition underlying an individual bank's demand for reserves. We then consider a banking system with many banks and sum their individual demands to derive the banking system's demand for reserves (see, e.g., Poole (1968), Ennis and Keister (2008) and references therein). Our formal derivations are relegated to Appendix A.

The bank's problem. Bank i's problem can be characterized as follows: At $t = 5$, bank i chooses between borrowing and lending in the interbank market given its current desired reserves, $\bar{R}_i + d_i$, and knowing that after the interbank market closes at the end of the period, it will receive a late shock u_i. Since the value of the late shock is not known when the interbank market is open, bank i's demand for reserves in the fed funds market depends

[10] If the supports of the random variables s and d are finite, then the fed funds rate always equals r_{IOR} for a sufficiently large (abundant) reserve supply, and the central bank has perfect interest rate control. If the supports of the demand and supply shocks are not finite, e.g., when s and d are normally distributed, then the probability that the fed funds rate does not equal r_{IOR} can be made arbitrarily small by choosing a sufficiently large reserve supply.

on the distribution of the late shocks (and not on the realization of the shock). As in Poole (1968), the shock may push bank i's reserves below its desired level.[11] In this case, bank i borrows from the central bank at a penalty rate, r_P, to bring its reserve holdings up to the desired level. If reserves exceed its desired level, then the bank does not borrow from the central bank. In either case, bank i deposits all of its reserves at the central bank and earns an interest on these balances equal to r_{IOR}.

Let us consider bank i's decision between lending and not lending an additional unit of reserves in the interbank market at time $t = 5$. If bank i lends, then the (gross) payoff associated with this trade is r, the interbank rate. In time period $t = 6$, bank i receives a shock u_i to its reserve holdings, R_i. If its new level of reserves $R_i + u_i$ falls short of its desired level, $\bar{R}_i + d_i$, i.e., if $R_i + u_i \leq \bar{R}_i + d_i$, then bank i must borrow reserves from the central bank at the rate r_P to make up for the shortfall. Hence, with probability $\Pr(R_i + u_i \leq \bar{R}_i + d_i)$ bank i's net payoff from lending a unit in the interbank market is the sum of (i) the interbank rate r from the unit of reserve it lent in the interbank market, (ii) the penalty rate, r_P, it pays from borrowing this unit back from the central bank, and (iii) the interest on reserves, r_{IOR}, it receives when the borrowed unit is deposited at the central bank. And with probability $\Pr(R_i + u_i > \bar{R}_i + d_i)$, the date $t = 6$ late shock does not push bank i's level of reserves below its desired level and bank i's net payoff is simply the interbank rate r it received on the unit it lent out. Thus, the expected net payoff from lending is

$$\Pr(R_i + u_i \leq \bar{R}_i + d_i)(r - r_P + r_{IOR}) + \Pr(R_i + u_i > \bar{R}_i + d_i)r. \tag{1}$$

Bank i is indifferent between lending and not lending an additional unit of reserves in the interbank market when the expected return to lending the additional unit (equation (1)) equals the expected return of not lending, which equals r_{IOR}. Rearranging this indifference condition, the competitive interbank market rate, r, can be expressed as a weighted average between the penalty rate, r_P, and the rate on reserves balances, r_{IOR}, where the weights capture the probability that bank i's reserves fall, or not, below its minimum level, i.e.,

$$r = r_P \Pr(R_i + u_i \leq \bar{R}_i + d_i) + r_{IOR} \Pr(R_i + u_i > \bar{R}_i + d_i). \tag{2}$$

[11] Some recent models use alternative modeling approaches, such as search and bargaining or preferred habitat, in contrast to the perfect competition assumption in Poole (1968). See for example Afonso and Lagos (2015), Afonso et al. (2019), Armenter and Lester (2017), Schulhofer-Wohl and Clouse (2018), and Chen et al. (2016). Kim et al. (2020) adds regulatory frictions while maintaining the assumption of perfect competition. Still, demand for reserves typically originates from a required quantity of reserves and timing of shocks that are broadly consistent with Poole (1968).

For simplicity, we assume that u_i is uniformly distributed over the interval $[-U_i, U_i]$ for all i. This assumption implies that the size of the late shock can take any value in the interval $[-U_i, U_i]$ with the same probability and is zero outside this interval. It also implies that the probability that the late shock, u_i, is: (i) less than some value $z \in [-U_i, U_i]$ is given by $\Pr(u_i \leq z) = (z + U_i)/2U_i$; ($ii$) less than some value $z < -U_i$ is $\Pr(u_i \leq z) = 0$; and (iii) less than some value $z > U_i$ is $\Pr(u_i \leq z) = 1$. We can then express the probability that the size of the late shock is such that reserves, $R_i + u_i$, fall short of bank i's desired level of reserves, $\bar{R}_i + d_i$—and hence the bank borrows from the central bank—as

$$\Pr(R_i + u_i \leq \bar{R}_i + d_i) = \begin{cases} 1 & \text{if } R_i < \bar{R}_i + d_i - U_i \\ \dfrac{\bar{R}_i + d_i + U_i - R_i}{2U_i} & \text{if } \bar{R}_i + d_i - U_i \leq R_i \leq \bar{R}_i + d_i + U_i \\ 0 & \text{if } R_i > \bar{R}_i + d_i + U_i \end{cases} \quad (3)$$

Substituting equation (3) in equation (2) and rearranging terms, the demand for reserves of bank i when the interbank market opens at time $t = 5$ is given by

$$R_i = \begin{cases} [0, \bar{R}_i + d_i - U_i] & \text{if } r = r_P \\ \bar{R}_i + d_i + U_i - 2U_i \, \dfrac{r - r_{IOR}}{r_P - r_{IOR}} & \text{if } r_{IOR} < r < r_P \\ [\bar{R}_i + d_i + U_i, \infty) & \text{if } r = r_{IOR} \end{cases} \quad (4)$$

When the interbank rate equals the penalty rate, bank i is indifferent between holding any amount of reserves between zero and $\bar{R}_i + d_i - U_i$, because in this region of the demand curve bank i's reserve balances will always be below its desired level for any realization of the late shock. This creates a flat demand curve at $r = r_P$. When the interbank rate falls below the penalty rate ($r < r_P$), bank i demands more reserves in the interbank market, since borrowing from the central bank at rate r_P is more costly than in the interbank market at rate r. This generates a downward slopping demand curve until the interbank rate equals the interest on reserves. When $r = r_{IOR}$, the opportunity cost of demanding reserves in the interbank market is zero and bank i becomes indifferent between holding any amount of reserves greater than $\bar{R}_i + d_i + U_i$; in this region of the demand curve, bank i's reserve balances will always exceed its desired level for any realization of the late shock. At rate $r = r_{IOR}$, the demand curve for reserves becomes flat again.[12]

[12] In practice, the flat region lies slightly below r_{IOR} due to regulatory costs faced by banks. Taking this into account will not change the structure of the model or its implications; see Kim et al. (2020).

Banking system demand for reserves. We can derive the banking system demand for reserves by simply summing the individual demand curves of the banks in the banking system, i.e., by summing the reserves demanded by the each bank for each interest rate. The aggregate demand for reserves, R^D, is given by

$$R^D = \begin{cases} [0, \bar{R} + d - U] & \text{if } r = r_P \\ \bar{R} + d + U - 2U \dfrac{r - r_{IOR}}{r_P - r_{IOR}} & \text{if } r_{IOR} < r < r_P \\ [\bar{R} + d + U, \infty) & \text{if } r = r_{IOR} \end{cases} \tag{5}$$

where $R^D \equiv \sum_i R_i$ is the aggregate quantity of reserves demanded by banks when the interbank market opens at $t = 5$; $\bar{R} = \sum_i \bar{R}_i$ is the initial aggregate quantity of of desired reserves (before the demand shocks, d_i, are realized); $d \equiv \sum_i d_i$ is the aggregate demand shock to the banking system's initial desired reserves (which is realized at $t = 4$, before the interbank market opens); and $U \equiv \sum_i U_i$ is sum of each bank i's maximum late period shock (the shock which is realized at $t = 6$, after the interbank market closes).

3.5 Reserve Supply

The equilibrium interest rate. The central bank understands the behavior of banks and the nature of the shocks s and d that hit the supply and demand for reserves of the banking sector. The time $t = 5$ equilibrium is characterized by the aggregate supply of reserves equating the aggregate demand for reserves, $R^D = R^S$, or, equivalently, $R^D = R + s + x$. Since R, s, x and d are known at $t = 5$, the equilibrium interbank interest rate is obtained by substituting $R + s + x$ for R^D in the aggregate demand function (5). Rearranging terms, the market-clearing interbank rate can be expressed as an implicit function of $R + s - d - x$, $r(R + s - d + x)$:

$$r = \begin{cases} r_P & \text{if } R + s - d + x < \bar{R} - U \\ r_{IOR} - c_0(R + s - d + x - \bar{R} - U) & \text{if } \bar{R} - U \leq R + s - d + x \leq \bar{R} + U \\ r_{IOR} & \text{if } R + s - d + x > \bar{R}_a + U \end{cases} \tag{6}$$

where $c_0 = (r_p - r_{IOR})/2U$. If $R + s - d + x \in (\bar{R} - U, \bar{R} + U)$, then the interbank rate is decreasing in the quantity of reserves in the banking system R. It is straightforward to show that the market clearing interbank rate has the expected comparative statics. In particular, $\partial r/\partial R \leq 0$, $\partial r/\partial r_p \geq 0$ and $\partial r/r_{IOR} \geq 0$.

The central bank's problem. When implementing monetary policy, the central bank incurs costs associated with: (*i*) losing close control over the policy rate by "missing" its time $t = 5$ target interbank (fed funds) rate, i.e., when the equilibrium interbank rate differs from the central bank's target; (*ii*) undertaking open market operations at time $t = 3$; and (*iii*) the size of its time $t = 1$ balance sheet.[13] We assume that these costs are linear and that the central bank's monetary policy implementation cost function is given by:

$$V \equiv E\left\{\alpha|r(R+s-d+x) - r(R)| + \beta|x|\right\} + \gamma R. \tag{7}$$

The cost function V is expressed from a time $t = 1$ perspective where s, d, and x are random variables. The term $\alpha|r(R+s-d+x) - r(R)|$ represents the cost associated with interest rate volatility, i.e., the cost of missing the target rate, $r(R)$. Intuitively, if there are no shocks to neither the reserve supply nor the desired reserve holdings, the equilibrium interest would be $r(R)$ since $s = d = x = 0$. Therefore, $r(R+s-d+x) - r(R)$ is the equilibrium deviation from the target interbank interest rate when the economy is characterized by uncertainty. The term $\beta|x|$ represents the operational cost of conducting open market operations. For simplicity, we assume that the cost is symmetric, i.e., draining and injecting reserves are equally costly. Finally, the term, γR, captures the political-economy cost associated with the size of the central bank's time $t = 1$ balance sheet, where we assume that a higher level of reserves, R, results in higher political-economy costs. Without loss of generality, we set $\gamma = 1$.

The central bank takes two policy implementation actions: at time $t = 1$, it chooses the level of reserves R, and, then, at time $t = 3$, it selects the size of the open market operation, $x(R, s)$, given its initial choice of reserves, R, and the realization of the time $t = 2$ reserve supply shock s. Taking into account how it conducts open market operations for all possible combinations of (R, s), the central bank's choice of the $t = 1$ reserve supply is given by the

[13] Concerns about the size of central bank balance sheets are often raised in policy normalization discussions. For example, in 2014, the Federal Reserve expressed its intention to 'hold no more securities than necessary' in its "Policy Normalization Principles and Plans," available at `https://www.federalreserve.gov/monetarypolicy/policy-normalization-discussions-communications-history.htm`. See also Bindseil (2016) and Borio (2023).

solution to

$$\min_{R\geq\bar{R}} E\{\alpha|r[R+s-d+x(R,s)]-r(R)|+\beta|x(R,s)|\}+R. \tag{8}$$

The restriction that $R \geq \bar{R}$ guarantees that the central bank supplies enough reserves to at least meet the initial desired reserves demanded by the banking system.[14] Although we treat $x(R, s)$ as an exogenous function, this function is constructed from the state-by-state time $t = 3$ optimization problem.[15]

The central bank's decisions regarding initial reserves, R, and open market operations, x, become trivial if the central bank does not face "real" trade-offs among the various costs. For example, if $\beta \geq \alpha c_0$, then the cost of an open market operation always exceeds the benefit associated with hitting the target rate, which implies that the central bank does not implement open market operations, i.e., $x = 0$. To rule out this pathological case, we assume that $\beta < \alpha c_0$ in the rest of the paper.

Parametrization of supply and demand shocks. We parameterize the shocks s and d in a stylized and convenient way to capture the uncertainty a central bank faces when it makes its reserve decisions at times $t = 1$ and $t = 3$. At time $t = 1$, the realizations of the supply and demand shocks are both unknown. At time $t = 3$, after the supply shock has been revealed, the demand shock still remains unknown. For simplicity, we assume that s and d follow independent uniform distributions:

$$s \sim \mathcal{U}\left(-S, S\right), \tag{9}$$

$$d \sim \mathcal{U}\left(-D, D\right), \tag{10}$$

where S and D are positive constants. Note that the demand shock d is an aggregate of the individual demand shocks, $d = \sum_i d_i$. In principle, we could allow s and d to be correlated

[14]From a technical perspective, we impose this constraint to rule out a solution where the central chooses a very low level of reserves, i.e., $R \ll \bar{R} - U$ can be the solution to the minimization problem in (8). This solution would be characterized by a very low political cost, very low interest rate volatility and no need for operations. The interest rate will be almost always equal to the penalty rate, r_P, and there would be significant borrowing from the central bank at time $t = 6$. Although unlikely in reality, the central bank's objective function does not prevent this outcome from occurring because the model does not place a cost associated with the central bank persistently lending reserves to banks at date $t = 6$.

[15]Alternatively, we could have represented the central bank's problem as choosing both R and x at time $t = 1$, where x is a function of s. This formulation, however, is not as convenient as the one we propose since it would not give rise to a unique functional form for $x(s)$ without additional restrictions. For example, one can make an arbitrary deviation over a measure zero set that does not change the expected value. Furthermore, given the smoothness of the problem at time $t = 3$, the measurability of x is not a concern if we write $x(R, s)$ as an optimizer for the time $t = 3$ problem.

without changing the structure of the problem, in which case the expected value of d given s would be accounted for by the central bank at $t = 3$ and the remaining portion of d would remain a random variable.

We now turn to the central bank's choice of monetary policy implementation regime.

3.6 Policy Implementation without Demand Uncertainty

We first consider the case with no uncertainty about the demand for reserves and assume that demand shocks are perfectly predictable, i.e., $D = 0$. In practice, this scenario describes the U.S. reserve market in the pre-crisis period. In that period, banks demanded reserves mainly to satisfy reserves requirements and the Trading Desk at the New York Fed was able to accurately estimate the banking system demand for reserves (Logan, 2017).

Perfect predictability of demand implies that all uncertainty is resolved by time the central bank conducts open market operations at time $t = 3$. Given the initial supply of reserves, R, and the realization of the supply shock, s (and $d = 0$), the central bank chooses the size of its open market operation, x, to minimize the cost of implementing monetary policy, $V(R)$, in (7), i.e.,

$$x(R, s) = \arg\min_{R \geq \bar{R}} \{E[\alpha|r(R + s + x) - r(R)|] + \beta|x| + R\},$$

where we impose $R \geq \bar{R}$ to ensure that the central bank supplies enough reserves to meet the desired amount demanded by the banking system. Proposition 1 summarizes our first result.

Proposition 1. *Assume that $D = 0$. If $\beta > 2$ and $\alpha c_0 > \frac{S}{U}$, the cost of implementing monetary policy, $V(R)$, has two local minima: one at $R = \bar{R}$ and another, denoted R_A, at $R > \bar{R} + U$.*

Otherwise, the cost function has local minima only within the steep portion of the demand curve, $R \in [\bar{R}, \bar{R} + U]$.

The proofs to all propositions are in Appendix B. Notice that $R_A > \bar{R} + U$ means that the higher value local minimum is located beyond the "kink" of the aggregate demand curve for reserves in equation (6).

Next, we describe the choice between scarce ($R = \bar{R}$) and ample ($R = R_A$) reserves, first using an illustrative example based on our model, and then more generally based on theoretical implications of our model. Then, we discuss the interpretation of the lower bounds for β and αc_0.

Scarce, ample and abundant reserves. In the following example, we define reserves as being scarce, ample, or abundant depending on how likely it is that the supply shock s pushes reserves below the threshold $\bar{R} + U$ that defines the transition from the flat to the downward-sloping region of the demand curve (see equation 6). When the supply of reserves exceeds this threshold the equilibrium interbank interest rate equals the rate that the central bank pays on reserves, r_{IOR}; when the supply of reserves falls short of this threshold, the equilibrium interbank interest rate exceeds r_{IOR}. We define reserves as *abundant* when there is a zero probability that the time $t = 1$ level of reserves, R, falls below $\bar{R} + U$ after the realization of the supply shock s; *ample* when the probability is "reasonably small"; and *scarce* when the probability is greater than reasonably small.

In the calibration exercise, we define a reasonably small probability to be below 25 percent; and greater than reasonably small probability as exceeding 25 percent. Figure 3 provides an illustration of Proposition 1. In the example described by the figure, the lower limit for an abundant reserve supply is equal to $\bar{R} + U + S$, and the lower limit for ample reserve supply is equal to $\bar{R} + U + 0.5 \times S$. Hence, an abundant reserve supply is at least $0.5 \times S$ larger than the lowest level of ample reserves in this example.

The central bank chooses between the two level of reserves—scarce or ample—in Proposition 1 by comparing the expected cost of implementing monetary policy with scarce reserves, $R = \bar{R}$, and with ample reserves, $R = R_A > \bar{R} + U$. In our calibration, the central bank chooses an ample level of reserves at time $t = 1$ (see Figure 3), since its expected cost is less than that associated with implementing monetary policy with scarce reserves $R = \bar{R}$. In general, however, the central bank's choice need not be $R = R_A$ and instead might be $R = \bar{R}$. Next, we discuss what determines this choice in our model.

Optimal level of reserves. Before discussing the optimal level of reserves, we first characterize the local minimum at R_A and then turn to the central bank's choice of reserves. We can determine the level of reserves R_A by equating the marginal benefit of choosing a higher level of initial reserves—which would be associated with smaller open market operations—

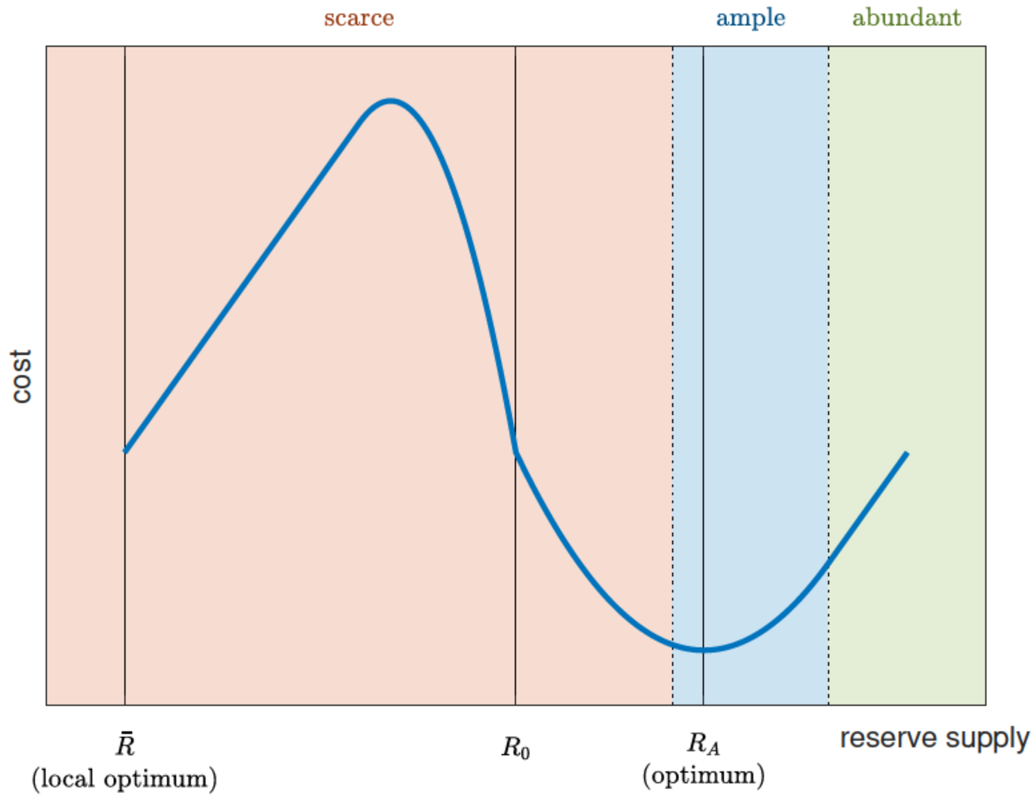

Figure 3: Central Bank's Cost with No Demand Shocks ($D = 0$)

The blue line shows the central bank's expected cost as a function of the initial reserve supply (equation (7)). The cost is generated under the assumption that there are no demand shocks, $D = 0$. The solid vertical lines mark the two local optima of the cost function—scarce ($\bar{R}$) and ample (R_A)—and the location of the kink (R_0) in the demand curve; the kink's location is deterministic because there are no demand shocks. The dotted lines mark the boundaries between scarce (red-shaded), ample (blue-shaded), and abundant (green-shaded) reserve supply based on the example discussed in Section 3.6.

to its marginal cost—which is the political-economy cost of a large balance sheet. After the supply shock, s, is realized, the central bank conducts an open market operation to inject reserves only if the new level of reserves, $R + s$, is less than the threshold $\bar{R} + U$; note that we are ignoring exceptional cases to focus on the intuition.[16] The size of the operation will be $x = -s - [R - (\bar{R} + U)]$ which equates the equilibrium interbank interest rate to the interest on reserve balances, i.e., $r(R + s + x) = r_{IOR}$. Clearly, increasing the time

[16]Exceptional cases can occur if S is large enough to make it possible that $R + s$ is on the other flat portion of the demand curve at $r = r_P$. The proof of Proposition 1 in Appendix B properly accounts for the exceptional cases.

$t = 1$ supply of reserves, R, reduces the size of open market operations at $t = 5$ only if $R + s < \bar{R} + U$; when $R + s > \bar{R} + U$ the central bank does not conduct any open market operations ($x = 0$). This implies that the marginal benefit of increasing R is equal to $\beta \Pr[R+s < \bar{R}+U] = \beta F_1[(\bar{R}+U-R)/S]$, where F_1 is the cumulative distribution function of a uniform distribution over $[-1, 1]$. Since the marginal cost of increasing reserves is equal to $\gamma = 1$, the level of reserves that minimizes the cost of implementing monetary policy is obtained by equating the marginal benefit to the cost of increasing reserves at date 1, which yields the following expression for R_A:

$$R_A = \bar{R} + U + SF_1^{-1}\left(1 - \frac{1}{\beta}\right). \tag{11}$$

Intuitively, R_A is increasing in β because a higher cost of open market operations incentivizes the central bank to increase the initial level of reserves R so as to move farther away from the threshold, $\bar{R} + U$. This threshold determines the transition between the negatively sloped and flat regions of the demand curve. R_A is also increasing in the volatility of the supply shock, S: As S increases so does the probability that reserves will be pushed below the threshold $\bar{R} + U$, incentivizing the central bank to increase R.

Interpretation of the lower bounds on β and αc_0. It is not surprising that we need β to be not too small—$\beta > 2$—to have a local optimum at $R > \bar{R} + U$; otherwise, the benefit from decreasing the expected size of open market operations will be too small for such a local optimum to exist.

The lower bound on αc_0, $\alpha c_0 > \frac{S}{U}$, is also an upper bound the size of the supply shock S: $S < \alpha c_0 U$. If S is too large, then the steep portion of the demand curve becomes relatively insignificant to the central bank. Accordingly, the benefit from decreasing the expected size of open market operations, which is associated with the steep portion of the curve, becomes less important to the central bank. We also note that such a large value of S seems irrelevant in practice—$S \geq \alpha c_0 U > 2U$ implies that the possible realization of $R+s$ can span the entire steep portion of the demand curve r even with the starting point of $R = \bar{R} + U$. As the following discussion shows, ample reserve supply is beneficial if S is large but only if it is not too large to render the kinked shape of the demand curve irrelevant.[17]

We now compare the central bank's cost of implementing a scarce reserve framework with $R = \bar{R}$ to the cost of implementing an ample reserve framework with $R_A > \bar{R} + U$ given by

[17]See Appendix B for a discussion of local minima when $\beta \leq 2$ or $S \geq \alpha c_0 U$.

equation (11):

- **Scarce reserves.** The central bank's implementation cost evaluated at $R = \bar{R}$ is approximately equal to
$$V(R^* = \bar{R}) \approx \frac{1}{2}\beta S + \bar{R}. \tag{12}$$
The first term captures the cost of fully offsetting reserve supply shocks, i.e., it is the expected value of $\beta|-s|$.[18]

- **Ample reserves.** The central bank's implementation cost evaluated at $R = R_A$ is
$$V(R^* = R_A) = \left(1 - \frac{1}{\beta}\right) S + \bar{R} + U. \tag{13}$$
As above, the first term is the expected cost associated with open market operations.

The coefficient on S in equation (13) is smaller than that in (12) because the central bank's open market operations are smaller and less frequent when reserves are ample. Since both of these terms are linear in S, if S is "large enough," the ample-reserve regime will have a lower expected implementation cost and, hence, will be preferred to the scarce-reserve regime. Proposition 2 formalizes this intuition:

Proposition 2. *Assume that $D = 0$ and $S < U$. Then, there exists a constant β_0 such that if $\beta > \beta_0$, then there exists some $S_0 < U$ such that the scarce-reserve regime is optimal for $S \in (0, S_0)$ and the ample-reserve regime is optimal for $S \in (S_0, U)$. Otherwise—if $2 < \beta \leq \beta_0$—then the scarce-reserve regime is optimal.*

We showed, with Proposition 1, that if $S \geq \alpha c_0 U$, there would be no locally optimal ample reserve supply. For certain large values of S, that is for S between U and $\alpha c_0 U$, the central bank optimally chooses not to conduct open market operations for large realizations of $|s|$ at $R = \bar{R}$ because the cost of interest rate deviation is bounded from above by $\alpha \times (1/2)(r_P - r_{IOR})$. This upper bound can make the scarce-reserve regime relatively more favorable for values of S close to but still below $\alpha c_0 U$, depending on other parameters of the model. By assuming $S < U$, the characterization of the central bank's behavior becomes much simpler.

[18] In the extreme case of a very large shock, the central bank will choose not to offset s. See the proof of Proposition 1 in Appendix B for a technical description of the conditions under which such an outcome will occur.

We can derive an expression for S_0 by finding the value of S that equates $V(\bar{R})$ in equation (12) to $V(R_A)$ in equation (13), which are exact for $S < U$:[19]

$$S_0 = \left[\frac{\beta}{2} - 1 + \frac{1}{\beta}\right]^{-1} U. \quad (14)$$

When demand shocks are predictable, $D = 0$, and the volatility of the shocks to the reserve supply is "low" ($S < S_0$), then a scarce-reserve regime will be optimal. This scenario is consistent with the monetary policy implementation regime that prevailed in the U.S. prior to the 2007-2009 financial crisis. On the other hand, when the volatility of the supply shock is "high," then the central bank will choose a regime with ample reserves. This scenario is consistent with monetary implementation policy in the U.S. since the 2007-2009 financial crisis. Figure 4 illustrates this intuition: When the volatility S is low—the orange line—it is less costly for the central bank to implement monetary policy with scarce reserves, $R = \bar{R}$; when volatility is high—blue line—ample reserves, $R \equiv R_A > \bar{R} + U$, provides the implementation framework with the lowest cost.

Intuitively, S_0 is higher if U is higher. If the threshold (kink) in the reserve demand is farther away from the minimum level, then the central bank has a stronger incentive to choose the minimum level of reserve supply to avoid incurring the cost associated with a larger reserve supply, which is proportional to U, while the benefit of smaller open market operations does not depend on U.

The cost of engaging in open market operations, β, is also an important determinant of the optimal monetary policy implementation framework that the central bank chooses. Intuitively, if the cost of conducting open market operations, β, is relatively small, then the central bank will choose a framework with scarce reserves even at higher levels of volatility, S, using open market to adjust reserves if needed. Alternatively, if the operational costs β are relatively high, then higher reserves will be chosen for even lower levels of volatility S. This intuition is verified in the following proposition,

Proposition 3. $\partial S_0 / \partial \beta < 0$*;* S_0 *is as defined in Proposition 2.*

Since the 2007-2009 financial crisis, reserve supply shocks and the required size of open market operations to offset those shocks have become substantially larger (see section 2.2).

[19]See the proof of Proposition 2 in Appendix B.

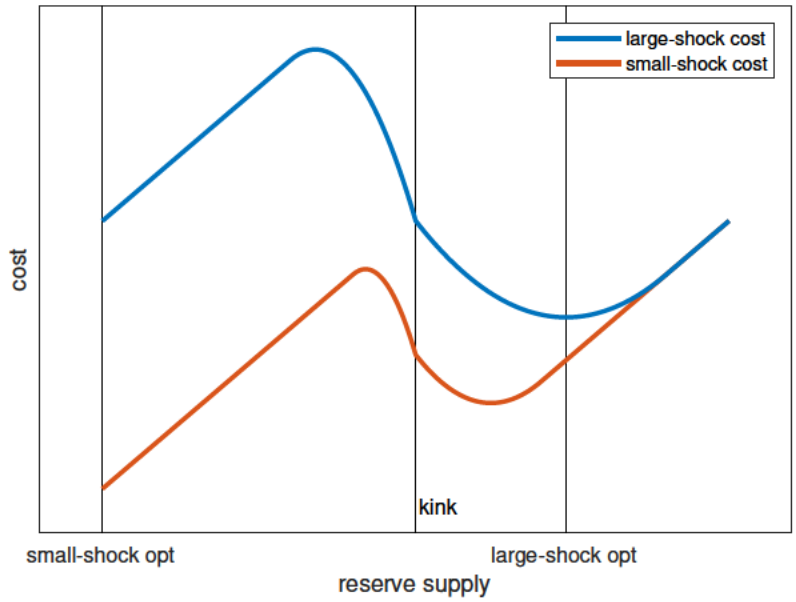

Figure 4: Central Bank's Cost for Different Values of Shocks (S)

The blue and orange lines show the central bank's expected cost as a function of the initial reserve supply (equation 7) for two values of the parameter characterizing the magnitude of supply shocks, S. The blue (orange) line corresponds to a large (small) value of S. For large S, the central bank chooses an implementation with ample reserves ("large-shock cost") while for small S, the implementation cost is lower with scarce reserves ("small-shock cost"). Costs are generated under the assumption that there are no demand shocks, D.

Even though the cost of open market operations is proportional to its size in the model, in practice this cost might be convex. For example, with larger operations, the central bank might be more concerned about risks associated with counterparty exposures, potential market distortions, footprint, and so on. If so, the post-crisis environment could be described one with a higher cost parameter β, which makes an ample-reserve regime preferable to the central bank.

The interpretation of the change in implementation regime in the U.S. relies on assuming that S has increased substantially but not "too much." In Proposition 1, we assume $S < \alpha c_0 U = \frac{\alpha}{2}(r_P - r_{IOR})$. We argue that this is not a very strong assumption since central banks tend to be averse to even relatively small interest rate volatility, and an interest rate movement of $r_P - r_{IOR}$ would be considered very large. Therefore, in practice, the upper bound on S, $\alpha c_0 U = \frac{\alpha}{2}(r_P - r_{IOR})$, is unlikely to be binding as it represents a large number.

However, in Proposition 2, we rely on a stronger assumption: $S < U$. Using the weaker assumption of $S < \alpha c_0 U$ makes the characterization of the optimal choice more complex. This is because when S is large, $R + s$ can reach the flat portion of the reserve demand curve even with scarce reserves at $R = \bar{R}$, setting an upper bound on the implementation cost of the scarce-reserve regime. A sufficient condition that allows us to dismiss this effect by making interest rate movements costly enough is the following: $\alpha c_0 \frac{1}{\beta} > 2$. The ratio $\alpha c_0 \frac{1}{\beta}$ represents the trade-off between tolerating interest rate movements and conducting open market operations to offset these movements; a value close to 1 indicates a central bank indifferent between the two.

Proposition 4. *Assume that $D = 0$, $\alpha c_0 > \frac{S}{U}$ and $\alpha c_0 \frac{1}{\beta} > 2$. Then, for any $\beta > 2$, there exists some $S_0 < \alpha c_0 U$ such that the scarce-reserve regime is optimal for $S \in (0, S_0)$ and the ample-reserve regime is optimal for $S \in (S_0, U)$.*

The proof is in Appendix B.

3.7 Environment with Demand Uncertainty

We now consider the case where there is uncertainty about the demand for reserves—i.e., $D > 0$—at the time $t = 5$ when the central bank conducts an open market operation. We can interpret this scenario as representative of the U.S. reserve market in the post-crisis period. As discussed in Section 2.2, since the 2007-2009 financial crisis, changes in bank regulation, banks' risk management practices, and liquidity stress tests, among other factors, have transformed the banking system's demand for reserves. We interpret the post-crisis period as characterized by an increase in the magnitude of the demand and supply shocks—larger S and D.

An increase in uncertainty may lead a central bank to prefer an ample reserve supply to a scarce one. In particular, when reserves are scarce and the reserve demand becomes less predictable (D increases), open market operations become less effective in stabilizing interest rate movements, making an ample-reserve regime more attractive to the central bank. This is because the effectiveness of open market operations in reducing interest rate deviations depends on the realization of the demand shock, d, i.e., the specific open market operation that the central banks chooses at $t = 3$ will reduce the distance between the equilibrium interest rate and the target rate $r(R)$ for some realizations of the demand shock d but will increase it for others. A wider distribution of d implies a more equal mixing of these outcomes, making open market operations less useful as a tool for minimizing the central bank's implementation cost. This, in turn, makes the ample-regime regime, which relies less on open market operations, more desirable.

The following simple example illustrates this point. Suppose that the central bank chooses an initial level of reserves that is scarce and equal to 3, i.e., $R = \bar{R} = 3$ at $t = 1$, and that a supply shock s of -1 has been realized at $t = 2$, where the supply shock can be either -1 or 1 with equal probability. Since the reserve demand curve in the scarce region is downward sloping, in the absence of reserve demand shocks, the central bank would inject $x = 1$ unit of reserves at $t = 3$. This operation costs the central bank β but saves αc_0 since the equilibrium interbank rate equals the target rate $r(R + s + x = 3)$.

Now suppose that, at $t = 4$, there are demand shocks d of either -1 or $+1$ with equal probability and that the central bank injects a unit of reserves at time $t = 3$. Then the equilibrium interest rate will be either $r(2) > r(3)$ if $d = 1$ or $r(4) < r(3)$ if $d = -1$. The time $t = 3$ expected cost associated with $x = 1$ is $\beta + \alpha c_0$. If, instead, the central does not inject a unit of reserve at time $t = 3$, then the equilibrium interest rate will be either $r(1) > r(3)$ if $d = 1$ or $r(3)$ if $d = -1$. The expected cost associated with not injecting or draining reserves, $x = 0$, is αc_0. Hence, the central bank's best response at time $t = 3$ is *not* to undertake an open market operation, i.e., $x = 0$ since the expected cost to the central bank is lower by β relative to the cost of conducting the operation ($x = 1$).

Next, consider the case where the central bank chooses ample reserves equal to $R_A = \bar{R}+U+2$ at $t = 1$ instead of scarce reserves for this example. Independently of the supply and demand shock realizations, the $t = 5$ equilibrium interbank rate is $r(R_A + s + x - d) = r_{IOR}$: hence, there is no interest rate variability. In this example, the central bank will choose ample reserves if the increase in balance sheet costs, $R_A - \bar{R}$, is less than the cost savings associated with interest rate variability, αc_0.

To see that the decrease in the predictability of demand makes ample reserves more attractive, we assume that the central bank learns what the demand shock at $t = 5$ will be at $t = 3$. Because the demand shock is known at $t = 3$, the central bank will offset the demand shock at $t = 3$ when reserves are scarce. The cost savings from choosing ample reserves at date $t = 1$ is only equal to β because the reserve injection at $t = 3$ would be 2 or 0 with equal probability, which is smaller than αc_0. Thus, cost savings from choosing ample reserves are larger with less predictable demand.

Qualitatively, key results and insights from the model with predictable reserve demand hold when the reserve demand is no longer perfectly predictable. Importantly, the central bank's $t = 1$ reserve supply decision is still a choice between a scarce-reserve regime and an ample-reserve regime. And due to the demand shock being unpredictable, it is more likely that an ample-reserve regime is preferred to a scarce-reserve regime. To facilitate the comparison between the cases with and without demand uncertainty, we state a more general version of Proposition 1:

Proposition 5. *Assume that $D = 0$ and s is a sum of two independent uniform distributions, $s = s_1 + s_2$, where s_1 and s_2 follow uniform distributions over $[-S_1, S_1]$ and $[-S_2, S_2]$, respectively. If $\beta > 2$ and $\alpha c_0 U > -\frac{\beta}{2}F_s^{-1}(\frac{1}{2} - \frac{1}{\beta})$ (a sufficient condition for which is $\alpha c_0 > \frac{S_1+S_2}{U}$), the cost of implementing monetary policy, $V(R)$, has two local minima: one at $R = \bar{R}$ and another, denoted R_A, at $R > \bar{R} + U$.*

Otherwise, the cost function has local minima only within the steep portion of the demand curve, $R \in [\bar{R}, \bar{R} + U]$.

The proof is in Appendix B. The intuition for this result is the same as in Proposition 1; setting $S_1 = 0$ or $S_2 = 0$ reduces Proposition 5 to Proposition 1.

Next, we show that with demand shocks, an ample level of reserve supply is more likely to be optimal:

Proposition 6. *Assume $S+D < U$ and define two economies 1 and 2 as follows: In economy 1, the supply shock s is a sum of two independent uniform distributions over $[-S, S]$ and $[-D, D]$ and the demand shock d is zero. In economy 2, the supply shock s is uniform over $[-S, S]$ and the demand shock d is uniform over $[-D, D]$.*

If there exists a local optimum $R > \bar{R}+U$ in economy 1, then there exists a local optimum R_A over $R > \bar{R}+U$ in economy 2, too. Furthermore, if the ample-reserve regime is optimal in economy 1—the local optimum at $R > \bar{R} + U$ is preferred to $R = \bar{R}$—then the ample-reserve regime is also optimal in economy 2.

The intuition behind this result is the following: The presence of demand shocks (economy 2) that are unknown at the time when the central bank chooses its operation makes open market operations less effective in economy 2, thus providing the central bank with a stronger incentive to preemptively choose a higher level of reserve supply relative to economy 1 and avoid a potential use of operations. Therefore, with the overall shock distribution $s + d$ fixed, having more uncertainty at the time of operation (economy 2) strengthens the relative preference for the ample-reserve regime over the scarce-reserve regime.[20] The formal proof of Proposition 6 is in Appendix B.

Through three examples, Figure 5 illustrates how uncertainty at the time of operation makes an ample reserve supply more favorable (relative to a scarce supply). When there is no uncertainty about the demand for reserves at the time of the open market operation—the orange line—the scarce reserve regime is preferred; however, when predictability is reduced and demand is uncertain—the blue and green lines—the ample reserve regime is preferred.

4 An Ample Reserve Regime in Practice

Our theory provides a rationale for the FOMC's decision in 2019 to remain in an ample reserve regime. In this section, we briefly discuss how effective this regime has been at maintaining interest rate control, which is the primary objective of the monetary policy implementation framework in the U.S. We also discuss financial stability implications of this type of framework.

4.1 Interest Rate Control

Measuring the effectiveness of monetary policy implementation can be approached in many ways. One approach is to track the position of the policy rate relative to the target rate or range, and determine how often the rate deviates from the target. In the U.S., the effective federal funds rate has printed outside the target range in only two instances since the FOMC announced the establishment of a target range for the federal funds rate in 2008.[21]

[20] Our findings are consistent with those in Afonso et al. (2023c). Building on a different modeling approach, Afonso et al. (2023c) show that, when the banking system demand for reserves is uncertain, the optimal level of reserves that a central bank supplies is greater than that absent uncertainty.

[21] The effective fed funds rate printed below the target range on December 31, 2015 and above the target range on September 17, 2019.

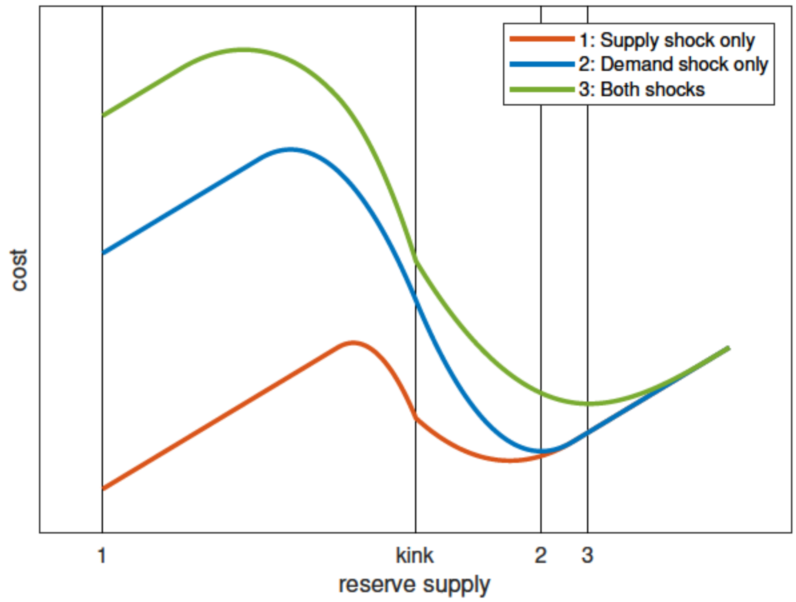

Figure 5: Central Bank's Cost: Cross-Model Comparison

The orange, blue, and green lines show the central bank's expected cost as a function of the initial reserve supply (equation 7) for different values of S and D. For a chosen constant W, the orange line corresponds to $S = W$ and $D = 0$ (model 1: supply shock only); the blue line to $S = 0$ and $D = W$ (model 2: demand shock only); and the green line to $S = W$ and $D = W$ (model 3: both shocks). The optimal supply of reserves is scarce for model 1 but ample for models 2 and 3. The vertical lines marked by numbers 1, 2 and 3 show the optimal reserve supply for models 1, 2 and 3, respectively. The expected location of the kink, $R = \bar{R} + U$, is also shown.

The post-financial crisis regime has been very effective at ensuring that the policy rate in the U.S. remains within its target range and has proved resilient through the Covid pandemic, the rapid hike in policy rates to tame inflationary pressures, and the increase in demand for precautionary liquidity during the 2023 banking turmoil. (Logan, 2017; Perli, 2023).

Other measures of effectiveness focus on rate dispersion. Duffie and Krishnamurthy (2016), for instance, propose an index intended to capture rate dispersion across different segments of money markets.[22] Building on the Duffie-Krishnamurthy index, Afonso et al. (2017) show that the implementation framework used by the Federal Reserve in the post-

[22] In particular, Duffie and Krishnamurthy (2016) consider the volume-weighted average absolute deviation from the volume-weighted average rate, which captures how much each market rate deviates from the average rate across markets. To implement the index, the authors adjust rates for term and credit spreads, and weight each instrument's influence by its outstanding amount.

crisis period has been effective and achieved good pass-through.

Another way of measuring the effectiveness of the operating regime is by the pass-through of administered rates to market rates. A particularly interesting approach to look at the effectiveness of the Federal Reserve's current implementation framework is to consider the effect of the "technical adjustments" on various money market rates. A technical adjustment is a change to the administered rates—the interest on reserve balances (IORB) rate and/or the ON RRP rate—that is intended to foster trading in the fed funds market at rates well within the target range, rather than change the stance of monetary policy. Afonso et al. (2022b) and Afonso et al. (2023a) show that changes in administered rates, through technical adjustments, effectively steer the policy rate within the target range.

Overall, the Federal Reserve's current implementation framework has been effective at interest rate control and at ensuring pass-through of the policy rate to short-term money markets rates.

4.2 Additional Financial Stability Considerations

Our paper focuses on the financial stability considerations associated with the size of the central bank balance sheet. An example of these considerations is political economy concerns that might cast some doubt over the independence of the central bank from the government. For instance, a large central bank balance sheet entails a large footprint on certain financial markets such as the market for government securities; a large presence in the government securities market might suggest a tight connection between monetary and fiscal policies. Another example arises from the remuneration of reserves balances—a key implementation tool in floor systems. A high level of reserves implies larger payments on the balances that banks hold at the central bank; these payments might be perceived as subsidies to the banking system.

Our stylized model abstracts from other financial stability considerations. Acharya et al. (2022) point to potential fragility concerns associated with central bank balance sheet expansions. During expansions of central bank balance sheets, which ultimately create reserve balances, banks finance these reserves with demandable deposits and lines of credit. The authors argue that this increase in deposits and lines of credit is not reversed during balance sheet shrinkage, creating an asymmetry responsible for tightening liquidity conditions and stress episodes during balance sheet normalization that can make the banking system more dependent on central bank liquidity infusions during stress (Acharya and Rajan, 2024).

There are also financial stability benefits of implementation frameworks with high levels of reserves that our model abstracts from. Higher reserves reduce the risk of disruptions in payments systems (Afonso et al., 2022a; Copeland et al., 2024) and increase the liquidity held by the banking sector, supporting the smooth functioning of financial markets. A monetary policy implementation regime with a sufficiently large supply of reserves allows banks to meet some of their needs for high-quality liquid assets (HQLA) with reserves. It also provides enough reserves for banks to meet their outflow needs with reserves during a time of stress and avoid the potential fire-sale effects from monetizing large quantities of assets (Bush et al., 2019). This makes the financial system safer, more resilient, and may reduce the need for banks to borrow from the central bank.

Another benefit of a regime with a large supply of reserves is that reserves as "money-like" short-term safe assets are particularly attractive to some investors and, for that reason, carry a premium that reduces their yield. When the supply of money-like assets is too small, private sector participants have an incentive to issue liabilities that have money-like properties because of their low cost. This can result in excessive maturity transformation, which makes the financial system more fragile (Greenwood et al., 2016; Carlson et al., 2016).

5 Conclusion

The 2007-2009 financial crisis, and its aftermath, have led to profound changes in the way many central banks implement monetary policy. In particular, large-scale asset purchases resulted in high levels of reserves balances at major central banks. Traditional implementation tools became ineffective, and central banks transitioned to control interest rates with administered rates, using a "floor" system. Some central banks, including the Federal Reserve, have indicated that they expect to continue using this type of implementation framework in the foreseeable future.

Our paper provides a framework to think through the costs and benefits of different implementation frameworks and discuss in which environments a level of reserves that is low, moderate, or high is preferable. We propose a model of the banking system demand for reserves to study the trade-offs that policy makers face when choosing a monetary policy implementation framework. We highlight potential trade-offs between rate control and the financial stability implications of using the central bank's balance sheet as a monetary policy implementation tool as well as considerations arising from active management of central bank reserve balances. We show that in the post-financial crisis environment, the optimal monetary policy regime is one where reserve balances lie between scarce and abundant.

Appendix

A Demand for Reserves

In this section we formally describe individual banks' problem and explain in more detail some of the results in Section 3.4. Bank i maximizes the following objective function:

$$\int_{-\infty}^{+\infty} V_i(u)\mu_i(u)du - R_i r. \tag{A-1}$$

μ_i is the probability density function of u_i and $V_i(u)$ is the value associated with the outcome $u_i = u$. $V_i(u)$ can be derived by integrating the marginal value of reserves up to $R_i + u_i$. The marginal value of reserves is r_P for $R_i + u_i < \bar{R}(i) + d_i$ and r_{IOR} for $R_i + u_i \geq \bar{R}(i) + d_i$. Integrating the marginal value from 0 (or any constant) to $R_i + u_i$, the expression for $V_i(u)$ is

$$V_i(u) = min(R_i + u_i, \bar{R}(i) + d_i)r_P + max(R_i + u_i - \bar{R}(i) - d_i, 0)r_{IOR}. \tag{A-2}$$

Note that

$$\begin{aligned} \frac{\partial V_i(u)}{\partial R_i} &= r_P \text{ if } R_i + u_i < \bar{R}(i) + d_i; \\ &= r_{IOR} \text{ otherwise.} \end{aligned} \tag{A-3}$$

The first-order condition (FOC) for bank i with respect to R_i is

$$\begin{aligned} 0 = \frac{\partial}{\partial R_i}[\int_{-\infty}^{+\infty} V_i(u)\mu_i(u)du - R_i r] &= \int_{-\infty}^{+\infty} \frac{\partial V_i(u)}{\partial R_i}\mu_i(u)du - r \\ = Prob(R_i + u_i < \bar{R}(i) + d_i)r_P &+ Prob(R_i + u_i \geq \bar{R}(i) + d_i)r_{IOR} - r. \end{aligned} \tag{A-4}$$

This is bank i's FOC described in Section 3.4.

Given the FOC of bank i, we can derive its demand for reserves. Recall that u_i is uniformly distributed over $(-U_i, U_i)$.

- If $R_i \leq \bar{R}(i)+d_i-U_i$, then $Prob(R_i+u_i < \bar{R}(i)+d_i) = 1$. Therefore, $r = 1 \cdot r_P + 0 \cdot r_{IOR} = r_P$.
- If $R_i \geq \bar{R}(i) + d_i + U_i$, then $Prob(R_i + u_i < \bar{R}(i) + d_i) = 0$. Therefore, $r = r_{IOR}$.

- If $\bar{R}(i) + d_i - U_i \leq R_i \leq \bar{R}(i) + d_i + U_i$, then $Prob(R_i + u_i < \bar{R}(i) + d_i) = -(R_i - \bar{R}(i) - d_i - U_i)/(2U_i)$. Therefore,

$$\begin{aligned} r &= r_P(-\frac{R_i - \bar{R}(i) - d_i - U_i}{2U_i}) + r_{IOR}(1 + \frac{R_i - \bar{R}(i) - d_i - U_i}{2U_i}) \\ &= r_{IOR} + (r_P - r_{IOR})(-\frac{R_i - \bar{R}(i) - d_i - U_i}{2U_i}). \end{aligned} \tag{A-5}$$

  Solving this equation for R_i, we have $R_i = \bar{R}(i) + d_i + U_i - 2U_i(r - r_{IOR})/(r_P - r_{IOR})$.

Next, we aggregate reserve demand across banks; recall that $R_{agg} = \sum_i R_i$, $\bar{R} = \sum_i \bar{R}(i)$, $d = \sum_i d_i$ and $U = \sum_i U_i$.

- If $R_{agg} \leq \bar{R} + d - U$, then $r = r_P$ in equilibrium and $R_i \leq \bar{R}(i) + d_i - U_i$ for every bank; note that R_i is not uniquely determined. To prove this, suppose that $R_j > \bar{R}(j) + d_j - U_j$ for some j. Then, to satisfy bank j's FOC, $r < r_P$. This implies $R_i > \bar{R}(i) + d_i - U_i$ for all i, implying $R > \bar{R} + d - U$, which is a contradiction.

- If $R_{agg} \geq \bar{R} + d + U$, then $r = r_{IOR}$ in equilibrium and $R_i \geq \bar{R}(i) + d_i + U_i$ for every bank; note that R_i is not uniquely determined. This can be proved by contradiction, similarly to how the previous case was proved.

- If $\bar{R} + d - U \leq R_{agg} \leq \bar{R} + d + U$, then $r_{IOR} < r < r_P$, because otherwise, R_{agg} would be outside the range. For any r in (r_{IOR}, r_P), bank i's choice of R_i is unique and given by $R_i = \bar{R}(i) + d_i + U_i - 2U_i(r - r_{IOR})/(r_P - r_{IOR})$, as shown earlier. Summing this expression across i, we have

$$R_{agg} = \bar{R} + d + U - 2U(\frac{r - r_{IOR}}{r_P - r_{IOR}}). \tag{A-6}$$

  Solving this for r, we have

$$r = r_{IOR} - (\frac{r_P - r_{IOR}}{2U})(R_{agg} - \bar{R} - d - U) \text{ if } \bar{R} + d - U < R_{agg} < \bar{R} + d + U. \tag{A-7}$$

In equilibrium, $R_{agg} = R + s + x$ and recall $y \equiv R + s + x - d$. Writing the expression in terms of y, we have

$$r = r_{IOR} - (\frac{r_P - r_{IOR}}{2U})(y - \bar{R} - U) \text{ if } \bar{R} - U < y < \bar{R} + U. \tag{A-8}$$

This is the functional form of $r(y)$ described in Section 3.4.

B Technical Assumptions and Proofs

Proof of Proposition 1: The proposition is as follows: Assume that $D = 0$. If $\beta > 2$ and $\alpha c_0 > (S/U)$, the cost of implementing monetary policy, $V(R)$, has two local minima: one at $R = \bar{R}$ and another, denoted R_A, at $R > \bar{R} + U$. Otherwise, the cost function has local minima only within the steep portion of the demand curve, $R \in [\bar{R}, \bar{R} + U]$.

Proving this proposition is somewhat redundant because we will prove a more general Proposition 6. Nonetheless we provide a proof because it is helpful in understanding the choice of the central bank in the equilibrium. Recall that the central bank's cost function from date 0 perspective is

$$V(R) = \mathbb{E}[\alpha|r(R + s - d + x(R, s)) - r(R)| + \beta|x(R, s)|] + R. \tag{B-9}$$

The central bank seeks to minimize this cost function under the constraint $R \geq \bar{R}$, with $x(R, s)$ optimal in each state s. To show the existence of two local minima, one at $R = \bar{R}$ and another over $R > R_0$, where R_0 is defined as $\bar{R}+U$, it is sufficient to show the following:

- $V'(\bar{R}) > 0$.
- $V'' \leq 0$ for $R < R_0$.
- $V'' \geq 0$ for $R > R_0$.
- $\lim_{R\to\infty} V'(R) > 0$.
- $V'(R_0^+) < 0$, where $V'(R_0^+)$ denotes the right limit of V' at R_0, if and only if $\beta > 2$ and $S < \alpha c_0 U$.

The first inequality shows that $R = \bar{R}$, the scarce supply, is a local optimum and the second inequality shows that there is no other local optimum below the kink level, $R < R_0$. The remaining inequalities show that there is a unique optimum with $R > R_0$ (the ample supply).

In what follows, we first prove the five inequalities and then discuss what happens if $\beta \leq \beta_0$. To prove the first inequality, $V'' < 0$ for $R < R_0$, we recognize that increasing R is less costly for R closer to R_0 (while still $\bar{R} \leq R < R_0$) because moving closer to the flat portion of the demand curve at $r = r_{IOR}$ helps lower the cost due to interest rate volatility. Recall that

$$\begin{aligned} r(y) &= r_{IOR} \text{ if } R_0 < y; \\ &= r_{IOR} + c_0(R_0 - y) \text{ if } R_0 - 2U \leq y \leq R_0; \\ &= r_{IOR} + 2c_0 U \text{ otherwise.} \end{aligned} \tag{B-10}$$

We first derive the optimal $x(R, s)$ for $\bar{R} \leq R < R_0$. Recall that the problem is

$$\min_x \alpha|r(R + s + x) - r(R)| + \beta|x|. \tag{B-11}$$

d is dropped because $D = 0$ and R is dropped from the cost function because this is a problem of determining x taking R and s is given. Given the standing assumption $\beta < \alpha c_0$ (section 3.5), we can characterize x:

- If $R_0 - 2U \leq R + s \leq R_0 - R$, then $x = -s$. $R + s$ is still in the steep portion of the demand curve and $\beta < \alpha c_0$ implies that the central bank finds it optimal to completely offset s.

- Otherwise, two cases are possible: either $x = -s$ or $x = 0$. Only x such that $R+s+x$ is on the steep portion of the demand curve can be better than $x = 0$, and any $R + s + x$ that is on the steep portion of the demand curve is dominated by $x = -s$ because $\beta < \alpha c_0$.

The preceding discussion shows that for any s, the optimal choice is either $x = 0$ or $x = -s$:

$$\min_x \alpha|r(R + s + x) - r(R)| + \beta|x| = \min(\alpha|r(R + s) - r(R)|, \beta|s|). \tag{B-12}$$

In other words, calculating the minimum cost requires just comparing $x = 0$ with $x = -s$. Given the functional form of r, $x = 0$ is chosen if (and only if) $|s|$ is large enough to take advantage of the flat portions of r: if $s \geq (\alpha c_0/\beta)(R_0 - R)$ or if $s \leq (\alpha c_0/\beta)(R_0 - 2U - R)$.

Recall that $V(R) = \mathbb{E}[\alpha|r(R+s-d+x(R,s)) - r(R)| + \beta|x(R,s)|] + R = \mathbb{E}\min(\alpha|r(R+s) - r(R)|, \beta|s|) + R$. Using this expression we can calculate the derivative dV/dR:

$$\begin{aligned} \frac{dV}{dR} &= \int \frac{\partial}{\partial R} \min(\alpha|r(R + s) - r(R)|, \beta|s|) f_s(ds) + 1 \\ &= -\alpha c_0[1 - F_s(\frac{\alpha c_0}{\beta}(R_0 - R))] + \alpha c_0 F_s(\frac{\alpha c_0}{\beta}(R_0 - 2U - R)) + 1. \end{aligned} \tag{B-13}$$

f_s and F_s are the probability density and the cumulative distribution functions of s, respectively. The contribution from the integrand is nonzero only for s such that $x(R,s) = 0$, leading to the expression.

Putting $R = \bar{R}$, we have the first two terms in the equation canceling each other out due to the symmetry of s around zero, giving us $V'(\bar{R}) = 1$, proving the first inequality, $V'(\bar{R}) > 1$.

As R increases, both the first and the second terms (weakly) decrease, proving the second inequality, $V''(R) \leq 0$ if $R < R_0$.

To prove the remaining three inequalities, we recognize that increasing R when $R \geq R_0$ reduces the cost of operations because the central bank does not need to respond if reserve supply after the shock still remains within the flat portion of the demand curve, $r(R+s) = r(R)$. Recall that, for the purpose of the proof, the central bank's problem can be written simply as

$$\min_x \alpha |r(R+s+x) - r(R)| + \beta |x|. \tag{B-14}$$

Now we characterize the optimal choice of x:

- If $R + s \geq R_0$, then $x = 0$ because $r(R+s) = r(R)$.
- If $R+s \leq R_0$, then the optimal choice is either $x = R_0 - R - s$ or $x = 0$. The argument is the same as that used in proving the first two inequalities, except that it is optimal to choose $x = R_0 - R - s$ rather than $x = -s$ because $x = R_0 - R - s$ is the minimum operation that achieves $r(R+s+x) = r(R)$.

The preceding discussion shows

$$\min_x \alpha |r(R+s+x) - r(R)| + \beta |x| = \min(\alpha |r(R+s) - r(R)|, \beta |R_0 - R - s|).$$

Given the functional form of r, $x = 0$ is optimal if $s \leq -(\alpha/\beta)(r_P - r_{IOR}) + R_0 - R$ or if $s \geq R_0 - R$.

Recall that $V(R) = \mathbb{E} \min(\alpha |r(R+s) - r(R)|, \beta |s|) + R$. Using this expression we can calculate the derivative dV/dR:

$$\begin{aligned} \frac{dV}{dR} &= \int \frac{\partial}{\partial R} \min(\alpha |r(R+s) - r(R)|, \beta |R_0 - R - s|) f_s(ds) + 1 \\ &= -\beta [F_s(R_0 - R) - F_s(-\frac{\alpha}{\beta}(r_P - r_{IOR}) + R_0 - R)] + 1. \end{aligned} \tag{B-15}$$

Note that the term multiplying β is the probability of $-(\alpha/\beta)(r_P - r_{IOR}) + R_0 - R \leq s \leq R_0 - R$, which is monotonically decreasing in R if $R_0 \leq R$, given the shape of the distribution of s. Therefore, (dV/dR) is monotonically increasing in R, proving the third inequality: $V''(R) \geq 0$ if $R_0 > R$.

Next, given the functional form of $V'(R)$, its limit as R goes to $+\infty$ is 1, which proves the fourth inequality.

Also, as R approaches R_0 from the right,

$$\lim_{R \to R_0^+} V'(R) = -\beta \left[\frac{1}{2} - F_s \left(-\frac{\alpha}{\beta}(r_P - r_{IOR}) \right) \right] + 1$$
$$= -\beta \left[\frac{1}{2} - F_s \left(-\frac{\alpha c_0}{\beta} \times 2U \right) \right] + 1. \tag{B-16}$$

If $S \leq (2\alpha c_0/\beta)U$, then this expression becomes $-(\beta/2) + 1$, which is negative if and only if $\beta > 2$. Otherwise, note that $F_s(-(2\alpha c_0/\beta)U) = (1/2)[1 - (2\alpha c_0/\beta)(U/S)]$. Thus, the preceding equation can be simply written as $-(\alpha c_0)(U/S) + 1$, which is negative if and only if $S < \alpha c_0 U$.

To summarize, $V'(R_0^+) < 0$ holds if and only if either (i) $\beta > 2$ and $S \leq (2\alpha c_0/\beta)U$; or (ii) $(2\alpha c_0/\beta)U < S < \alpha c_0 U$. Note that condition (i) implies $S < \alpha c_0 U$ and condition (ii) implies $\beta > 2$ (in the sense that otherwise the set of (β, S) satisfying (ii) is empty). Therefore the condition implies $\beta > 2$ and $S < \alpha c_0 U$. Conversely, if $\beta > 2$ and $S < \alpha c_0 U$, either condition (i) or (ii) is satisfied depending on the value of S.

Therefore, the fifth inequality, $V'(R_0^+) < 0$, holds if and only if $\beta > 2$ and $S < \alpha c_0 U$.

If $V'(R_0^+) \geq 0$, the inequality along with $V''(R) \geq 0$ for $R > R_0$ (the third inequality in the earlier list) shows that there is no local minimum in $R > R_0$. This completes the proof of Proposition 1.

Local minima in case $\beta \leq 2$ or $S \geq \alpha c_0 U$. Notice that $V''(R) < 0$ for $R < R_0$ implies that there can be a local minimum only at $R = R_0$ other than $R = \bar{R}$. And $R = R_0$ is a local minimum if and only if $V'(R_0^-) < 0$, where $V'(R_0^-)$ is the limit of $V'(R)$ as R approaches R_0 from the left. Using the expression for $V'(R)$ derived while proving the first inequality, we can calculate the limit:

$$\lim_{R \to R_0^-} V'(R) = -\alpha c_0 \left[\frac{1}{2} - F_s \left(-2 \left(\frac{\alpha c_0}{\beta} \right) U \right) \right] + 1. \tag{B-17}$$

If $S \leq (2\alpha c_0/\beta)U$, then this expression can be simplified as $-(\alpha c_0/2) + 1$, which is negative if and only if $\alpha c_0 > 2$. If $S > (2\alpha c_0/\beta)U$, then this expression can be simplified as $-[(\alpha c_0)^2/\beta](U/S) + 1$, which is negative if and only if $S < [(\alpha c_0)^2/\beta]U$.

Proof of Proposition 2: The proposition is as follows: Assume that $D = 0$ and $S < U$. Then, there exists a constant β_0 such that if $\beta > \beta_0$, then there exists some $S_0 < U$ such that the scarce-reserve regime is optimal for $S \in (0, S_0)$ and the ample-reserve regime is optimal for $S \in (S_0, U)$. Otherwise—if $2 < \beta \leq \beta_0$—then the scarce-reserve regime is optimal.

As explained in Section 3.6, the intuition behind this result is clear—the cost to the central bank loads more heavily on S in the scarce-reserve regime, thus a larger S makes the ample-reserve regime more favorable relative to the scarce-reserve regime.

For $R = \bar{R}$, note that $S < U$ implies $min(\alpha|r(R+s) - r(R)|, \beta|s|) = \beta|s|$ for any $|s| < S$. Therefore,

$$V(\bar{R}) = \int min(\alpha|r(R+s)-r(R)|, \beta|s|) f_s(s)ds + \bar{R} = \int \beta|s| f_s(s)ds + \bar{R} = \frac{1}{2}\beta S + \bar{R}. \quad \text{(B-18)}$$

Next, we derive an expression for R_A. Note that $R_0 < R_A < R_0 + S$, where $R_0 \equiv \bar{R} + U$ as before. This is because $V(R)$ is convex over $R > R_0$ and $V'(R_0 + S) = 1$; for $R \geq R_0 + S$, $R + s$ always stays in the flat portion of the demand curve r and there is no cost associated with interest rate movements or open market operations.

For $R_0 < R < R_0 + S$, as explained in the proof of Proposition 1,

$$\frac{dV}{dR} = -\beta[F_s(R_0 - R) - F_s(-\frac{\alpha}{\beta}(r_P - r_{IOR}) + R_0 - R)] + 1.$$

Given $S < U$, we can simplify this expression as

$$\frac{dV}{dR} = -\beta F_s(R_0 - R) + 1 = -\beta F_1(\frac{R_0 - R}{S}) + 1.$$

As in the main text, F_1 denotes the cumulative distribution function of a uniform distribution over $(-1, 1)$. Setting this expression at 0 to derive the value of R_A, we have

$$R_A = F_1^{-1}\left(1 - \frac{1}{\beta}\right) S + R_0 = \left(1 - \frac{2}{\beta}\right) S + R_0. \quad \text{(B-19)}$$

Following the proof of Proposition 1 and using $S < U$,

$$\begin{aligned} V(R_A) &= \int \min(\alpha|r(R_A+s) - r(R_A)|, \beta|R_0 - R_A - s|) f_s(ds) + R_A \\ &= \int \beta|R_0 - R_A - s| f_s(ds) + R_A \\ &= \frac{1}{2}\beta|R_0 - R_A - S| Prob(R_0 - R_A - s \leq 0) + R_A \\ &= \frac{1}{2}\beta \times \frac{2}{\beta} S \times \frac{1}{\beta} + \left(1 - \frac{2}{\beta}\right) S + R_0 = \left(1 - \frac{1}{\beta}\right) S + U + \bar{R}. \end{aligned} \tag{B-20}$$

The difference between these two cost functions is

$$V(\bar{R}) - V(R_A) = \frac{1}{2}(\beta - 2 + 2\beta^{-1})S - U. \tag{B-21}$$

The coefficient on S is positive if $\beta > 2$. If $S = U$, the expression becomes

$$\frac{1}{2\beta}(\beta^2 - 4\beta + 2). \tag{B-22}$$

If $2 < \beta \leq \beta_0$, where $\beta_0 = 2 + \sqrt{2}$, this expression is non-positive, and $V(\bar{R}) - V(R_A)$ is negative for any $S < U$, implying that the scarce-reserve regime is optimal.

If $\beta > \beta_0$, then the expression is positive, which means that for some S, $V(\bar{R}) > V(R_A)$. Given that the coefficient on S is larger in the expression for $V(\bar{R})$ than in the expression for $V(R_A)$, the condition for $V(\bar{R}) > V(R_A)$, implying that the ample-reserve regime is optimal, can be expressed as $S > S_0$.

We can derive the value of S_0 as the value of S that satisfies the equation $V(\bar{R}) - V(R_A) = 0$. Using the explicit expression for $V(\bar{R}) - V(R_A)$, we see

$$S_0 = \left[\frac{\beta}{2} - 1 + \frac{1}{\beta}\right]^{-1} U. \tag{B-23}$$

This completes the proof of Proposition 2.

Proof of Proposition 3: The proposition is as follows: $\partial S_0 / \partial \beta < 0$; S_0 is as defined in Proposition 2.

Note that, from the proof of Proposition 2,

$$S_0 = \left[\frac{\beta}{2} - 1 + \frac{1}{\beta}\right]^{-1} U. \tag{B-24}$$

We only need to prove that the expression inside the brackets, $(\beta/2) - 1 + (1/\beta)$, is increasing in β. Its derivative is $(1/2) - (1/\beta^2)$, which is positive if $\beta > 2$. This completes the proof of Proposition 3.

Proof of Proposition 4: The proposition is as follows: Assume that $D = 0$, $\alpha c_0 > (S/U)$ and $(\alpha c_0/\beta) > 2$. Then, for any $\beta > 2$, there exists some $S_0 < (\alpha c_0)U$ such that the scarce-reserve regime is optimal for $S \in (0, S_0)$ and the ample-reserve regime is optimal for $S \in (S_0, U)$.

First, we calculate $V(\bar{R})$. Note that if $|s|$ is large enough, then the central bank chooses $x = 0$ instead of $x = -s$ (see the proof of Proposition 1). The central bank is indifferent if $\alpha|r(R + s) - r(R)| = \beta|s|$, which occurs at $|s| = (\alpha c_0/\beta)U$.

Therefore, if $S \leq (\alpha c_0/\beta)U$, then $V(\bar{R})$ has the same expression as before (see the proof of Proposition 2):

$$V(\bar{R}) = \frac{1}{2}\beta S + \bar{R}. \tag{B-25}$$

If $S > (\alpha c_0/\beta)U$, then conditional on $s \leq (\alpha c_0/\beta)U$, the expected cost is given by the preceding expression evaluated at $S = (\alpha c_0/\beta)U$. Conditional on $s > (\alpha c_0/\beta)U$, the expected cost is constant at $\alpha c_0 U$. Therefore,

$$\begin{aligned} V(\bar{R}) &= \frac{1}{2}\alpha c_0 U \times \frac{1}{S}\frac{\alpha c_0}{\beta}U + \alpha c_0 U \times \left(1 - \frac{1}{S}\frac{\alpha c_0}{\beta}U\right) + \bar{R} \\ &= \left(1 - \frac{\alpha c_0}{2\beta}\frac{U}{S}\right)\alpha c_0 U + \bar{R}. \end{aligned} \tag{B-26}$$

Next, we calculate R_A. Note that the expression derived earlier (in the proof of Proposition 2) is still valid, as long as the central bank is still choosing $x = R_0 - R_A - s$ for the largest negative realization of s, $s = -S$:

$$R_A = \left(1 - \frac{2}{\beta}\right) S + \bar{R} + U. \tag{B-27}$$

Note that, for $s = -S$, $R_A + s = -(2/\beta)S + \bar{R} + U > \bar{R} + U - (2\alpha c_0/\beta)U$. The implied offset x by the central bank is $x = (R_0 - R_A - s) < (2\alpha c_0/\beta)U$, with associated implementation cost of $\beta x < 2\alpha c_0 U = \alpha(r_P - r_{IOR})$. Thus, the central bank optimally chooses $x = (R_0 - R_A - s)$ over $x = 0$ for $s = -S$, and the expression for R_A is valid.

An immediate corollary of the preceding discussion is that the earlier expression for

$V(R_A)$ is also valid (from the proof of Proposition 2):

$$V(R_A) = \left(1 - \frac{1}{\beta}\right) S + U + \bar{R}. \tag{B-28}$$

The difference between these two costs, say $D_V(S) \equiv V(\bar{R}) - V(R_A)$, which depend on S (holding all other parameters constant), is

$$D_V(S) = \left(\frac{\beta}{2} - 1 + \frac{1}{\beta}\right) S - U. \tag{B-29}$$

if $S < (\alpha c_0/\beta)U$. Otherwise—if $(\alpha c_0/\beta)U \leq S < (\alpha c_0)U$—

$$D_V(S) = \left(1 - \frac{\alpha c_0}{2\beta}\frac{U}{S}\right) \alpha c_0 U - \left(1 - \frac{1}{\beta}\right) S - U. \tag{B-30}$$

Notice that $D_V(0) = -U < 0$ and D_V is concave, $D_V'' < 0$. This means that if $D_V(\alpha c_0 U) > 0$, then D_V crosses 0 exactly once, such that there exists some S_0 such that $D_V > 0$ if and only if $S > S_0$.

We can simplify the expression for $D_V(\alpha c_0 U)$ as follows:

$$D_V(\alpha c_0 U) = \left(\frac{\alpha c_0}{2\beta} - 1\right) U. \tag{B-31}$$

This is positive if $(\alpha c_0/\beta) > 2$, which completes the proof of Proposition 4.

Proof of Proposition 5: The proposition is as follows: Assume that $D = 0$ and s is a sum of two independent uniform distributions, $s = s_1 + s_2$, where s_1 and s_2 follow independent uniform distributions over $[-S_1, S_1]$ and $[-S_2, S_2]$, respectively. If $\beta > 2$ and $\alpha c_0 U > -(\beta/2)F_s^{-1}(1/2 - 1/\beta)$ (a sufficient condition for which is $\alpha c_0 > (S_1 + S_2)/U$), the cost of implementing monetary policy, $V(R)$, has two local minima: one at $R = \bar{R}$ and another, denoted R_A, at $R > \bar{R} + U$. Otherwise, the cost function has local minima only within the steep portion of the demand curve, $R \in [\bar{R}, \bar{R} + U]$.

As in the proof of Proposition 1, it is sufficient to prove the following:

- $V'(\bar{R}) > 0$.
- $V'' \leq 0$ for $R < R_0$.
- $V'' \geq 0$ for $R > R_0$.

- $\lim_{R\to\infty} V'(R) > 0$.
- $V'(R_0^+) < 0$, where $V'(R_0^+)$ denotes the right limit of V' at R_0, if and only if $\beta > 2$ and $\alpha c_0 U > -(\beta/2)F_s^{-1}(1/2 - 1/\beta)$.

Following the proof of Proposition 1, for $R < R_0$,

$$\frac{dV}{dR} = -\alpha c_0[1 - F_s(\frac{\alpha c_0}{\beta}(R_0 - R))] + \alpha c_0 F_s(\frac{\alpha c_0}{\beta}(R_0 - 2U - R)) + 1. \tag{B-32}$$

Since the distribution of s is symmetric around zero, $V'(\bar{R}) = 1 > 0$. Also, both instances of F_s decreases in R, implying $V'' \leq 0$. This proves the first two inequalities.

For $R > R_0$, the optimal choice of x is $x = 0$ if $s \leq -(\alpha/\beta)(r_P - r_{IOR}) + R_0 - R$ or if $s \geq R_0 - R$; and is $x = -s$ otherwise, following the characterization in the proof of Proposition 1. This implies that the derivative of V has the following form:

$$\begin{aligned}\frac{dV}{dR} &= -\beta[F_s(R_0 - R) - F_s(-\frac{\alpha}{\beta}(r_P - r_{IOR}) + R_0 - R)] + 1 \\ &= -\beta Prob(-\frac{\alpha}{\beta}(r_P - r_{IOR}) + R_0 - R \leq s \leq R_0 - R) + 1.\end{aligned} \tag{B-33}$$

Since the density of s is maximum at 0 and (weakly) decreases as s moves away from 0, the probability in the expression decreases in s, implying $V'' \geq 0$. In addition, as $R \to \infty$, $V'(R) = 1 > 0$.

We only need to prove the last inequality. Note that

$$V'(R_0^+) = -\beta\left[\frac{1}{2} - F_s(-\frac{2\alpha c_0}{\beta}U)\right] + 1. \tag{B-34}$$

If $S_1 + S_2 \leq (2\alpha c_0/\beta)U$, this expression becomes $-(\beta/2) + 1$, which is negative if and only if $\beta > 2$. Otherwise, the expression is negative if and only if $F_s(-2\alpha c_0 U/\beta) < 1/2 - 1/\beta$, which is equivalent to $-(\beta/2)F_s^{-1}(1/2 - 1/\beta) < \alpha c_0 U$.

To summarize, $V'(R_0^+) < 0$ holds if and only if either (i) $\beta > 2$ and $S_1 + S_2 \leq (2\alpha c_0/\beta)U$; or (ii) $S_1 + S_2 > (2\alpha c_0/\beta)U$ and $-(\beta/2)F_s^{-1}(1/2 - 1/\beta) < \alpha c_0 U$. Condition (i) implies $S_1 + S_2 < \alpha c_0 U$. For condition (ii), note that $F_s^{-1}(1/2 - 1/\beta) \geq F_t^{-1}(1/2 - 1/\beta) = -2(S_1 + S_2)/\beta$ for t that follows a uniform distribution over $[-S_1 - S_2, S_1 + S_2]$. Therefore, $-(\beta/2)F_s^{-1}(1/2 - 1/\beta) \leq S_1 + S_2$, as stated by the proposition. This implies that if $\beta \leq 2$, then no set of coefficient will satisfy condition (ii), thus condition (ii) implies $\beta > 2$. Also, $F_s^{-1}(1/2 - 1/\beta) \leq$

$F_u^{-1}(1/2 - 1/\beta) = (S_1 + S_2)(\sqrt{1 - 2/\beta} - 1)$ for a random variable u defined as $u = u_1 + u_2$, where u_1 and u_2 each follow i.i.d uniform distribution over $[-(S_1+S_2)/2, (S_1+S_2)/2]$. Thus, $-(\beta/2)F_s^{-1}(1/2 - 1/\beta) < \alpha c_0 U$ implies $(S_1 + S_2)(1 - \sqrt{1 - 2/\beta}) < (2\alpha c_0/\beta)U$, which is a necessary condition for $S_1 + S_2 \leq (2\alpha c_0/\beta)U$.

Therefore, $V'(R_0^+) < 0$ if and only if $\beta > 2$ and $-(\beta/2)F_s^{-1}(1/2 - 1/\beta) < \alpha c_0 U$, which completes the proof of Proposition 5.

Proof of Proposition 6: The proposition is as follows: Assume $S + D < U$ and define two economies 1 and 2 as follows: In economy 1, the supply shock s is a sum of two independent uniform distributions over $[-S, S]$ and $[-D, D]$ and the demand shock d is zero. In economy 2, the supply shock s is uniform over $[-S, S]$ and the demand shock d is uniform over $[-D, D]$.

If there exists a local optimum $R > \bar{R} + U$ in economy 1, then there exists a local optimum R_A over $R > \bar{R} + U$ in economy 2, too. Furthermore, if the ample-reserve regime is optimal in economy 1—the local optimum at $R > \bar{R} + U$ is preferred to $R = \bar{R}$—then the ample-reserve regime is also optimal in economy 2.

This is a generalized version of Propositions 1 and 5. The basic ideas underlying the proof are the same, but we rely less on deriving closed-form expressions because they become more complicated with the presence of demand shocks. To prove the existence of local optimum in $R > R_0$, it is sufficient to prove the following five inequalities for economy 2 as in Proposition 1:

- $V'(\bar{R}) > 0$.
- $V'' \leq 0$ for $R < R_0$.
- $V'' \geq 0$ for $R > R_0$.
- $\lim_{R\to\infty} V'(R) > 0$.
- $V'(R_0^+) < 0$, where $V'(R_0^+)$ denotes the right limit of V' at R_0, if $V'(R_0^+) < 0$ in economy 1.

We write the central bank's cost function as follows:

$$V(R) = \int W(R, s) f_s(s) ds + R. \tag{B-35}$$

$W(R,s)$ is the expected cost due to interest rate volatility and central bank operations given (R,s) and f_s is the probability density function of s. We can write

$$W(R,s) = \min_x \left[\beta|x| + \int \alpha|r(R+s+x+e) - r(R)|f_d(e)de \right]. \tag{B-36}$$

We denote the demand shock as e instead of d to avoid confusion of notation involving integrals. $f_d(e)$ is the probability density function of d. Also, we write $R+s+x+e$ instead of $R+s+x-e$, which we can do because e's distribution is symmetric around zero.

Taking the derivative of $W(R,s)$ with respect to R, we have, for $\bar{R} \le R \le \bar{R}+U$,

$$\begin{aligned}
\frac{\partial W}{\partial R} &= \frac{\partial}{\partial R}\left[\beta|x| + \int \alpha|r(R+s+x+e) - r(R)|f_d(e)de \right] \\
&= \int \alpha \frac{\partial}{\partial R}|r(R+s+x+e) - r(R)|f_d(e)de \\
&= -\alpha c_0 Prob(R+s+x+e \ge \bar{R}+U) + \alpha c_0 Prob(R+s+x+e \le \bar{R}-U) \\
&= -\alpha c_0[1 - F_d(\bar{R}-R-s-x+U) - F_d(\bar{R}-R-s-x-U)].
\end{aligned} \tag{B-37}$$

F_d denotes the cumulative distribution function of demand, e, and x is an optimal choice that is implicitly a function of R and x. In the second line, however, note that x is not treated as a function of R in calculating the partial derivative. The terms including $\partial x/\partial R$ cancel out because x is either an optimal choice satisfying the first-order condition—envelope property—or $x=0$ is optimal, in which case $\partial x/\partial R = 0$.

At $R=\bar{R}$, given the symmetry of r around R, $x(-s) = x(s)$. Therefore,

$$\begin{aligned}
\frac{\partial W}{\partial R}(s) + \frac{\partial W}{\partial R}(-s) &= \alpha c_0[F_d(-s-x(s)+U) + F_d(-s-x(s)-U) \\
&\quad + F_d(s-x(-s)+U) + F_d(s-x(-s)-U) - 2] \\
&= \alpha c_0[F_d(-s-x(s)+U) + F_d(s+x(s)-U) \\
&\quad + F_d(-s-x(s)-U) + F_d(s+x(s)+U) - 2] \\
&= 0.
\end{aligned} \tag{B-38}$$

Due to the symmetry of the distribution of e around 0.

Therefore,

$$V'(R) = \int \frac{\partial W}{\partial R} f_s(s)ds + 1 = 1. \tag{B-39}$$

This proves the first inequality.

To calculate V'', we note

$$\frac{\partial^2 W}{\partial R^2} = -\alpha c_0[f_d(\bar{R} - R - s - x + U) + f_d(\bar{R} - R - s - x - U)]\left(1 + \frac{\partial x}{\partial R}\right). \tag{B-40}$$

Therefore,

$$\begin{aligned} V'' &= \int \frac{\partial^2 W}{\partial R^2} f_s(s) ds \\ &= \int -\alpha c_0[f_d(\bar{R} - R - s - x + U) + f_d(\bar{R} - R - s - x - U)]\left(1 + \frac{\partial x}{\partial R}\right) f_s(s) ds. \end{aligned} \tag{B-41}$$

To show $V'' \leq 0$, it is sufficient to show that $1 + \partial x/\partial R \geq 0$ for any (R, s). Recall that x is an optimizer of the following problem:

$$\min_x \left[\beta|x| + \int \alpha|r(R + s + x + e) - r(R)| f_d(e) de\right]. \tag{B-42}$$

If $x = 0$, then $\partial x/\partial R = 0$ because it is a boundary solution. Therefore, $1 + \partial x/\partial R = 1 \geq 0$.

Suppose that $x \neq 0$. Then, the FOC for x is

$$\begin{aligned} 0 &= \beta sign(x) + \int \alpha \frac{\partial}{\partial x}|r(R + s + x + e) - r(R)| f_d(e) de \\ &= \beta sign(x) + \alpha c_0[-Prob(-s - x + \bar{R} - U - R < e < -s - x) \\ &+ Prob(-s - x < e < -s - x + \bar{R} + U - R)]. \end{aligned} \tag{B-43}$$

$sign(x)$ is 1 if $x > 0$ and -1 if $x < 0$. The second line can be understood as follows: If $e < -s - x$, then increasing x moves $r(R + s + x + e)$ toward $r(R)$, reducing the cost. However, if $e > -s - x$, then increasing x moves $r(R + s + x + e)$ away from $r(R)$; exceptions are when $r(R + s + x + e)$ is at the flat portion of the demand curve, which occurs if $e < -s - x + \bar{R} - U - R$ or $e > -s - x + \bar{R} + U - R$.

Taking the total derivative of the FOC with respect to R, we have

$$\begin{aligned} 0 = \alpha c_0[&[2f_d(-s - x) - f_d(-s - x + \bar{R} + U - R) - f_d(-s - x + \bar{R} - U - R)]\frac{\partial x}{\partial R} \\ &- f_d(-s - x + \bar{R} + U - R) - f_d(-s - x + \bar{R} - U - R)]. \end{aligned} \tag{B-44}$$

Given that f_d is the probability density for a uniform distribution, the coefficient on $\partial x/\partial R$ is non-negative, which implies $\partial x/\partial R \geq 0$, sufficient for the proof, except when the coefficient is zero. This can happen if the densities in the expression for the coefficient are either all zero or all positive.

If they are all zero, it implies that the probabilities in the expression for the FOC (for x) are 0 or 1, implying that the FOC will not hold. If they are all positive, it means that the probabilities in the FOC are not a function of x, and the FOC can hold only for a single value of R. Therefore, we can ignore this case for the purpose of the proof. This completes the proof of the second inequality, $V'' \leq 0$ for $R < \bar{R} + U$.

We turn to characterize V and its derivatives for $R > R_0$. Expressions for W and its derivatives are simpler because $r(R)$ is at the floor and there are only one-sided deviations:

$$\begin{aligned}
\frac{\partial W}{\partial R}(R,s) &= \frac{\partial}{\partial R}\left[\beta|x| + \int \alpha|r(R+s+x+e) - r(R)|f_d(e)de\right] \\
&= \int \alpha\frac{\partial}{\partial R}|r(R+s+x+e) - r(R)|f_d(e)de \\
&= -\alpha c_0 Prob(\bar{R} - U \leq R+s+x+e \leq \bar{R}+U) \\
&= -\alpha c_0[F_d(\bar{R} - R - s - x + U) - F_d(\bar{R} - R - s - x - U)]. \qquad \text{(B-45)}
\end{aligned}$$

As discussed earlier, we used the envelope property to remove terms involving $\partial x/\partial R$.

Therefore,

$$\frac{\partial^2 W}{\partial R^2} = \alpha c_0[f_d(\bar{R} - R - s - x + U) - f_d(\bar{R} - R - s - x - U)]\left(1 + \frac{\partial x}{\partial R}\right). \qquad \text{(B-46)}$$

For $R > R_0$, the central bank chooses only $x \geq 0$; $x < 0$ is not optimal because it increases the interest rate cost term $|r(R+s+x+e) - r(R)|$. Thus the optimal x is either 0 or satisfies the following FOC, derived earlier:

$$\begin{aligned}
0 &= \beta + \int \alpha\frac{\partial}{\partial x}|r(R+s+x+e) - r(R)|f_d(e)de \\
&= \beta - \alpha c_0[F_d(\bar{R} - R - s - x + U) - F_d(\bar{R} - R - s - x - U)]. \qquad \text{(B-47)}
\end{aligned}$$

Differentiating the FOC with respect to R, we have

$$0 = \alpha c_0[f_d(\bar{R} - R - s - x + U) - f_d(\bar{R} - R - s - x - U)]\left(1 + \frac{\partial x}{\partial R}\right). \qquad \text{(B-48)}$$

Note that the right-hand side is identical to the expression for $\partial^2 W/\partial R^2$, implying $\partial^2 W/\partial R^2 = 0$.

Let A_R denote the set of $s \in [-S, S]$ such that $\partial^2 W/\partial R^2 < 0$; for any such s, the interior solution $x = 0$ is optimal. The expression for the second derivative implies that $|R + s - (\bar{R} - U)| \le D$ and $|R + s - (\bar{R} + U)| > D$, so that $\partial^2 W/\partial R^2 = -\alpha c_0/(2D)$.

Note that $s < \bar{R} - R$ given that $R + s$ is closer to $\bar{R} - U$ than to $\bar{R} + U$. For any $s \in A_R$, we define $t(s) = s - 2(R + s - \bar{R}) = 2\bar{R} - 2R - s$, a function of s. This makes $R + t(s) - \bar{R} = \bar{R} - (R + s)$, implying $|t(s)| < |s|$ and $t(s) \in [-S, S]$. Also, it implies $Prob(\bar{R} - U \le R + s + e \le \bar{R} + U) = Prob(\bar{R} - U \le R + t(s) + e \le \bar{R} + U)$. This makes the derivative of the implementation cost with respect to x—the right-hand side of the FOC derived earlier—evaluated $x = 0$ the same for s and $t(s)$, which means that it is positive. Furthermore, the derivative of the cost increases in x at $t(s)$ over $x > 0$; the derivative can be written as $\beta - \alpha c_0 Prob(\bar{R} - U \le R + t(s) + x + e \le \bar{R} + U)$, and the probability in the expression decreases in x given that $R + t(s) + x + e \ge \bar{R} - U$ with probability 1 for $x > 0$.

This implies that $x = 0$ is optimal for $t(s)$. Therefore, $\partial^2 W/\partial R^2(t(s)) = \alpha c_0[f_d(\bar{R} - R - t + U) - f_d(\bar{R} - R - t - U)]\left(1 + \frac{\partial x}{\partial R}\right) = \alpha c_0/(2D)$, which equals $-\partial^2 W/\partial R^2(s)$. Let us define $B_R \equiv \{t(s) | s \in A_R\}$.

Then,

$$
\begin{aligned}
V'' &= \int \frac{\partial^2 W}{\partial R^2} f_s(s) ds \\
&= \int_{[-S,S]-A_R-B_R} \frac{\partial^2 W}{\partial R^2} f_s(s) ds + \int_{A_R} \frac{\partial^2 W}{\partial R^2} f_s(s) ds + \int_{B_R} \frac{\partial^2 W}{\partial R^2} f_s(s) ds \\
&= \int_{[-S,S]-A_R-B_R} \frac{\partial^2 W}{\partial R^2} f_s(s) ds \ge 0. \qquad \text{(B-49)}
\end{aligned}
$$

Next, we prove the fourth inequality, $\lim_{R \to \infty} V'(R) > 0$. Recall that

$$
W(R, s) = \min_x \left[\beta |x| + \int \alpha |r(R + s + x + e) - r(R)| f_d(e) de \right]. \qquad \text{(B-50)}
$$

If $R > \bar{R} + U + S + D$, then by simply choosing $x = 0$ for all $s \in [-S, S]$, the central bank can minimize W at 0. This implies that for $R > \bar{R} + U + S + D$, $\partial W/\partial R = 0$ for all $s \in [-S, S]$. Therefore, for $R > \bar{R} + U + S + D$,

$$
V' = \int \frac{\partial W}{\partial R} f_s(s) ds + 1 = 1. \qquad \text{(B-51)}
$$

This proves $\lim_{R \to \infty} V'(R) > 0$.

Lastly, we consider the fifth inequality, $V'(R_0^+) < 0$. For convenience, we a random variable u as the sum of s and e, so that it represents the distribution of the supply shock in economy 1.

We can write

$$V(R) = Y(R) + \int [W(R,s) - Z(R,s)] f_s(s) ds. \tag{B-52}$$

Note that $Y(R)$ and $Z(R,s)$ denote the expected implementation cost functions that are unconditional—like $V(R)$—and conditional on s—like $W(R,s)$—for economy 1, respectively. By assumption, $Y'(R_0^+) < 0$, so it is sufficient to prove that $(d/dR)\int[W(R,s) - Z(R,s)]f_s(s)ds \leq 0$ for $R > R_0$.

Note that

$$Z(R,s) = \int \min_x [\beta|x| + \alpha|r(R+s+x+e) - r(R)| f_d(e) de]. \tag{B-53}$$

In the expression, x depends on both s and e. Since $S + D < U$ by assumption, the optimal choice of operation x for economy 1 is $x = [R_0 - (R+s+e)]^+$, where $[y]^+$ denotes the positive part of y, $max(y, 0)$, as discussed in the proof of Proposition 1. Therefore,

$$Z(R,s) = \int \beta[R_0 - (R+s+e)]^+ f_d(e) de. \tag{B-54}$$

Note that in the expression for W, x can only depend on s:

$$W(R,s) = \min_x \left[\beta|x| + \int \alpha|r(R+s+x+e) - r(R)| f_d(e) de\right]. \tag{B-55}$$

The optimal choice of x is nonnegative because $r(R)$ is at the lower bound of r. The derivative of the second term with respect to x is $-\alpha c_0 Prob(R+s+x+e < R_0|s) = -\alpha c_0 Prob_s(R+s+x+e < R_0)$, which linearly increases in $R+s+x$ over $-D < R+s+x-R_0 < D$ from $-\alpha c_0$ to 0 and is constant at $-\alpha c_0$ over $R+s+x-R_0 < -D$ and 0 over $R+s+x-R_0 > D$. Therefore, the optimal solution x satisfies the following:

$$-\alpha c_0 Prob_s(R+s+x+e < R_0) = -\beta \text{ if } -\alpha c_0 Prob_s(R+s+e < R_0) \leq -\beta; \tag{B-56}$$

and $x = 0$ otherwise.

Let δ be the solution to $-\alpha c_0 Prob(R_0 + \delta + e < R_0) = -\beta$. Given the density of e,

$$\delta = \left(1 - \frac{2\beta}{\alpha c_0}\right) D. \tag{B-57}$$

Then, we can write the optimal choice of x as follows:

$$x(R, s) = [R_0 + \delta - (R + s)]^+. \tag{B-58}$$

And the expression for W becomes

$$\begin{aligned} W(R, s) &= \int \left[\beta[R_0 + \delta - (R + s)]^+ + \alpha|r(R + s + [R_0 + \delta - (R + s)]^+ + e) - r(R)|\right] f_d(e)de \\ &= \int w(R + s, e) f_d(e)de. \end{aligned} \tag{B-59}$$

The function w is defined as the integrated expression between the brackets, which is a function of $R + s$ and e. Then, using y to denote the first argument of w,

$$w(y, e) = \begin{cases} \beta[R_0 + \delta - y] + \alpha c_0[-\delta - e]^+ \text{ if } y \leq R_0 + \delta; \\ \alpha c_0[R_0 - (R + s + e)]^+ \text{ otherwise.} \end{cases} \tag{B-60}$$

Then,

$$\begin{aligned} \int [W(R, s) - Z(R, s)] f_s(s)ds &= \int \int [w(R + s, e) - \beta[R_0 - (R + s + e)]^+] f_d(e) f_s(s)deds \\ &= \int \int [w(y, e) - \beta[R_0 - (y + e)]^+] f_d(e) f_s(y - R)dedy. \end{aligned} \tag{B-61}$$

Let $v(y, e)$ denote the term in the brackets, $w(y, e) - \beta[R_0 - (y + e)]^+$. Then, the expression for v can be simplified as follows:

$$v(y, e) = \begin{cases} (\alpha c_0 - \beta)[-\delta - e - [y - R_0 - \delta]^+]^+ & \text{if } e \leq -\delta; \\ \beta min(\delta + e, [-y + R_0 + \delta]^+) & \text{otherwise.} \end{cases} \tag{B-62}$$

The expression makes it clear that for any given e, $v(y, e)$ is monotonically decreasing in y.

Recall that we need to prove $(d/dR) \int [W(R, s) - Z(R, s)] f_s(s)ds \leq 0$. Using D to denote the Dirac delta function (to avoid confusion with δ defined earlier), we have

$$\begin{aligned} \frac{d}{dR} \int [W(R, s) - Z(R, s)] f_s(s)ds &= \frac{d}{dR} \int \int v(y, e) f_d(e) f_s(y - R)dedy \\ &= \int \int v(y, e) f_d(e) \frac{1}{2S}[-D(y - (R - S)) + D(y - (R + S))]dedy \\ &= \frac{1}{2S} \int [v(R + S, e) - v(R - S, e)]de \leq 0. \end{aligned} \tag{B-63}$$

This proves the first part of the proposition about the existence of a local optimum R_A over $R > \bar{R} + U$ in economy 2.

Next, we prove that the optimality of the ample-reserve regime in economy 1 (over the scarce-reserve regime) is sufficient for the optimality of the ample-reserve regime in economy 2. The optimality of the ample-reserve regime in economy 1 implies $Y(\bar{R}) \geq Y(R_A)$ for some $R_A > R_0$, where $Y(R)$ is the implementation cost in economy 1 as defined earlier. It is sufficient to prove that $V(\bar{R}) \geq V(R_A)$. A sufficient condition for this result is $V(\bar{R}) - Y(\bar{R}) \geq V(R_A) - Y(R_A)$, which we now prove.

Given $S + D < U$, the optimal choice of x in economy 1 for $R = \bar{R}$ completely offsets shocks: $x = -s - e$. Therefore,

$$Y(\bar{R}) = \int\int \beta|s + e| f_d(e) f_s(s) deds. \tag{B-64}$$

In economy 2, the optimal choice of x can only depend on s:

$$V(\bar{R}) = \int\int [\beta|x(s)| + \alpha c_0|s - x(s) + e|] f_d(e) f_s(s) deds. \tag{B-65}$$

Therefore,

$$\begin{aligned} V(\bar{R}) - Y(\bar{R}) &= \int\int [\alpha c_0|s - x(s) + e| + \beta|x(s)| - \beta|s + e|] f_d(e) f_s(s) deds \\ &\geq \int\int [\alpha c_0|s - x(s) + e| - \beta|s - x(s) + e|] f_d(e) f_s(s) deds \\ &\geq \int\int [\alpha c_0|e| + \beta|e|] f_d(e) f_s(s) deds \\ &= (\alpha c_0 - \beta) \int |e| f_d(e) de \\ &= \frac{1}{2}(\alpha c_0 - \beta) D. \end{aligned} \tag{B-66}$$

Using the notation defined earlier,

$$\begin{aligned} V(R_A) - Y(R_A) &= \int\int v(R_A + s, e) f_d(e) f_s(s) deds \\ &\leq \int\int v(R_0 + s, e) f_d(e) f_s(s) deds \end{aligned} \tag{B-67}$$

From the expression for $v(y, e)$ derived earlier, notice that $v(y, e) \leq (\alpha c_0 - \beta)(-\delta - e)$ if

$e \leq -\delta$ and $v(y,e) \leq \beta(\delta + e)$ otherwise. Therefore,

$$\begin{aligned}
V(R_A) - Y(R_A) &\leq \int\int v(R_0 + s, e) f_d(e) f_s(s) deds \\
&\leq \frac{1}{2}\left[(\alpha c_0 - \beta)(D-\delta)Prob(e \leq -\delta) + \beta(D+\delta)Prob(e > -\delta)\right] \\
&= \frac{1}{4D}\left[(\alpha c_0 - \beta)(D-\delta)^2 + \beta(D+\delta)^2\right] \\
&= \left[(1 - \frac{\beta}{\alpha c_0})(\frac{\beta}{\alpha c_0})^2 + \frac{\beta}{\alpha c_0}(1 - \frac{\beta}{\alpha c_0})^2\right]\alpha c_0 D \\
&= (\alpha c_0 - \beta)\frac{\beta}{\alpha c_0}D. \qquad \text{(B-68)}
\end{aligned}$$

Clearly, $V(\bar{R}) - Y(\bar{R}) \geq V(R_A) - Y(R_A)$ if $\alpha c_0 \geq 2\beta$.

To complete the proof, we assume $\beta < \alpha c_0 < 2\beta$. Notice that

$$\begin{aligned}
V(R_A) - Y(R_A) &\leq \int\int v(R_0 + s, e) f_d(e) f_s(s) deds \\
&= \int_{s\in[-S,0]}\int [v(R_0 + s, e) + v(R_0 - s, e)] f_d(e) f_s(s) deds \qquad \text{(B-69)}
\end{aligned}$$

The last line follows from the fact that s follows a symmetric distribution around zero.

Note that with $\beta < \alpha c_0 < 2\beta$, $\delta < 0$. For $s < 0$, the expression for v derived earlier implies that

$$v(R_0 - s, e) = \begin{cases} (\alpha c_0 - \beta)(-e + s) & \text{if } e \leq s; \\ 0 & \text{otherwise.} \end{cases} \qquad \text{(B-70)}$$

Therefore,

$$\int v(R_0 - s, e) f_d(e) de = \frac{1}{4}D(\alpha c_0 - \beta)([1 + \frac{s}{D}]^+)^2. \qquad \text{(B-71)}$$

For $s \leq \delta$, the expression for v is such that

$$v(R_0 + s, e) = \begin{cases} (\alpha c_0 - \beta)(-e - \delta) & \text{if } e \leq -\delta; \\ \beta(e + \delta) & \text{if } -\delta < e \leq min(-s, D); \\ \beta(-s + \delta) & \text{otherwise.} \end{cases} \qquad \text{(B-72)}$$

This implies

$$\begin{aligned}
\int v(R_0 + s, e) f_d(e) de &= D(\alpha c_0 - \beta)(\frac{\beta}{\alpha c_0})^2 + D\beta(1 - \frac{\beta}{\alpha c_0})^2 - \frac{1}{4}D\beta([1 + \frac{s}{D}]^+)^2 \\
&= (\alpha c_0 - \beta)\frac{\beta}{\alpha c_0}D - \frac{1}{4}D\beta([1 + \frac{s}{D}]^+)^2. \qquad \text{(B-73)}
\end{aligned}$$

Thus,

$$\int [v(R_0+s,e)+v(R_0-s,e)]f_d(e)de = (\alpha c_0-\beta)\frac{\beta}{\alpha c_0}D + \frac{1}{4}D(\alpha c_0 - 2\beta)([1+\frac{s}{D}]^+)^2$$
$$\leq (\alpha c_0-\beta)\frac{\beta}{\alpha c_0}D. \qquad \text{(B-74)}$$

For $\delta < s < 0$, the expression for v implies

$$v(R_0+s,e) = \begin{cases} (\alpha c_0-\beta)(-e-s) & \text{if } e \leq -s; \\ 0 & \text{otherwise.} \end{cases} \qquad \text{(B-75)}$$

This implies

$$\int v(R_0+s,e)f_d(e)de = \frac{1}{4}D(\alpha c_0-\beta)(1-\frac{s}{D})^2. \qquad \text{(B-76)}$$

Therefore,

$$\int [v(R_0+s,e)+v(R_0-s,e)]f_d(e)de = \frac{1}{4}D(\alpha c_0-\beta)\left[(1-\frac{s}{D})^2 + ([1+\frac{s}{D}]^+)^2\right]$$
$$= \frac{1}{4}D(\alpha c_0-\beta)\left[(1-\frac{s}{D})^2 + (1+\frac{s}{D})^2\right]$$
$$\leq \frac{1}{2}D(\alpha c_0-\beta) < (\alpha c_0-\beta)\frac{\beta}{\alpha c_0}D. \qquad \text{(B-77)}$$

Finally, integrating these differences over s, we have

$$V(R_A)-Y(R_A) = \int_{s\in[-S,0]}\int [v(R_0+s,e)+v(R_0-s,e)]f_d(e)f_s(s)deds$$
$$\leq \int_{s\in[-S,0]} (\alpha c_0-\beta)\frac{\beta}{\alpha c_0}Df_s(s)ds$$
$$= \frac{1}{2}(\alpha c_0-\beta)\frac{\beta}{\alpha c_0}D$$
$$< V(\bar{R})-Y(\bar{R}). \qquad \text{(B-78)}$$

This completes the proof.

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