Fundamental Disagreement about Monetary Policy and the Term Structure of Interest Rates

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Abstract

Using a unique data set of individual professional forecasts, we document disagreement about the future path of monetary policy, particularly at longer horizons. The stark differences in short rate forecasts imply strong disagreement about the risk-return trade-off of longer-term bonds. Longer-horizon short rate disagreement co-moves with term premiums. We estimate an affine term structure model in which investors hold heterogeneous beliefs about the long-run level of rates. Our model fits U.S. Treasury yields and the short rate paths predicted by different groups of professional forecasters very well. About one-third of the variation in term premiums is driven by short rate disagreement.

Key words: disagreement, heterogeneous beliefs, noisy information, speculation, survey forecasts, yield curve, term premium
1 Introduction

Bond yields reflect investors’ expectations about the future path of short rates as well as their attitudes toward risk. Most term structure models specify these two components of interest rates for a representative investor. While this provides a reasonable starting point for many analyses, it may mask important dynamics among investors and thus fail to provide a complete account of the driving forces behind bond yields. In this paper, we propose and estimate a term structure model which explicitly incorporates differences in beliefs about future short rates.

It has been widely documented that economic agents hold heterogeneous beliefs about future macroeconomic outcomes. This is not only true for households and firms, but also for professional forecasters who are, arguably, among the best informed economic agents.¹ To determine the fair value of longer-term bonds, investors need to make forecasts of short rates far into the future. Andrade, Crump, Eusepi, and Moench (2016) document that professional forecasters hold strongly different beliefs about the path of short rates, particularly at longer horizons. Such fundamental disagreement about short rates implies differences in the perceived risk-return tradeoff of longer-term bonds. These should induce speculative trading which, in turn, contributes to time-variation in term premiums (Xiong and Yan (2010)).

This paper makes two contributions. We start by using a unique and novel dataset of professional forecasters’ individual longer-run expectations from Blue Chip Economic Indicators (BCEI) to document the following facts. First, we confirm that disagreement about future short rates is substantial, particularly at intermediate to long horizons. Second, since term premiums are defined as the difference between observed yields and average expected future short rates, the stark differences in short rate forecasts imply strong disagreement about the risk-return tradeoff of longer-term bonds. Third, disagreement about short rates comoves strongly with the term premium of the consensus forecaster as well as with estimates of the term premium from a reduced-form no-arbitrage model. To the best of our knowledge, no such individual longer-term forecast data has previously been studied in the literature.

In a next step, we build a term structure model that can match these facts. Our model features two agents who disagree about the future path of short rates. We assume that the comovement of bond yields is fully captured by three factors: level, slope, and curvature. While the slope and curvature factors are stationary, the level factor has a time-varying long-run mean which itself follows a random walk. Both agents perfectly observe the three yield curve factors, know the parameters of their data-generating process, have identical preferences and perceive the same volatility of shocks. However, in line with the documented evidence about short-rate disagreement, we assume that they hold different beliefs about the

long-run mean of the level factor. As they trade bonds at equilibrium prices, their pricing kernels and hence their perceived risk-return trade-off of longer-term bonds differ. We follow the term structure literature and assume that bond prices are a time-invariant function of the three pricing factors which follow a stationary VAR with a constant mean. We then show that no-arbitrage restrictions imply that each investor’s price of level risk moves one-to-one with her belief about the future level of rates.

We fit our model using zero coupon Treasury yields as well as the term structure of survey forecasts of the federal funds rate for two different hypothetical investors: the top-10 and bottom-10 average responses of the Blue Chip Financial Forecasts (BCFF) survey. Our model fits yields and the two survey forecast paths of the short rate very well in our sample starting in 1983. In the estimated model, investors expecting higher future short rates perceive term premiums to be negative on average for most maturities. In contrast, investors predicting short rates to be low perceive average term premiums to range from about 50 basis points at the one-year to about two percent at the ten-year maturity.

To study the implications of heterogeneous beliefs for the term premium, we introduce a representative investor whose term premium is a weighted-average of the term premium of each individual investor. We obtain the weights by solving a simple portfolio allocation problem under the assumption of mean-variance preferences. Intuitively, an investor expecting higher (lower) future short rates assigns a lower (higher) expected return to longer-term bonds and will tilt her portfolio accordingly. As the weights track the relative wealth of each agent and since short-rates were on a secular downward trend throughout our sample period, this implies that the investor expecting lower future short rates becomes more important for the marginal pricing of risk in the economy as time evolves.

Strikingly, the representative investor’s term premium does not display any trend unlike common estimates of the term premium in the literature. The ten-year term premium is essentially rangebound between 0% and 2% throughout our sample. This stationarity is the result of a composition effect. The top-10 investor who expects short rates to be relatively high and thus perceives a lower term premium, begins the sample with a large wealth share. In contrast, the term premium of the bottom-10 investor shows a strong downward trend in the first part of the sample, reflecting only a modest decline in expected average future short rates relative to observed yields. However, initially this downward trend has little impact on the representative investor’s term premium because her wealth share starts out small. While the wealth share of the bottom-10 investor is increasing over time, her term premium continues to decline, leading to a stationary term premium of the representative investor.

We quantify the importance of short rate disagreement for risk premiums by focusing on the wedge between the term premium of the representative investor and that of an econometrician who knows the model structure and parameters but filters the time-varying
long-run level of rates from the yield curve factors. This wedge has two components. The first reflects investors’ heterogeneous repricing of risk in response to changes in their beliefs. The second arises because of endogenous wealth fluctuations. In our estimated model, the two components account for about one third of the overall variation in the econometrician’s term premium across maturities. Updates in the pricing of risk, driven by changes in the long-run belief about short rates, are the dominant source of medium and high-frequency movement in the term premium. In contrast, changes in relative wealth primarily contribute at low frequencies.

Our paper contributes to the small but growing literature on bond pricing with heterogeneous beliefs. Conceptually, our approach is most closely related to Xiong and Yan (2010) who propose an equilibrium model of bond markets with two groups of investors who hold different beliefs about the long-run mean of fundamentals. We deviate from their analysis in two important ways. First, while Xiong and Yan (2010) consider heterogeneous beliefs about the inflation target, in our model investors disagree about the expected path of the nominal short rate. Indeed, as shown in Andrade, Crump, Eusepi, and Moench (2016), disagreement about the inflation target is not sufficient to account for the sizable long-run disagreement about the short rate. Second, and most importantly, we embed these heterogeneous beliefs about the long-run level of rates into an estimated affine term structure model. This allows us to assess its empirical relevance and study the implications of heterogeneous policy expectations for term premiums.

Other authors have also studied bond pricing with heterogeneous beliefs. Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018) consider a model in which investors with habit formation utility disagree about the distribution of inflation, not just expected inflation. This disagreement induces heterogeneity in investors’ consumption and investment decisions and, on average, raises real and nominal bond yields. They further document empirically that inflation disagreement has a strong effect on real and nominal bond yields over and above the impact of expected inflation, consistent with their theoretical model. Buraschi and Whelan (2016) study the interactions between risk aversion and disagreement. In their model heterogeneous beliefs arise because agents have different views about the (constant) long-run growth rate of consumption and because their perceptions of the correlation of shocks differs. They find that disagreement has larger effects on equilibrium bond prices when risk aversion is low. More recently, Buraschi, Piatti, and Whelan (2019) aggregate individual expected excess bond returns based on forecasters’ past accuracy in predicting interest rates. In line with our findings, they document that disagreement about bond risk premia is time-varying and persistent. While they show that their measure of aggregate expected bond returns is highly correlated with disagreement about future real growth and inflation, they do not study its comovement with disagreement about nominal short rates that is the focus of our analysis.
Barillas and Nimark (2019) build a model of the term structure in which investors with heterogeneous information sets form higher-order expectations about the beliefs of all other investors. Equilibrium bond prices then reflect a speculative component which depends on investors’ beliefs about the error that the average investor makes when predicting future short rates. Their model suggests that the speculative component explains a sizable fraction of the variation in U.S. Treasury yields. Barillas and Nimark (2017) generalize this model to allow for richer price of risk specifications as used in the empirical term structure literature. In their model, investors observe heterogeneous signals of the state variables driving bond yields. They forecast the forecasts of other investors and engage in speculative trading. In equilibrium, individual investors’ prices of risk then reflect idiosyncratic signals, higher order expectations of the true state variables, as well as investor-specific expectations of maturity-specific shocks. Importantly, in their model the pricing factors follow stationary vector autoregressions under both the risk-neutral and the physical measure implying that investors do not disagree about short rates in the long-run, in contrast, with empirical evidence.

Our paper is also related to the term structure literature using survey information in the model estimation. For example, Kim and Wright (2005) and Piazzesi, Salomao, and Schneider (2015) use consensus survey forecasts to discipline the time-series dynamics under the physical measure. Giacoletti, Laursen, and Singleton (2020) build a dynamic term structure model in which a representative investor updates her beliefs about future bond yields. They find that when this updating is conditioned on the dispersion in bond yield forecasts, the model produces substantially smaller forecast errors. We provide a structural interpretation to their findings by explicitly relating term premium dynamics to investors’ differences in beliefs and resulting relative wealth fluctuations. Finally, Crump, Eusepi, and Moench (2018) use the universe of surveys of professional forecasters to infer the consensus expected path of future short rates. They show that although these short rate expectations show sizable variation, term premiums obtained as the simple difference between yields and expected short rates account for the bulk of yield variation at high and medium-term frequencies.

The remainder of this paper is structured as follows. Section 2 documents some novel facts about short rate disagreement and term premiums. In Section 3, we describe our affine term structure model with heterogeneous beliefs about the long-run level of rates. Section 4 presents the estimation results and uses the model to quantify the importance of belief heterogeneity and relative wealth changes for term premium dynamics. In Section 5 we quantify the importance of disagreement for term premium dynamics. Section 6 concludes.
2 Stylized Facts of Disagreement and Term Premiums

In this section, we motivate our subsequent analysis by providing some novel stylized facts on disagreement about future policy rates and term premiums. Our results are based on the Blue Chip Economic Indicators (BCEI) and the Blue Chip Financial Forecasts (BCFF) surveys. The BC surveys have been conducted monthly since the early 1980s. They ask two partly overlapping panels, of about 40 professional forecasters, to provide forecasts of the quarterly average of a variety of economic and financial variables. Since the mid 1980s, the surveys have also biannually been collecting forecasts from two years as far as 7-to-11 years ahead. While the BCFF survey publishes the individual forecasts for horizons up to six quarters into the future at a monthly frequency, they only report three quantities for the biannual forecasts of horizons of two years and above. These are the average across all forecasters, which we label the “consensus forecast,” as well as the average of the top-10 and bottom-10 responses for a given forecasted variable at a given horizon. While the estimated term structure model with disagreement in Sections 3 and 4 relies on these three forecast series for the federal funds rate from the BCFF survey, we set the stage in this section by providing information also on individual longer term three-month Treasury bill forecasts from the BCEI survey. To the best of our knowledge, no such individual longer-term forecast data has previously been studied in the literature.

Our analysis is motivated by Andrade, Crump, Eusepi, and Moench (2016) who show that the term structure of disagreement about future short rates is upward sloping. This implies that while forecasters agree to a large extent about monetary policy in the near-term, they have strongly opposing views about the medium and longer-term policy outlook, which at least partially reflects disagreement about the fundamentals of the economy. Here, we expand on these results by showing that fundamental or long-term disagreement about short term interest rates $i)$ is not driven by outlier predictions; $ii)$ is a persistent phenomenon in the sense that individual forecasters tend to see high or low future short rates across all forecast horizons; $iii)$ implies sizable fundamental disagreement about term premiums; and $iv)$ is strongly correlated with the term premium of the consensus forecaster and that implied by an affine term structure model.

Figure 1 shows the individual predictions for the three-month Treasury bill at horizons of two and 7-11 years into the future. The left-hand chart shows that while individual longer-term forecasts broadly move together there is a considerable degree of disagreement among forecasters already at the two-year horizon. Specifically, they disagree by as much as six percentage points about the level of the three-month TBill. The strong disagreement is particularly pronounced just after the start of the large-scale asset purchase programs by the Federal Reserve in 2009, but drops considerably when calendar-based forward guidance
Figure 1: Disagreement about short rates at medium and long horizons

Notes:
This figure plots individual forecasts from the Blue Chip Economic Indicators survey for the three-month Treasury Bill at forecast horizons of two and 7-11 years into the future. The red and blue dots represent the top and bottom-10 average responses, respectively. The sample is from 1999-2016.

was introduced in the summer of 2011 (Crump, Eusepi, and Moench (2013)). The chart also shows that the width of the forecast distribution as measured by the difference between the top-10 and bottom-10 average responses is wide and varies considerably over time.

The right-hand chart of Figure 1 provides the predictions of the three-month TBill of the same individuals in the long-run. As expected, there is less of a cyclical element in these forecasts. That said, the chart also shows that the entire distribution of long-run forecasts of the short rate has trended down over the sample. This strongly suggests that the long-run level of the short rate is perceived to vary over time. This feature of forecasters’ beliefs will be a central element of our modeling strategy. Interestingly, while there clearly is a strong common element in the individual forecasts, the distribution at this very long horizon is also quite wide. This indicates that forecasters disagree to a considerable extent about the long-run (fundamental) value of the short term interest rate. Quite strikingly, at the end of our sample some forecasters believe the long-run value of the TBill will remain below two percent while others see it go back to a level of around four percent. These heterogeneous assessments likely reveal sharply different views of the steady state of the economy.

As it is inherently difficult to predict far into the future, one might worry that individual forecasters’ responses are to some extent arbitrary and do not necessarily reflect their views of the world. While we do not observe the names of individual forecasters in our sample of long-term predictions, we are able to trace their forecast paths at any given point in time. We can thus check whether the individual medium to long-run predictions are consistent in
Notes:
This figure plots the rank correlation among individual forecasts from the Blue Chip Economic Indicators (BCEI) survey for the three-month Treasury Bill at adjacent forecast horizons between two and 7-11 years into the future. The dashed lines provide the 5th and 95th probability bands. The sample is from 1999-2016.

The term premium is defined as the difference between the yield on a government bond and the average short rate expected to prevail over the life of the bond. Since we observe survey participants’ individual forecast paths for the short rate, we can compute their individual
Figure 3: Disagreement about term premiums at medium and long horizons

Notes:
This figure plots forward term premiums implied by individual forecasts of the three-month Treasury Bill from the Blue Chip Economic Indicators (BCEI) survey at forecast horizons 1-2 and 7-11 years into the future. The red and blue dots represent the top and bottom-10 average responses, respectively. The sample is from 1999-2016.

term premiums for various forward horizons. Figure 3 displays the evolution of forward term premiums implied by the individual TBill forecasts for the one-to-two year and 7-11 year forward horizons.

The figure clearly shows that the assessment of the compensation that long-term bond investors command differ widely across forecasters. Moreover, especially in the latter part of the sample quite a few survey participants see term premiums in negative territory, possibly suggesting that they view longer term Treasuries as hedges against adverse states of the economy. As before, while individuals’ views about term premiums are quite heterogeneous, the top and bottom-10 average predictions appear to represent well the dispersion of beliefs across the forecaster distribution.

Figure 4 displays the one-two year and the 7-11 year forward term premium implied by the consensus forecaster as well as the difference between the top and bottom-10 average forecasts of the short rate for the corresponding horizon. The charts clearly show that the two measures comove at both horizons, providing suggestive evidence that disagreement is informative for the term premium. The model presented later in the paper generates this comovement.

The positive correlation between a measure of the disagreement about future short rates and the term premium shown above is not restricted to the term premium implied by the consensus forecast. The upper panel of Figure 5 shows the time series of the two and ten-year
Figure 4: Consensus term premium and disagreement about short rates

Notes:
This figure plots forward term premiums implied by the consensus forecast along with the difference between the top and bottom-10 average forecasts for the three-month Treasury Bill from the Blue Chip Economic Indicators (BCEI) survey at forecast horizons 1-2 and 7-11 years into the future. The sample is from 1999-2016.

Treasury term premium obtained from the Adrian, Crump, and Moench (2013) (ACM) model. This no-arbitrage term structure model uses the first five principal components of Treasury yields as pricing factors and does not include survey forecasts in the estimation. We superimpose the difference between the top and bottom-10 average responses at the two-year horizon and the five and 7-11 year ahead horizons, respectively. The charts show a strong comovement of the ACM term premium and short rate disagreement at both horizons. This becomes even more apparent when considering scatter plots of the same series in the bottom panel of Figure 5. Both survey-based and statistical term premiums are strongly correlated with measures of longer-term disagreement about the short rate.

In sum, the results provided in this section show that individual forecasters’ views about future short rates differ quite substantially at all forecast horizons including the very long-run. Moreover, the long-run level of short rates as perceived by individual forecasters drifts slowly over time. The top and bottom-10 average forecasts represent well the differences in beliefs across individuals. Individuals’ forecasts across horizons are strongly correlated suggesting that these forecasts reflect different fundamental views about the economy. Finally, forecast disagreement comoves strongly with different measures of term premiums.

See https://www.newyorkfed.org/research/data_indicators/term_premia.html.
Figure 5: Disagreement about short rates and term premiums

Notes:
This figure plots disagreement measures calculated using survey forecasts and ACM term premiums obtained from the model described in Adrian, Crump, and Moench (2013). The upper two charts display the difference between top-10 and bottom-10 average forecasts of the federal funds rate obtained from the Blue Chip Financial Forecasts (BCFF) survey. One- to four-quarter ahead, five-year ahead and long-horizon (7-11 years) survey forecasts are used. The lower two charts compare two-year and ten-year ACM term premiums with disagreement over similar horizons. Asterisks and circles in the charts are long-horizon forecasts from surveys conducted biannually.

3 A GATSM with Fundamental Disagreement

In this section, we introduce a Gaussian affine term structure model (GATSM) with two investors who hold different beliefs about the future level of rates. As is common in such models, we assume that yields are affine functions of three pricing factors which follow a stationary vector autoregression under the pricing measure. The pricing factors can be portfolios of yields and can be interpreted as level, slope, and curvature. We extend the standard affine model in two ways. First, we assume different dynamics of the pricing factors under the physical measure. Specifically, while the slope and curvature factors are assumed to be stationary, we assume that the level has a time-varying long-run mean which itself follows a random walk. Second, we allow agents to have different beliefs about that long-run level of rates. This assumption is motivated by the evidence presented in the previous section. In the following, we introduce the individual pieces of this specification which will form the
basis for our empirical analysis in Section 4.

### 3.1 The Model

We assume that there are two agents \(i \in \{A, B\}\) who interact in a complete market setting, trading bonds at different maturities. Three factors determine the evolution of bond prices, which we label ‘L’, ‘S’ and ‘C’. We assume that these factors evolve according to the following data generating process:

\[
\begin{align*}
    x_{t}^{L} &= \left(1 - \beta^{L}\right) \mu_{t}^{L} + \beta^{L} x_{t-1}^{L} + \sigma_{L} \epsilon_{t}^{L}, \\
    x_{t}^{S} &= \left(1 - \beta^{S}\right) \mu_{t}^{S} + \beta^{S} x_{t-1}^{S} + \sigma_{S} \epsilon_{t}^{S}, \\
    x_{t}^{C} &= \left(1 - \beta^{C}\right) \mu_{t}^{C} + \beta^{C} x_{t-1}^{C} + \sigma_{C} \epsilon_{t}^{C}, \\
    \mu_{t}^{L} &= \mu_{t-1}^{L} + \sigma_{\mu} \eta_{t}^{L}.
\end{align*}
\]

We can write the system more compactly as

\[
X_{t} = (I - \beta) \mu_{t} + \beta X_{t-1} + \Sigma_{1/2} \epsilon_{t},
\]

where \(X_{t} = (x_{t}^{L}, x_{t}^{S}, x_{t}^{C})'\), \(\mu_{t} = (\mu_{t}^{L}, \mu_{t}^{S}, \mu_{t}^{C})'\), \(\beta = \text{diag} \left(\beta^{L}, \beta^{S}, \beta^{C}\right)\), \(\epsilon_{t} = (\epsilon_{t}^{L}, \epsilon_{t}^{S}, \epsilon_{t}^{C})'\), and \(\Sigma_{\epsilon} = \text{diag} \left(\sigma_{L}^{2}, \sigma_{S}^{2}, \sigma_{C}^{2}\right)\). The short rate is assumed to be linear in the pricing factors:

\[
r_{t} = \delta_{0} + \delta_{1}' X_{t}.
\]

Both agents perfectly observe the three pricing factors \(X_{t}\). They also know the data generating process and the parameters \((\beta^{j}, \sigma_{j})\) with \(j \in \{L, S, C\}\) as well as \(\{\mu^{S}, \mu^{C}, \sigma_{\mu}\}\). However, we assume that the two agents have different beliefs about the drift \(\mu_{t}^{L}\) which determines the long-run mean of the level factor. Both agents believe \(\mu_{t}^{L}\) evolves according to a random walk. Their estimates of the drift evolve according to

\[
\mu_{t}^{i} = \mu_{t-1}^{i} + \eta_{t}^{i}, \quad \text{for } i \in \{A, B\}.
\]

This equation parsimoniously captures the updating of investors’ beliefs about slow-moving trends and reflects agents’ attempts to disentangle short-run macroeconomic developments from long-term changes in the economy. This difference in beliefs across the two agents could be driven, for example, by idiosyncratic signals or some other form of informational friction.\(^3\)

Importantly we assume that the innovations \(\eta_{t}^{i}\) are correlated among each other and with the

\(^3\)For example, Andrade, Crump, Eusepi, and Moench (2016) show that the observed term structures of disagreement can be rationalized in a model with informational frictions where the state variables follow a VAR with slow-moving long-run means which agents filter from the imperfectly observed data.
innovations $\eta^L_t$. This correlation structure embeds the assumption that both agents’ private signals about the innovation to the drift are noisy measures of its true innovation. As such, their signals will also be correlated with each other.

We now turn to equilibrium bond prices. Absence of arbitrage implies that

$$P_t^{(n)} = E_t^i \left( M_{t+1}^i P_{t+1}^{(n-1)} \right), \quad \text{for } i \in \{A, B\}. $$

where $P_t^{(n)}$ is the price of an $n$-period bond in time $t$ and $M_t^i$ denotes the stochastic discount factor of agent $i$. As is common in the affine term structure literature we assume an exponentially affine functional form:

$$M_{t+1}^i = \exp \left( -r_t - \frac{1}{2} \lambda_t^i \Sigma \lambda_t^i - \lambda_t^\epsilon^i \epsilon_{t+1} \right), \quad (3.8)$$

where $\epsilon_t$ are the innovations to the pricing factors and $\lambda_t^i$ denotes the agent-specific vector of market prices of risk associated with these pricing factors. Following Duffee (2002) and many others, we assume that prices of risk are linear in the pricing factors:

$$\lambda_t^i = \lambda_{0,t}^i + \Lambda_1^i X_t, \quad (3.9)$$

where $\lambda_{0,t}^i$ is a $3 \times 1$ vector and $\Lambda_1^i$ a $3 \times 3$ matrix. We assume a time-varying intercept in the market prices of risk to embed the notion that investors’ perceived risk-return tradeoff can fluctuate with their assessment of the long-run mean of the risk factors.

We assume both agents trade at equilibrium bond prices and conjecture that they are exponentially affine in the observed pricing factors:

$$\ln P_t^{(n)} = A_n + B'_n X_t$$

We can then use this in agent $i$’s first order condition to get

$$P_t^{(n)} = E_t^i \left( M_{t+1}^i P_{t+1}^{(n-1)} \right) = E_t^i \left( M_{t+1}^i \exp \left( A_{n-1} + B'_{n-1} X_{t+1} \right) \right) \quad \text{for } i \in \{A, B\}. \quad (3.10)$$

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$$P_t^{(n)} = E_t^i \left( M_{t+1}^i P_{t+1}^{(n-1)} \right) = E_t^i \left( M_{t+1}^i \exp \left( A_{n-1} + B'_{n-1} X_{t+1} \right) \right) \quad \text{for } i \in \{A, B\}. \quad (3.11)$$

\[4\] In line with the existing affine term structure literature, we only consider a solution with a constant intercept under the pricing measure. While alternative solutions with a time-varying intercept are conceivable, specific parametric assumptions about the intercept would be required to ensure a closed-form system of Ricatti equations as the one we derive below.
This yields the typical Ricatti equations:

\[
A_n = A_{n-1} + B'_{n-1} \left[ (I - \beta) \mu_t^i - \Sigma \lambda^i_{0,t} \right] \Psi_0 + \frac{1}{2} B'_{n-1} \Sigma \epsilon_{n-1} \delta_0 \\
B'_{n} = B'_{n-1} \left[ \beta - \Sigma \Lambda^i_1 \Psi_1 \right] - \delta'_{n} \quad \text{for } i \in \{A, B\}.
\] (3.12) (3.13)

An equilibrium where these pricing equations hold and where both agents trade therefore requires that

\[
(I - \beta) \mu_t^i - \Sigma \lambda^i_{0,t} = \Psi_0 \quad i \in \{A, B\} \quad \forall t \in \{1, \ldots, T\} \quad \text{(3.14)}
\]

and

\[
\beta - \Sigma \lambda^i_1 = \Psi_1 \quad i \in \{A, B\}.
\] (3.15)

Hence we can express

\[
\ln P_t^{(n)} = A_n (\Psi_0) + B'_n (\Psi_1) X_t
\] (3.16)

where \(\Psi_0\) is a \(3 \times 1\) vector and \(\Psi_1\) a \(3 \times 3\) matrix denoting the intercept and autoregressive coefficient of \(X_t\) under the pricing measure.

The restrictions in equation (3.14) imply that

\[
\sigma^2 L \lambda^i_{0,t} = \left(1 - \beta^L\right) \mu_t^i - \Psi_0^L \quad \text{for } i \in \{A, B\} \quad \forall t \in \{1, \ldots, T\},
\]

where \(\lambda^i_{0,t}\) is the first element of the vector \(\lambda^i_{0,t}\) and \(\Psi_0^L\) is the first element of \(\Psi_0\). Hence, it follows that

\[
\mu_t^A - \mu_t^B = \frac{\sigma^2 L}{1 - \beta^L} \left(\lambda^A_{0,t} - \lambda^B_{0,t}\right) \quad \forall t \in \{1, \ldots, T\}.
\] (3.17)

Conversely, the restrictions in equation (3.15) imply that

\[
\beta - \Sigma \lambda^A_1 = \beta - \Sigma \lambda^B_1 = \Psi_1
\]

or

\[
\lambda^A_1 = \lambda^B_1 = \lambda_1
\] (3.18)

Combined, we therefore obtain

\[
\frac{\sigma^2 L}{1 - \beta^L} \left(\lambda^A_{t} - \lambda^B_{t}\right) = \mu_t^A - \mu_t^B.
\] (3.19)

In other words, any disagreement about the long-run mean of the level factor translates
one-for-one into a difference in risk attitudes. This is intuitive. Assume investor A anticipates a higher long-run level of rates than investor B. This implies that A expects the future level of short rates to be higher than B which, in turn, makes longer-term bonds a relatively less attractive investment for A than for B. Hence, if they trade bonds at equilibrium prices, the price of risk that A requires for bearing level risk has to be higher than for B.

We now have all the ingredients to write our model in state-space form. In addition to data on observed bond yields, \( y^o_t \), we will use observations of short-term rate forecasts of two agents, labeled \( y_{t}^{E,A} \) and \( y_{t}^{E,B} \) to identify the parameters of the model. The observation equation is given by:

\[
\begin{bmatrix}
y^o_t \\
y_t^{E,A} \\
y_t^{E,B}
\end{bmatrix}
= \begin{bmatrix}
A^X \\
H_0^A(\beta, \mu) \\
H_0^B(\beta, \mu)
\end{bmatrix}
+ \begin{bmatrix}
B^X & 0 & 0 \\
H_X^A(\beta) & H_\mu^A(\beta) & 0 & 0 \\
H_X^B(\beta) & 0 & H_\mu^B(\beta) & 0
\end{bmatrix}
\times
\begin{bmatrix}
X_t \\
\mu_t^A \\
\mu_t^B \\
\mu_t^L
\end{bmatrix}
+ e_t, \quad (3.20)
\]

where the first few entries of \( e_t \) are yield pricing errors \( e^{(n)}_t \) and \( e_t \) has a diagonal variance-covariance matrix. Moreover, \( H_0^A(\beta, \mu), H_0^B(\beta, \mu), H_X^A(\beta), H_\mu^A(\beta), H_X^B(\beta), H_\mu^B(\beta) \) determine the model-implied loadings of future short rate expectations of agents A and B on the observed model factors \( X_t \) as well as their respective updates of the drift of the level factor, \( \mu_t^A \) and \( \mu_t^B \). These loadings are known, nonlinear transformations of the model parameters, see also Crump, Eusepi, and Moench (2018). Appendix C provides further details of the state space model.

Combining equations (3.4), (3.5) and (3.7), the transition equation is given by

\[
\begin{bmatrix}
X_t \\
\mu_t^A \\
\mu_t^B \\
\mu_t^L
\end{bmatrix}
= \begin{bmatrix}
\alpha^X \\
0 \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
\beta & 0 & 0 & l \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
X_{t-1} \\
\mu_{t-1}^A \\
\mu_{t-1}^B \\
\mu_{t-1}^L
\end{bmatrix}
+ u_t, \quad (3.21)
\]
where $\alpha^X = (0, (1 - \beta^S) \mu^S, (1 - \beta^C) \mu^C)'$ and $l = (1 - \beta^L, 0, 0)$ and where

$$u_t = \begin{pmatrix} \Sigma^{1/2} & \Gamma \\ 0 & \Sigma^{1/2}_\eta \end{pmatrix} \begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 0 & 0 & 1 - \beta^L \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

with $u_t = (\epsilon_t^L, \epsilon_t^S, \epsilon_t^C, \eta_t^A, \eta_t^B, \eta_t^L)'$. Hence, the innovations of different beliefs about the level factor can be correlated with each other, while the innovations of the pricing factors $X_t$ are uncorrelated with the innovations of these beliefs.

The vector of parameters to be estimated is given by

$$\Theta = \begin{pmatrix} \beta^L & \beta^S & \beta^C & \mu^S & \mu^C \mu_0 & \text{vech} (\Sigma_\epsilon) & \Psi_0 & \text{diag} (\Psi_1) & \sigma_y & \sigma_{\text{short}} & \sigma_{\text{long}} \end{pmatrix}'$$

where $\Sigma_\eta$ is a variance-covariance matrix capturing the correlation between innovations to the drift and agents’ beliefs, and the last three elements of $\Theta$ are the variances of the i.i.d. measurement error on the yields and survey data. We estimate the parameters via maximum likelihood and filter the perceived drifts of the level factor $\mu^A_t$ and $\mu^B_t$, and as a result, estimates of the prices of risk $\lambda^A_t$ and $\lambda^B_t$. Note that while the level factor is fully spanned by the cross-section of bond yields, its time-varying long-run mean $\mu_t$ as perceived by different investors is not. At the same time, the different beliefs about the long-run mean bear information about future expected short rates and expected bond returns. Hence, we can interpret the long-run means perceived by the two investors as “hidden” or “unspanned” factors in the spirit of Duffee (2011b) and Joslin, Priebsch, and Singleton (2014). As disagreement about future short rates in our model is fully captured by the difference in beliefs about the long-run mean of the level factor, disagreement is itself unspanned in our model. In contrast to these papers which use observable macroeconomic variables, here we use observable information on survey forecasts as unspanned factors.5

### 3.2 Modeling the Representative Investor

Standard term structure models assume a representative agent and decompose bond yields into short rate expectations and term premiums from the perspective of that agent. As such, a term premium decomposition of the representative investor (RI) is a useful benchmark for our model with heterogenous beliefs. Assume the RI has a pricing kernel with the same

---

5This is similar in spirit to Chernov and Mueller (2012) who also filter an unspanned factor from survey forecasts of inflation.
functional form as the two investors $A$ and $B$ above, but with different prices of risk:

$$M_{t+1}^R = \exp \left( -r_t - \frac{1}{2} \lambda_t^R \lambda_t^R - \lambda_t^R \epsilon_{t+1} \right)$$

(3.22) where we assume that prices of risk of the representative investor follow the same affine function of the pricing factors $X_t$:

$$\lambda_t^R = \lambda_{0,t}^R + \Lambda_1^R X_t.$$  

We further assume that the representative investor faces the same equilibrium bond prices as the two investors in our model:

$$\ln P_t^{(n)} = A_{n} + B_{n}'X_t.$$  

Then, following the algebra above the equilibrium would restrict market prices of risk to be related to the perceived drift

$$(I - \beta) \mu_t^R - \Sigma \epsilon_\lambda_{0,t} = \Psi_0,$$  

(3.23)

where $\mu_t^R$ is the conditional expectation of the drift based on the time $t$ information set. We also have that

$$\beta - \Sigma \epsilon_\Lambda_1^R = \Psi_1$$  
or  
$$\Lambda_1^R = \Lambda_1$$

(3.24)

Assume now that the representative investor’s prices of risk are a weighted average of the individual agents’ prices of risk, with weights given by $w_{A,t}$ and $w_{B,t}$:

$$\lambda_t^R = w_{A,t} \lambda_{0,t}^A + w_{B,t} \lambda_{0,t}^B + \Lambda_1 X_t,$$  

(3.25)

where $w_{A,t} + w_{B,t} = 1 \ \forall \ t$

Hence,

$$(I - \beta) \mu_t^R - \left( w_{A,t} \Sigma \epsilon_\lambda_{0,t}^A + w_{B,t} \Sigma \epsilon_\lambda_{0,t}^B \right) = \Psi_0,$$

Now use the equilibrium restrictions in equations (3.14) to obtain

$$\Psi_0 = (I - \beta) \mu_t^R - w_{A,t} \left( (I - \beta) \mu_t^A - \Psi_0 \right) - w_{B,t} \left( (I - \beta) \mu_t^B - \Psi_0 \right).$$  

16
and therefore
\[ \mu^R_t = w_{A,t}\mu^A_t + w_{B,t}\mu^B_t. \] (3.26)

Hence, the long-run mean of the level factor perceived by the representative investor follows the same weighted average of the filtered drifts of the two agents.

We can therefore fully characterize the equilibrium of the two agent economy as the outcome of an asset pricing decision of a representative investor with prices of risk and beliefs that are described as a weighted average of individual investors. This is consistent with Jouini and Napp (2007) and Bhamra and Uppal (2014) who show that in heterogeneous asset pricing models with general preferences the discount factor and beliefs of the representative investor can be represented as weighted averages of the discount factors and beliefs of the individual agents. Since we use reduced-form pricing kernels, as is common in affine term structure models, we cannot directly derive these weights. Instead, we assume an underlying model where the two investors have mean-variance preferences with the conditional mean driven by \( \lambda^i_t, i \in \{A, B\} \). In this case, the weights track the relative wealth of agents which in turn reflects past disagreement between the two agents (see Jouini and Napp (2007) and Appendix A for further details). Intuitively, the greater the share of wealth an agent accumulates, the more impact she will have on the marginal pricing of bonds, and the more closely the representative investor tracks her belief. We thus henceforth refer to the weights as relative wealth weights. As a robustness check we also provide a model-free, nonparametric approach to estimate the weights relying on realized excess returns and the individual prices of risk for each agent (see Appendix A.1 for full details). These results show that this nonparametric procedure generates estimated weights which are similar to those based on our baseline approach.

### 3.3 Parsing the Channels

We are now in a position to turn to the main question of our analysis: what is the impact of disagreement about future monetary policy on the term structure of interest rates and on term premiums in particular? Specifically, we assess to what extent term premiums are driven by differences in beliefs that trigger speculative trading, or through movements in the wealth distribution as a consequence of past disagreement.

Recall that the market prices of risk of the representative investor are given by
\[ \lambda_t^R = \Lambda_1X_t + w_{A,t}\lambda^A_{0,t} + w_{B,t}\lambda^B_{0,t}. \] (3.27)
Expected one-month excess holding period returns of the representative investor are given by

\[ E_t[r_{t+1}^{(n-1)}] = \mathbf{B}_{n-1} \Lambda_1 \mathbf{X}_t + \mathbf{B}_{n-1} \Lambda_0^R, \]  

(3.28)

where \( \mathbf{B}_{n-1} \) is the vector of loadings of the log price of a bond with maturity \( n \) on the pricing factors \( \mathbf{X}_t \) derived above. Term premiums are averages of expected future one period excess returns with declining maturity. Thus,

\[
TP_t^{(n)} = \left( \frac{E_t[r_{t+1}^{(n-1)}] + r_{t+2}^{(n-2)} + \ldots + r_{t+n-2}^{(2)}}{n-1} \right) = \tilde{\mathbf{B}}_{n,1} \Lambda_1 \mathbf{X}_t + \tilde{\mathbf{B}}_{n,2} \Lambda_0^R, \quad (3.29)
\]

where \( \tilde{\mathbf{B}}_{n,1} \) and \( \tilde{\mathbf{B}}_{n,2} \) collect the coefficients mapping current variables to their forecasts of excess returns.

It is instructive to also introduce the term premium of an econometrician who knows the structure and parameters of the model, observes the three pricing factors and filters the long-run mean \( \mu^L_t \) of the level factor from these data. \( \lambda^L_{0,t} \) captures the evolution of prices of risk that the econometrician would attribute to perceived shifts in the long-run level of rates, over and above the time variation of prices of risk \( \Lambda_1 \mathbf{X}_t \) implied by the affine pricing assumption. \( \lambda^L_{0,t} \) is constructed using the pricing restrictions described above (e.g., equation 3.30) for the econometrician, i.e., \( (I - \beta) \mu^L_t - \sum_e \lambda^L_{0,t} = \Psi_0 \).

The first component is the one that would be present in any standard affine term structure model. The second component captures changes in prices of risk driven by shifts in the estimated, time-varying long-run level of the term structure of interest rates. We can then express the difference between the representative investor’s term premium and that of an econometrician as

\[
D_t^{(n)} = TP_{t,\text{econ}}^{(n)} - TP_t^{(n)} = \tilde{\mathbf{B}}_{n,2} \left( w^A_t \lambda^A_{0,t} + w^B_t \lambda^B_{0,t} \right), \quad (3.31)
\]

where \( \lambda^i_{0,t} = \lambda^L_{0,t} - \lambda^i_{0,t} \) for \( i \in \{ A, B \} \). In differences this can be approximated as:

\[
\Delta D_t^{(n)} \approx \left( \tilde{\mathbf{B}}_{n,2} \left( w^A_t \Delta \lambda^A_{0,t} + w^B_t \Delta \lambda^B_{0,t} \right) \right) + \left( \tilde{\mathbf{B}}_{n,2} \left( \Delta w^A_t + \Delta w^B_t \right) \right), \quad (3.32)
\]

\[ ^6 \text{In this model, the initial observation of } \mu^L_t \text{ is undefined. We initialize it by using the first two observations of consensus forecasts for the longest-available forecast horizon at the beginning of the sample.} \]

\[ ^7 \text{The econometrician’s term premium comprises of two terms,} \]

\[ TP_{t,\text{econ}}^{(n)} = \tilde{\mathbf{B}}_{n,1} \Lambda_1 \mathbf{X}_t + \tilde{\mathbf{B}}_{n,2} \lambda^L_{0,t}. \]  

(3.30)
The wedge between the term premium and the econometrician’s term premium, $\mathcal{D}_{t}^{(n)}$, is driven by disagreement and approximately consists of two components. The first reflects the heterogeneous responses of investors’ risk attitudes to changes in their perceptions of the long-run mean of the level factor. The second arises because of endogenous wealth fluctuations driven by past disagreement. Holding fixed investors’ current risk attitudes, any change in the relative wealth ratio will induce changes in the representative investor’s belief. It is worth noting that as these two disagreement-driven effects interact with each other, the relation between term premiums and disagreement about future short rates is not constant, in line with the empirical evidence in Giacoletti, Laursen, and Singleton (2020) who detect a time-varying impact of disagreement on expected excess returns.

Intuitively, the wealth effect arises because investors disagree about expected excess returns and therefore choose different portfolio allocations. Given the previous period’s portfolio, the realization of returns then changes the relative market power of the two investors which, in turn, affects the term premium of the representative investor. The heterogeneous response to changes in beliefs does not affect the wealth distribution contemporaneously. However, as investors change their beliefs about future short rates and thus term premiums, the representative investor’s term premium will also change as long as the relative wealth ratio is different from one.

4 Empirical Results

In this section, we present an empirical analysis based on our two-investor GATSM with fundamental disagreement. We first describe the data used and the estimation approach. We then discuss the model fit and provide decompositions of Treasury yields into expected short rates and term premiums. In Section 5, we then use the model to quantify the importance of disagreement for term premium dynamics.

4.1 Data and Estimation

We jointly estimate our model using zero-coupon Treasury yields as well as survey expectations of short rates for two different groups of investors. We obtain the latter from the Blue Chip Financial Forecasts (BCFF) survey. Specifically, we use two- and four-quarter-ahead, one-to-two, four-to-five year-ahead, and long-term forecasts which cover horizons between six and ten or seven and eleven years into the future, depending on when the survey was taken. The short-term forecasts are observed monthly and this is our frequency of observation also for Treasury yields. The medium-term and long-term forecasts are observed biannually. The missing monthly observations in between biannual survey observations can be accommodated.
in our state-space framework. The BCFF provides medium and long-term forecasts for three different cross-sectional averages of the forecaster distribution: the average across all responses (the “consensus” forecast), the average of the top-10 responses and the average of the bottom-10 responses. We employ the top-10 and bottom-10 average responses as proxies representing two investors at the opposite spectrum of the belief distribution about future short rates. As discussed in Section 2, the difference between the top-10 and bottom-10 average responses is closely correlated with common measures of forecaster disagreement, such as the cross-sectional standard deviation or the interquartile range.

We obtain zero coupon Treasury yields from Gürkaynak, Sack, and Wright (2007) (GSW henceforth). The GSW zero coupon yields are based on fitted Nelson-Siegel-Svensson curves, the parameters of which are published along with the estimated zero coupon curve. We use these parameters to back out the cross-section of zero-coupon yields for maturities up to ten years, using end-of-month values. In our estimation, we use $N = 8$ Treasuries with maturities $n = 3, 6, 12, 24, 36, 60, 84, 120$ months. Our sample period is $1983 : 03 - 2015 : 08$ for a total of 400 monthly observations. We employ the normalization scheme in Joslin, Singleton, and Zhu (2011) to estimate our model, where the bond pricing parameters $A_n (\Psi_0)$ and $B_n (\Psi_1)$ are uniquely determined by a parsimonious risk-neutral parameter set, see Appendix B for details.

Our model fits both yields and survey forecasts of the short rate very precisely, as displayed in Figure 6 which shows the time series and cross section of observed and model-implied yields. All parameter estimates and associated standard errors are provided in Appendix D. The average of yield pricing errors is no more than 5 basis points in absolute value and is thus well in line with previous studies. The bottom two panels of Figure 6 provide a plot of the unconditional mean and standard deviation of yields across maturities as observed and fitted by the model. The charts show that the model fits both moments well.

We next turn to the model fit of survey forecasts of the short rate. The top two charts in Figure 7 show the observed and fitted top-10 and bottom-10 average survey forecasts of the federal funds rate, where actual values are plotted by solid lines. These two charts document that with only the perceived long-run mean of the level factor being different across investors, our model is able to capture the substantial time variation in investors’ disagreement about future short rates. The bottom four panels of Figure 7 provide a plot of unconditional first and second moments of the two groups’ survey forecasts as observed and fitted by the model, again documenting that the model fits survey forecasts at all horizons quite precisely.

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8See [http://www.federalreserve.gov/econresdata/researchdata.htm](http://www.federalreserve.gov/econresdata/researchdata.htm)
Figure 6: Time-series and cross-sectional fit of yields

Notes:
This figure provides plots of observed and model-implied yields for the two- and ten-year maturities in the upper two charts. Observed yields are displayed by solid lines, dashed lines correspond to model-implied yields. The bottom panels plot unconditional averages and standard deviations of observed yields against those implied by the model.
Figure 7: Time-series and cross-sectional fit of survey forecasts

Notes:
This figure provides plots of observed and model-implied survey forecasts of the fed funds rate. Observed survey forecasts are displayed as solid lines, dashed lines correspond to model-implied survey forecasts. The top two charts show the Blue Chip Financial Forecasts (BCFF) four-quarter ahead and long-range (7-11 years) top-10 and bottom-10 average forecasts of the federal funds rate. Asterisks in the top right chart are long-term forecasts which are observed biannually. The bottom four panels plot unconditional means and standard deviations of survey forecasts of the top-10 and bottom-10 average responses against those implied by the model.
5 Disagreement and Term Premiums

Having shown that our model fits both yields and survey data on future short rates precisely, we now study how disagreement about monetary policy affects term premiums and the pricing of risk. We start by showing the term premiums for the two investors. The upper panel of Figure 8 displays these term premiums for the two and ten-year maturity, respectively. The investor expecting high future short rates (top-10) implicitly sees term premiums on two-year notes to hover between minus one and one percent over the last 30-years. Similarly, this investor sees ten-year Treasury term premiums fluctuate around zero in a somewhat wider range. In stark contrast, the (bottom-10) investor expecting low future policy rates, saw term premiums consistently positive across time and maturity, declining from about three (six) percent at the two-year (ten-year) maturity in the early 1980s to just below one percent for both maturities at the end of the sample.

The bottom-left panel of Figure 8 provides the time series average of the implied term premiums across maturities for the two agents. While the bottom-10 investor has an upward sloping term structure of term premiums ranging between 50 basis points at the one-year maturity and 200 basis points at the ten-year maturity, the top-10 investor essentially sees term premiums on average slightly negative across all except the very long maturities. In other words, to this investor Treasuries provide insurance for which she is willing to pay a premium. The bottom-right chart presents the differences between the two term premium estimates across time and maturities, showing that these differences have been much less pronounced since around the year 2000, consistent with the decline in disagreement about the longer-term level of rates.

Figure 9 visualizes the degree of belief heterogeneity about the long-run mean of the short rate as well as the evolution of the relative wealth ratio among the two investors. The left-hand chart displays the wealth weights $w^A_t$ and $w^B_t$ of the two investors. The right-hand chart shows the evolution of the long-run means as perceived by the top-10 and the bottom-10 investors, in addition to the model-implied long-run mean of the representative investor, $\mu^R_t$. Recall that the latter is itself a wealth-weighted average of the two investors’ beliefs. As the relative wealth ratio is tilted towards the top-10 investor at the beginning of the sample, the representative investor’s estimate of the long-run mean of the policy rate is initially similar to that investor. Since the top-10 investor consistently overestimates future short rates across the sample, they invest a smaller share of their wealth in longer-maturity bonds. In contrast, the bottom-10 investor overweights longer-maturity bonds which outperform on average in the face of a secular decline in overall interest rates. Therefore, the bottom-10 investor gradually increases her wealth share over time, and so the representative investor’s belief moves away from the belief of the top-10 investor and converges toward that of the
Figure 8: Term premiums for the two investors

Notes:
This figure plots the term premiums implied by the top-10 and bottom-10 investors’ beliefs about future short rates. The upper panels plot the term premium estimates for two- and ten-year treasury notes. The lower left panel plots the sample averages of term premium estimates in the investors’ beliefs for different maturities. The lower right panel displays time-varying differences across maturities between the two investors’ beliefs.

5.1 The Term Premium in a Heterogeneous Beliefs Economy

Given these differences in beliefs across investors and fluctuations in relative wealth, what is the term premium of the representative investor in this heterogeneous beliefs economy? Figure 10 provides time series of the two-year (top panel) and ten-year (bottom panel) term premium estimates of the representative investor as solid lines. We compare these to two other measures of the term premium. The first is the term premium based on the no-arbitrage term structure model by Adrian, Crump, and Moench (2013) (ACM) which does not use any survey information (dashed lines). The second (dotted line) is the term premium obtained
Notes:
This figure provides graphs exhibiting the slow-moving components in different beliefs and wealth weights. The left chart sets out the wealth weights of two investors \( w_A = \frac{W_A}{W_A + W_B} \) and \( w_B = \frac{W_B}{W_A + W_B} \), i.e., top-10 (A) and bottom-10 (B) investors. The right-hand chart plots the estimates of the two investors’, the econometrician’s, and the representative investor’s beliefs about the long-run mean, i.e., filtered estimates of \( \mu_{A_t} \), \( \mu_{B_t} \), \( \mu_L \), and \( \mu_R \).

from Crump, Eusepi, and Moench (2018) (CEM). They obtain term premiums as the simple difference between observed Treasury yields and the average expected short rate path obtained from surveys. Their approach does not impose no-arbitrage, but instead uses all available surveys of professional forecasters in the US to fit the joint term structures of consensus expectations for output growth, inflation and the short rate.

The most striking feature of the term premium in the heterogenous-beliefs economy is that it is stationary throughout the sample – fluctuating primarily between -0.5% and 1% for the two-year term premium and 0% and 2% for the ten-year term premium. In contrast, estimated term premiums using yields only (ACM) or consensus survey-data only (CEM), show a pronounced downward trend in the 1980s. Figures 8 and 9 provide the intuition for this result. First, observe that the term premium of the bottom-10 investor shows a strong downward trend reflecting a much more modest decline in the average future path of interest rates relative to observed yields. In contrast, the top-10 investor expects higher average future short rates which are broadly in-line with observed yields leading to a stationary term premium. Because the top-10 investor begins the sample with a large wealth share, the term premium of the representative investor which is given by the wealth-weighted average of the individual term premiums is low. The rise in the wealth share of the bottom-10 investor then
coincides with the convergence of the two individual term premiums. After about 1995, the wedge between the two term premiums is roughly constant with the top-10 investor showing a broadly negative term premium and the bottom-10 investor a broadly positive term premium.

In the second half of the sample, the term premium in this heterogeneous-beliefs economy broadly co-moves with the estimates of ACM and CEM, especially after around 2000. In particular, at the two-year maturity all three term premium estimates track each other closely after mid 2011 which is when the Federal Open Market Committee (FOMC) announced that it would keep the fed funds rate exceptionally low “at least through mid-2013,” marking the Committee’s first use of date-based forward guidance. This date-based forward guidance seems to have played an important role in stabilizing short-term yields through term premiums alike, as shown in the upper panel of Figure 10 (see also King (2019)).

These results show that investor disagreement plays an important role in measuring and understanding the dynamics of term premiums. It is important to emphasize that we obtain stationary term premiums which are based on real-time measures of expected interest rates through survey data. Our results show that the distribution of short-rate forecasts are highly informative about the pricing of risk.

5.2 Decomposing Term Premiums

We now turn to investigate the channels through which disagreement affects term premiums. Specifically, we assess to what extent term premiums are driven by differences in beliefs that trigger speculative trading, or through movements in the wealth distribution as a consequence of past disagreement.

We begin with the decomposition introduced in equation (3.32). As a point of comparison we contrast the term premium for the two investors and their wealth-weighted average with that of the econometrician. Recall that the wedge between the term premium of the representative investor and the econometrician’s term premium reflects the role of investor disagreement. Figure 11 presents these different elements for the ten-year maturity. The top panel shows the econometrician’s term premium along with the wedge between the econometrician’s and the representative investor’s term premium (the representative investor’s term premium can be obtained by subtracting the red line from the blue line). As the figure shows, investor disagreement about the long-run level of rates explains a sizable share of the variance of term premiums in a representative investor economy. For the ten-year maturity, the disagreement contribution to the term premium, $D_t^{(n)}$, explains about a third

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9In January 2012, the FOMC replaced “mid-2013” with “late-2014” and in September 2012, replaced “late-2014” with “mid-2015”.

10The results for the two-year maturity (not shown) are qualitatively similar.
Figure 10: Term Premium Estimates

Notes:
This figure provides plots of the term premium estimates for two- and ten-year Treasuries. The term premium estimates of the representative investor (RI) are plotted as solid lines. The dotted lines correspond to the term premium based on survey data from Crump, Eusepi, and Moench (2018) and the dashed lines to the term premium the affine term structure model of Adrian, Crump, and Moench (2013).

of the variation in the econometrician’s term premium:

\[
1 = \frac{\text{cov}(TP_t^{(n)}, TP_{t,\text{econ}}^{(n)})}{\text{V}(TP_{t,\text{econ}}^{(n)})} \approx 0.67 + \frac{\text{cov}(D_t^{(n)}, TP_{t,\text{econ}}^{(n)})}{\text{V}(TP_{t,\text{econ}}^{(n)})} \approx 0.33.
\] (5.1)

In the middle panel of Figure 11 we decompose the disagreement contribution to the representative investor’s term premium into the contribution from (i) the changing wealth weights and (ii) the changing price of risk. We see that updates in the pricing of risk, driven by changes in the long-run belief about short rates, are the dominant source of medium and high-frequency movements in the term premium. In contrast, the contribution from changes in relative wealth shows a pronounced trend and so primarily contributes at low frequencies. The econometrician’s term premium appears to move broadly in sync with
Notes:
This figure provides plots of the econometrician’s term premium for the ten-year Treasury along with the components of the representative investor’s term premium that is driven by disagreement. The bottom plot provides a scatterplot of the econometrician’s term premium along with disagreement on long-horizon survey forecasts from the BCFF.

At the beginning of the sample, we see a sharp decline in the econometrician’s term premium which is mimicked by the decline in the disagreement component following the Volcker disinflation period when investors strongly updated their views about future nominal short rates. Similarly, the first half of the 1990s saw a substantial increase in the disagreement component, which coincided with a rise in the econometrician’s term premium. In contrast, the econometrician’s term premium reaches its minimum around 2001, which corresponds to the minimum recorded disagreement about the future stance of monetary policy.\(^{11}\) The correlation between the econometrician’s term premium and the wedge, \(D_i^{(n)}\), is 0.74.\(^{12}\) According to the BCFF, the difference between average top-10 and bottom-10 forecasts rose by 1.5 percentage points.
long-run disagreement in the survey data since the start of the sample. This is consistent with
the Federal Reserve moving towards greater transparency during that time (e.g., announcing
changes in the target rate in statements after each FOMC meeting, publishing transcripts
of past meetings with a five year lag). These efforts contributed to an improved private
sector understanding of U.S. monetary policy (see, e.g. Swanson (2006), Crump, Eusepi,
and Moench (2011)). Finally, we see a steady decline in the econometrician’s term premium
starting in around 2005 and ending at the onset of the Great Recession. This movement
is again accompanied by a drop in disagreement from a local maximum of about 2% down
to about 1% over the same period. To visualize these patterns fully, in the bottom panel
of Figure 11, we present a scatterplot of the econometrician’s term premium versus survey
disagreement about long-horizon forecasts for the short rate. This shows that our model
replicates one of the main stylized facts we introduced in Section 2 (see Figure 4): term
premiums and forecaster disagreement are strongly positively correlated.

6 Conclusion

Bond investors disagree about the future path of policy rates, and particularly about their
long-run level. Accordingly, they disagree about the risk-return tradeoff of longer-term bonds
and engage in speculative trading. This induces shifts in their relative wealth which, in
turn, affects the marginal pricing of risk in the economy. Hence, the term premium of
an econometrician observing only yields partly reflects disagreement-driven changes in the
marginal pricing of risk.

In this paper, we have formalized this intuition in an affine term structure model with
heterogeneous beliefs. In our model investors perfectly observe the level, slope, and curvature
of the yield curve but have different beliefs about the long-run level of rates. Our model fits
yields and survey forecasts of future short rates very well. It generates sizable movements in
the relative wealth ratio and implies subjective and objective term premiums which are in
line with other estimates. We use the model to show that a sizable fraction of the variation
of term premiums – about one third – is disagreement-driven.
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Appendix A  Portfolio Allocation and Weights

In our economy, the representative investor’s belief is a wealth weighted average of different investors, A and B. We define the relative wealth ratio \( \kappa_A^B(t) = \frac{w_A^t}{w_B^t} \) as the ratio of B’s wealth relative to A’s. The wealth weights are thus specified as

\[
w_A^t = \frac{1}{\kappa_A^B(t) + 1}, \quad w_B^t = 1 - w_A^t.
\]

(A.1)

The law of motion of the relative wealth ratio is given by

\[
\ln(\kappa_A^B(t+1) = \kappa_A^B(t)) = \ln(\frac{w_B^{t+1}}{w_B^t}) - \ln(\frac{w_A^{t+1}}{w_A^t}) = r_x^B - r_x^A,
\]

(A.2)

where \( r_x^A \) and \( r_x^B \) denote the gross returns on the portfolios chosen by A and B, respectively.

In our affine term structure model, the evolution of the vector of excess log bond returns is given by

\[
r_x^{t+1} = B_X \lambda_i^t + B_X \Sigma_{\epsilon}^{1/2} \epsilon_{t+1} - \frac{1}{2} B_X \Sigma_{\epsilon} B_X',
\]

(A.3)

In a one-period portfolio allocation problem with power utility, the optimal portfolio weights for investor \( i \), denoted \( \alpha_i^t \), are given by the following standard solution (Cochrane (2014)):\(^{13}\)

\[
\alpha_i^t = \frac{1}{\gamma} \Sigma^{-1} E_t[r_x^{t+1}], \quad i = A, B,
\]

(A.4)

where \( r_x^{t+1} \) is the \( N \times 1 \) vector of log excess returns and \( \Sigma \) their corresponding conditional variance covariance matrix with \( \Sigma = B_X \Sigma_{\epsilon} B_X' \). Hence, given the realization of excess returns and optimal portfolio weights \( \alpha_i^t \), the realization of \( i \)’s excess portfolio returns at time \( t+1 \) is given by

\[
r_x^{i,t+1} = \alpha_i^t B_X \lambda_i^t + \alpha_i^t B_X \Sigma_{\epsilon}^{1/2} \epsilon_{t+1} - \frac{1}{2} \alpha_i^t B_X \Sigma_{\epsilon} B_X' \alpha_i^t \quad i = A, B
\]

(A.5)

where the standard solution for \( \alpha_i^t \) can be written as a linear combination of investor \( i \)’s market prices of risk:

\[
\alpha_i^t = \frac{1}{\gamma} \Sigma^{-1} B_X \lambda_i^t
\]

\[
= \frac{1}{\gamma} (B_X \Sigma_{\epsilon} B_X')^{-1} B_X \lambda_i^t.
\]

\[
= \frac{1}{\gamma} B_X^{-1} \Sigma_{\epsilon}^{-1} \lambda_i^t, \quad i = A, B.
\]

(A.6)

Combining the above two equations, we obtain the solution for realized excess returns:

\[
r_x^{i,t+1} = (\frac{1}{\gamma} - \frac{1}{2\gamma^2}) \lambda_i^t \Sigma_{\epsilon}^{-1} \lambda_i^t + \frac{1}{\gamma} \lambda_i^t \Sigma_{\epsilon}^{1/2} \epsilon_{t+1}, \quad i = A, B
\]

(A.7)

Finally, the law of motion of the relative wealth ratio given in Equation (A.2) can be written in closed

\(^{13}\)As it is difficult to obtain a closed-form solution of the portfolio problem for power utility, a Taylor expansion method can be used to approximate the utility function up to second order accuracy and, therefore, a simple solution is available in closed form.
Finally, note that the initial relative weight ratio $\eta_B^A(0)$ is undefined. We calibrate this value by minimizing the average of the squared differences between $\mu^R_t$ and $\mu^L_t$. This approach results in a large initial wealth share for the top-10 investor which is arguably consistent with the persistent rise in the level of interest rates in the late 1970s and early 1980s leading into our sample period.

### A.1 Nonparametric Estimation of Time-Varying Weights

In our model we have that

$$r_{x,t+1} = \mathcal{B}_{n-1} \Lambda_1 \mathbf{X}_t + \mathcal{B}_{n-1} (w_t^A \lambda_{0,t}^A + w_t^B \lambda_{0,t}^B) + \mathcal{B}_{n-1} v_{t+1},$$

where we have used that $\lambda^R_t = w_t^A \lambda_{0,t}^A + w_t^B \lambda_{0,t}^B + \Lambda_1 \mathbf{X}_t$ for some weights $w_t^A + w_t^B = 1$. Importantly we have that $\mathbb{E}[v_{t+1}] = 0$. Suppose that we assume that $w_t^A$ is a smooth process over time. Then it can be well approximated by a linear combination of an appropriate set of basis functions:

$$w_t^A \approx \sum_{j=0}^m \zeta_j \cdot P_{j,t},$$

where

$$P_{0,t} = 1$$
$$P_{i,t} = \sqrt{2} \cos (i\pi (t - .5) / T),$$

and $m$ is a user-chosen parameter which governs how much smoothness to impose (see, e.g., Bierens and Martins (2010)). For example if $m = 0$ then we would set $w_t^A$ to be constant over time. Next note that,

$$\begin{align*}
   r_{x,t+1}^{(n-1)} = & \quad \mathcal{B}_{n-1} \Lambda_1 \mathbf{X}_t + \mathcal{B}_{n-1} \lambda_{0,t}^A + \mathcal{B}_{n-1} v_{t+1} \\
   = & \quad \mathcal{B}_{n-1} \Lambda_1 \mathbf{X}_t + \mathcal{B}_{n-1} \lambda_{0,t}^B + \mathcal{B}_{n-1} \lambda_{0,t}^A (\lambda_{0,t}^A - \lambda_{0,t}^B) + \mathcal{B}_{n-1} v_{t+1}
\end{align*}$$

We observe one-period excess returns across time and maturity. From our model outputs we have $(\mathcal{B}_{n-1}, \Lambda_1, \lambda_{0,t}^A, \lambda_{0,t}^B)$. However, we do not observe $\mathcal{B}_{n-1} v_{t+1}$ because this is the realized return error relative to the representative investor. Thus, we can estimate the coefficients on each basis function directly by linear regression. To choose $m$ in practice, we use the BIC,

$$BIC = nT \log \left( \sum_{n,t} (\xi_t^{(n)})^2 \right) + m \log (nT),$$
where
\[ \xi_t^{(n-1)} = r_{t+1}^{(n-1)} - B_{n-1} \lambda_1 X_t - B_{n-1} \lambda_0^{B} - \hat{c}_0 \cdot B_{n-1} \left( \lambda_0^A - \lambda_0^B \right) P_{0,t} - \cdots - \hat{c}_m \cdot B_{n-1} \left( \lambda_0^A - \lambda_0^B \right) P_{m,t} - \hat{c} \]

and \( \hat{c} \) is the estimated constant. The red line in Figure 12 shows the nonparametrically estimated weights using the optimal choice based on BIC \((m = 1)\). As we observe, these model-free estimated weights are consistent with the weights we use in our baseline analysis.

![Figure 12: Smooth Weight Estimates](image)

**Notes:**
This figure provides graphs exhibiting the smooth weight estimates and 99% confidence intervals from the best regression model \((m = 1)\) based on the BIC criterion. The structural weights are generated from our model with a power utility function \((\gamma = 6)\).

**Appendix B Normalization Scheme**

To estimate the model, we employ the normalization scheme proposed by Joslin, Singleton, and Zhu (2011) (henceforth, JSZ). Under the JSZ normalization scheme, we have a risk-neutral parameter set \(\Theta^Q \equiv (\Sigma_{XX}, \lambda^Q, k^Q_{\infty})\). Let \(\tilde{X}_t\) denote a set of risk factors with
\[ r_t = 1' \tilde{X}_t, \]
\[ \tilde{X}_{t+1} = C(k^Q_{\infty}) + J(\lambda^Q) \tilde{X}_t + \Sigma_{XX}^{1/2} e_{X,t+1}. \]

JSZ show that there exists a unique rotation of \(\tilde{X}_t\) so that the factors are portfolios (or principal components) of bond yields:
\[ P_t = v(\Theta^Q, W) + L(\lambda^Q, W) \tilde{X}_t, \]

where \(W\) denote weights used to construct factor-mimicking portfolios such that the latent states are portfolios.
of yields.\textsuperscript{14} That is, \( P_t = y_t^\sigma \cdot W' \). It can be shown that the parameters controlling the risk neutral dynamics \((\Psi_0, \Psi_1, \delta_0, \delta_1 \text{ and } \Sigma_c)\) are all functions of the elements in \( \Theta_Q \).

The physical dynamics can be written in a similar normalized form

\[
\tilde{X}_{t+1} = C(\mu_t) + J(\lambda^\beta)\tilde{X}_t + \Sigma_{X'X}^{1/2} \epsilon_{t+1}, \tag{B.4}
\]

where we impose that the coefficient matrix \( J(\lambda^\beta) \) is diagonal.

\section*{Appendix C \ State-Space Representation}

Our model can be written in a state-space representation in terms of pricing factors \( X_t \)

\[
\begin{bmatrix}
  y_t^o \\
  y_t^{E,A} \\
  y_t^{E,B}
\end{bmatrix}
= \begin{bmatrix}
  A^X \\
  A^E.A \\
  A^E.B
\end{bmatrix}
+ \begin{bmatrix}
  B^X \\
  B^E.A \\
  B^E.B
\end{bmatrix} \times X_t + \epsilon_t, \tag{C.1}
\]

\[
X_t = (I - \beta) \mu_t + \beta X_{t-1} + \Sigma^{1/2} \epsilon_t, \tag{C.2}
\]

where \( y_t^o \) is a vector of zero coupon yields, \( y_t^{E,i} = A, B \) is a vector of investor-\( i \)'s survey forecasts of future short rates,\textsuperscript{15} \( A^X \) and \( B^X \) are explicit functions of \( \Theta_Q \equiv (\Sigma_{X,X}, \lambda^Q, k_{\infty}^Q) \), and \( (A^E,i, B^E,i) \) can be mapped from short rate parameters and investor-\( i \)'s physical dynamics. Note that \( [B^X, B^{E,A}, B^{E,B}]' \) is the coefficient matrix that is time-homogeneous, but \( A^{E,A}_t = A^E(\mu^A_t) \) and \( A^{E,B}_t = A^E(\mu^B_t) \), as linear functions of slow-moving drifts, are time-varying. We assume the slow-moving component \( \mu^i_t \) follows a random walk

\[
\mu^i_t = \mu^i_{t-1} + u^i_t, \quad i = A, B. \tag{C.3}
\]

The transition equation is

\[
\begin{bmatrix}
  X_t \\
  \mu^A_t \\
  \mu^B_t \\
  \mu^L_t
\end{bmatrix}
= \begin{bmatrix}
  \alpha^X \\
  0 \\
  0 \\
  0
\end{bmatrix}
+ \begin{bmatrix}
  \beta & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\times \begin{bmatrix}
  X_{t-1} \\
  \mu^A_{t-1} \\
  \mu^B_{t-1} \\
  \mu^L_{t-1}
\end{bmatrix}
+ u_t. \tag{C.4}
\]

For description we assume there are three pricing factors and the slow-moving components are one-

\textsuperscript{14}We choose the portfolio weights following Duffee (2011a). The portfolios can be interpreted as empirical Level, Slope and Curvature.

\textsuperscript{15}Specifically, \( y_t^{E,A} \) and \( y_t^{E,B} \) are top- and bottom-10 average survey forecasts.
dimensional, and then the dynamics about $X_t$ under the physical measure are given by

$$ \begin{bmatrix} x_t^L \\ x_t^S \\ x_t^C \end{bmatrix} = \begin{bmatrix} 0 & \beta^L & 0 & 0 & (1 - \beta^L) \\ \alpha^S & 0 & \beta^S & 0 & 0 \\ \alpha^C & 0 & 0 & \beta^C & 0 \end{bmatrix} \times \begin{bmatrix} x_{t-1}^L \\ x_{t-1}^S \\ x_{t-1}^C \\ \mu_{t-1}^L \end{bmatrix} + u_t^X. \quad (C.5) $$

The measurement equation is

$$ \begin{bmatrix} y_t^A \\ y_t^{E,A} \\ y_t^{E,B} \end{bmatrix} = \begin{bmatrix} A^X \\ H_0^A (\beta, \mu) \\ H_0^B (\beta, \mu) \end{bmatrix} + \begin{bmatrix} B^X & 0 & 0 \\ H_X^A (\beta) & H_\mu^A (\beta) & 0 \\ H_X^B (\beta) & H_\mu^B (\beta) & 0 \end{bmatrix} \times \begin{bmatrix} X_t \\ \mu_t^A \\ \mu_t^B \\ \mu_t^L \end{bmatrix} + e_t. \quad (C.6) $$

Firstly, we have $A_n^X = -\frac{1}{n} A_n, B_n^X = -\frac{1}{n} E_n$, where $A_n^X \in A^X, B_n^X \in B^X$. $A_n$ and $B_n$ can be obtained from the recursions:

$$ A_n = A_{n-1} + E_{n-1}' \Psi_0 + \frac{1}{2} E_{n-1}' \Sigma \epsilon B_{n-1} + \delta_0, \quad (C.7) $$

$$ E_n' = E_{n-1}' \Psi_1 - \delta_1', \quad (C.8) $$

$$ A_0 = 0, \quad E_0' = 0. \quad (C.9) $$
### Table 1: Model Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k^Q$</td>
<td>2.67E-04</td>
<td>(4.29E-5)</td>
</tr>
<tr>
<td>$X^L$</td>
<td>0.000</td>
<td>(2.82E-6)</td>
</tr>
<tr>
<td>$X^S$</td>
<td>-0.029</td>
<td>(1.73E-2)</td>
</tr>
<tr>
<td>$X^C$</td>
<td>-0.091</td>
<td>(3.81E-2)</td>
</tr>
<tr>
<td>diag($\Sigma_{XX}$)</td>
<td>1.16E-05</td>
<td>(1.64E-5)</td>
</tr>
<tr>
<td>$\alpha^X$</td>
<td>4.65E-04</td>
<td>(3.64E-4)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.941</td>
<td>(3.52E-2)</td>
</tr>
<tr>
<td>$\sigma_{short}$</td>
<td>7.49E-03</td>
<td>(9.82E-3)</td>
</tr>
<tr>
<td>$\sigma_{long}$</td>
<td>1.87E-03</td>
<td>(1.03E-3)</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>8.93E-04</td>
<td>(2.67E-4)</td>
</tr>
<tr>
<td>chol($\Sigma_{\eta}$)</td>
<td>1.20E-05</td>
<td>(1.15E-5)</td>
</tr>
</tbody>
</table>

**Notes:** This table reports parameter estimates for our affine term structure model. The sample period is 1983:03-2015:08, and standard errors are reported in parentheses. $\sigma_y$ is the standard deviation of bond yield observational errors. $\sigma_{short}$ and $\sigma_{long}$ denote observational error standard deviations of short-horizon forecasts (less than one year) and long-horizon forecasts, respectively. $L$, $A$ and $B$ denote the econometrician’s estimate of $u^L_t$, $\sigma_{short}$, and $\sigma_{long}$ and the respective beliefs of the top-10 and bottom-10 investor. Other parameters are defined in Appendix B.