Federal Reserve Bank of New York
Staff Reports

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Staff Report No. 958
January 2021

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Federal Reserve Bank of New York Staff Reports, no. 958
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JEL classification: E42, L11, L15

Abstract

In a market where consumers choose between payment options and firms compete with products and prices, we show that payment data drives the formation of a market monopoly. A data-sharing policy can successfully restore and maintain a competitive market, but often at the expense of both efficiency and consumer welfare. The introduction of a low-cost anonymous means of electronic payment, or digital cash, preserves the market structure and improves consumers' welfare by enabling them to monetize their private information. We discuss the potential role of central banks in providing digital cash.

Key words: customer data, privacy, market structure, digital cash, payments

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To view the author disclosure statement(s), visit https://www.newyorkfed.org/research/staff_reports/sr958.html.

1 Introduction

The majority of retail payments in the United States are digital (Stavins, 2017). With some exceptions, virtually all digital payments are tracked, collected, and aggregated by financial institutions, payment providers, and vendors. There are a variety of reasons why firms want payments data.[1] Digital payments require identification, which allows for purchases to be augmented with detailed demographic and financial information. In addition, an individual's transaction history can provide an indication of their willingness to pay for certain items and is a measure of their disposal income. This information can potentially be exploited by firms when making pricing decisions.[2] Alternatively, information about a consumers' willingness to pay for different items can be used to design better products that more closely match consumer preferences.

While firms value the private information of consumers, they typically do not have to pay for this information. One simple explanation for this is that consumers are unaware that their data is valuable.[3] Or, in instances where private information is verifiable, theory predicts that firms with uncontested monopoly power will extract all of the surplus.[4] It may also be the case that information about consumer preferences is only valuable in the aggregate. That is, in some scenarios a single individual's private information is not worth anything to a firm, but the information from a group (or representative sample) of individuals is valuable. If consumers could somehow bargain collectively they might be able to extract some of the surplus generated by the collection of private information, but such arrangements typically do not exist.[5]

There is, however, another possibility that emerges in markets where trade occurs at posted prices. In such environments, the benefits of price reductions accrue to all consumers and hence, innovations that increase each individual's ability to preserve

[1]For example, see Agarwal et al. (2018) and Parlour et al. (2019).

[2]The use of payments data by firms to price discriminate and the implications of introducing a privacy-preserving form of digital cash when that occurs is studied in Garratt and van Oordt (2019).

[3]See Gambaro (2017).

[4]For instance, see the discussion of Example 2 in Acquisti et al. (2016).

[5]For theoretical discussions on this issue see Ibarra et al. (2018), who propose treating data as labor and contemplate the formation of "data labor unions". Formal data markets are also considered in Bergemann et al. (2020) who investigate the role of data intermediaries.

their privacy, may impact market outcomes in ways that are similar to what we might expect to see under collective bargaining. This may be true even if the innovation is not used. The potential for consumers to switch to a privacy preserving payment method could induce merchants to offer preferred terms of trade in order to induce consumers not to make this switch.[6]

This paper considers an environment where firms use private information that is revealed when consumers pay, to design future goods that more closely match these consumers' ideal design. Having more data provides firms with a better understanding of the distribution of consumer preferences. The collection of private information, therefore, has the potential to increase social surplus. Our focus is on how much surplus is generated through the collection of private information and on how that surplus is divided. This depends on market structure, and the set of available payment instruments, and their privacy-preserving characteristics.

Firms compete by producing differentiated goods and setting prices. We allow for the possibility that these prices will depend on the type of payment instrument used by the consumer, or in particular, on whether or not it is privacy-preserving. Initially we assume there are two payment options: cash, which is privacy-preserving but costly to use, and an electronic payment method that reveals the consumer's private information. Consumers all prefer to consume goods that more closely match their ideal design, but they differ in terms of how much they value their privacy. Hence whether or not consumers will seek to pay in a way that preserves their privacy will depend on their own preferences for privacy, the quality of the good and the prices under the different payment options.

Our first result is that without a low-cost privacy-preserving digital payment option, there is a unique steady-state market outcome in which one firm captures the entire market. Payment data catalyzes the formation of a monopoly by enabling a firm with small advantages in information maintain and build its dominant position in the market. We refer to this equilibrium outcome as a data monopoly. Since the data monopolist gains access to all of the consumer data and hence can produce supe-

[6] Arguing by analogy, it is well documented that the *threat* of unionization may be sufficient to obtain the gains of collective bargaining, without the actual formation of a union (Taschereau-Dumouchel, 2020).

rior products, this solution maximizes total surplus. Most of the gains from consumer data do not go to consumers. Rather, consumers only receive the share of surplus that is necessary to induce the marginal consumer to use electronic payments over cash.

Interestingly, policies aimed at breaking up the data monopoly are not typically beneficial. We show that a data-sharing policy restores competition in the market, but this reduces total surplus and can even harm consumer welfare. With data sharing, competition allows consumers to extract the entire surplus generated by their data. However, at competitive prices for electronic purchases, cash becomes preferred by consumers with a high preference for privacy. Consequently less data is collected and firms are unable to produce products that maximize consumption value.

If the goal is to improve consumer welfare, then a policy is needed that enables consumers to extract more of the surplus generated by their private data, while at the same time incentivizing them to provide it. This can be achieved through the provision of a low cost, privacy-preserving digital payment option, which we refer to as *digital cash*. If digital cash allows consumers to protect their privacy and is not costly to use, then it strictly dominates cash and provides consumers with a low cost alternative to paying with the electronic payment method that reveals their private information.

Digital cash improves consumer welfare under the monopoly solution even though it is not used. This is because, the monopolist faces potential competition from the other firm, who despite having less (or no) access to private information and thus makes an inferior product, may make an attractive offer to consumers by accepting digital cash. This allows their consumers to enjoy (albeit inferior) goods without revealing their identity, and hence derive utility from privacy. We show that in equilibrium, this allows the firm with no market share to exert competitive pressure on the dominant firm. In order to induce consumers to provide access to data, the monopolist offers concessions that are linked to the payment method – discounted prices are offered to consumers that allow the firm to collect data. These discounted prices coincide with the marginal consumer's utility derived from privacy. In this way, the introduction of digital cash allows consumers to monetize their private data.

Our results are relevant to the current debate surrounding central bank digital currencies (CBDCs). CBDCs can come in many different forms (Bech and Garratt, 2017), however current interest focuses on whether or not central banks should respond to declining use of cash for transaction purposes by offering a digital cash substitute; namely central bank money in digital form that is available to the general public. Central banks such as Sweden and Canada, have published position papers explaining their thinking with regards to a CBDC; see Riksbank (2017, 2018) and Lane (2020). The message from these reports is that central banks might consider offering non-interest bearing digital accounts to consumers that offer privacy aspects.

Central banks are uniquely positioned to provide consumers with a method of payment that offers privacy. This is because a central bank can credibly commit to safeguarding data from outside vendors (Lagarde, 2018). The central bank has no profit motive to exploit payments data.[7] Previous work that explores potential implications of CBDC suggests that the introduction of a non-interest bearing, cash-like CBDC would have no effect on equilibrium allocations or welfare; see, for example, Keister and Sanches (2018) and Chiu et al. (2018).[8] Our results suggest the introduction of a non-interest bearing, cash-like CBDC would improve consumer welfare through a previously unrecognized channel.

2 Related literature

There are three strands of literature that are relevant to our analysis. First, there is a large body of work that looks at the economics of data and privacy; see the survey by Acquisti et al. (2016). This literature often focuses on the role of data in targeted advertising (Johnson, 2013), acquisition of data and externalities associated with data collection, (Choi et al., 2019; Garratt and van Oordt, 2019; Bergemann et al., 2020) and personalized pricing (Odlyzko, 2004; Rayna et al., 2015; Acquisti and Varian, 2005). A variety of papers look at whether or not consumers benefit from

[7]There are, of course, considerations that may arise when the government has access to additional personal information, such as has been discussed in the context of China's proposed Social Credit System. However, the extent to which the central bank shares payment data with the government can be limited.

[8]These models consider a Lagos and Wright (2005) search-theoretic framework in which a non-interest bearing CBDC acts as a perfect substitute for cash in meetings where the seller, perhaps for reasons related to privacy, will not accept bank deposits.

revealing their private information. For instance, Bourreau et al. (2017) point to the trade-off that arises when firms use data to price discriminate: some consumers with a high willingness to pay can be worse off, but others with low willingness to pay can be made better off due to market expansion. They point out that in cases where only certain firms are able to engage in personalised pricing, the availability of consumer data can increase market concentration (pg 43). Liu et al. (2020) studies the how weakening of privacy can improve firms' abilities to identify consumers vulnerable to temptation. However, we know of no other papers that look specifically at how consumers are compensated for their private information through payment-device specific prices.

Second, there are multiple papers that look specifically at data collected through payments. Parlour et al. (2019) highlights complementarities between payment and credit information, studies how fintech entrants, who may disrupt the flow of payment information to banks, may potentially impact consumer welfare. Garratt and van Oordt (2019) examines the potential for vendors to exploit payment information to price discriminate against wealthier consumers with higher reservation prices. These papers were written against a backdrop where the collection of private data through payments has become an increasingly important issue, as the line between big tech and big finance becomes increasingly blurred.[9] Facebook's recent Libra proposal offers, perhaps, the clearest glimpse of what a future might look like with little or no separation between individuals' social and financial status (Libra Association, 2019).

Third, there is a burgeoning literature on data and market structure. Farboodi et al. (2019) studies how data, which can be used to enhance decision-making, impacts the distribution of firm size in an economy with monopolistic competition. Much of this literature focuses on data mergers and whether or not these are anti-competitive (Wasastjerna, 2018). A primary concern with data mergers is that they can lead to concentrated market structures in which a single firm controls most or all of the market share. People (Furman et al., 2019) have also raised concerns that data collection,

[9] See, for example, the recent discussion surrounding the draft proposal of "Keep Big Tech Out of Finance Act" circulated by the House Financial Services Committee. https://financialservices.house.gov/calendar/eventsingle.aspx?EventID=404001

even in the absence of merges, can lead to "winner-takes-most" outcomes and that market competition alone is insufficient to prevent market concentration.

3 Model

Consumers and Firms. Time is discrete and infinite and discounted at rate $\beta \in (0,1]$. In each period, there is a continuum of consumers indexed by $i \in [0,1]$, two data-driven firms indexed $j = 1,2$, and $n \geq 1$ traditional firms.

Data. In each period, there is a continuum of states $\theta \in [0,1]$. Goods are defined by the realizations of random variables $x_\theta \in [0,1]$, which are drawn each period for each of these states. We denote the set of all possible goods by $\mathbf{X}$. The realization $\mathbf{x} \in \mathbf{X}$, is not directly observed by firms. However, data-driven firms $j = 1,2$ each own a technology to forecast the realization $\mathbf{x}$, which improves with the amount of data they collected in the past. Specifically, firm j learns x_θ for a measure $\rho(\mu^e_{t-1})$ of states θ, where μ^e_{t-1} is the measure of consumers that made an electronic purchase from firm j in the previous period, for some ρ such that $\rho(0) = 0$, $\rho(1) = 1$, $\rho'(\mu) > 0$ and $\rho''(\mu) > 0$ over the interval $\mu \in [0,1]$.The data of each firm is private.[10] Let the initial stock of firm j, μ^e_{j0} to be drawn from some probability distribution G with support $[0,1/2]$. The traditional firms do not own technology to process data.

Data enables firms to produce goods that are more attractive to consumers, and also provides an edge against their competitors. An important characteristic is the returns to scale on data. If access to more consumers' data increasingly improves the predictive power of data for analyzing consumers' tastes and producing a more desirable good, then data exhibits *network effects*, formally defined below:

Definition 1. *Data said to exhibit* network effects *if* $\rho'' > 0$ *over* $\mu \in [0,1]$. *Data is said to exhibit stronger network effects if* ρ'' *is larger.*

This property of data is essential to understanding the potential social value for high concentration of market share in data-driven sectors.

[10] Since we are interested in impact of data on the intensive margin, we can but do not explicitly consider the volume of publicly available data. If public data is accessible to all firms, the v term in Equation 1 can be interpreted as the utility value of good produced using publicly available data.

Payment Vehicles. There are three potential payment vehicles: cash (c), electronic (e) and privacy-preserving electronic, which we denote by (d) for "digital cash". Physical cash offers privacy to consumers but at a convenience disutility of $-\kappa$, where $\kappa \in (0, \alpha)$. Electronic payments allow the vendor to collect data from consumers and therefore do not offer privacy. Privacy-preserving electronic money provides consumers with an anonymous digital payment vehicle, enabling privacy. Consumer i derives individual utility from privacy equal to $\alpha_i \sim U[0, \alpha]$, which is not observable to firms.

Consumer Preferences. Each period, a measure of consumers, normalized to 1, seek a purchase. Individual consumers derive utility from consuming a single unit of a good that is produced by one of the firms. The amount of utility each consumer receives from consuming a good depends on how close the good type $\mathbf{y}$ is to $\mathbf{x}$. Given a pair ($\mathbf{y}$,$\mathbf{x}$), let $\nu(\mathbf{y}, \mathbf{x}) = \int \mathbb{1}_{y_\theta = x_\theta} d\theta$ denote the measure of states over which $y_\theta = x_\theta$.[11] Each consumer's utility, if they purchase the good $\mathbf{y}$, is given by

$$v + \gamma \cdot \nu(\mathbf{y}, \mathbf{x}) - p^m + \alpha_i \mathbb{1}_{m \in \{c,d\}} - \kappa \mathbb{1}_{m=c}, \tag{1}$$

where v is the reservation utility from consumption, γ is a taste parameter from consuming a higher quality good (i.e., a good that matches the state values more often), and p^m is the price the consumer pays for good $\mathbf{y}$, which may depend on the payment vehicle m. Here, γ captures the potential for data to enhance the consumption value of goods. We focus on an set of parameters such that γ is sufficiently high relative to α:

Assumption 1 (Data is Valuable.)**.** $\gamma > \frac{2\alpha}{\beta}$.

We use subscripts $j \in \{1, 2\}$ to denote firm specific goods and prices. Upon observing the set of goods and payment vehicle contingent prices, $(\mathbf{y}_j, p_j^m)$, each consumer chooses whether to consume a firm's good or forgo consumption. In the latter case, utility is zero.

Firms' Objective. Firms face a unit cost c to produce an invisible good. Firms' information sets are determined by an endowment of historical data μ_{jt-1}, where by

[11]Judd (1985) pointed out that there are severe measurability problems associated with this integration. See Uhlig (1996) for a solution.

assumption $\mu_{0t-1} = 0$. After observing their forecasts $\rho(\mu_{jt-1})$, each firm independently choose a single good type $\mathbf{y}_j$. After observing each firm's choices, each firm chooses the set M_j of payment vehicles to accept and price p_j^m associated with each payment vehicle $m \in M_j$ to maximize their discounted expected profit, where the current period expected profit is given by:

$$\pi_j(\mu_{jt-1}^e) = \mathbb{E}\left[\sum_{m \in M_j} (p_j^m - c) \cdot \mu_t^m\right],$$

where μ_t^m is the measure of consumers that choose to consume firm j's good using payment m. We assume that if two goods yield equal utility (net of price), then the consumer buys from the firm with greater μ_{t-1}.[12] It is also assumed that firms are subject to a non-negative profit condition. The value function of firm j is given by:

$$\Pi_j(\mu_j^e, \{\mu_k^e\}_{k \neq j}) = \max_{M_j, (\mathbf{y}_j, p_j^m)} \pi_j(\mu_j^e) + \beta \Pi_j(\mu_j^{e\prime}, \{\mu_k^{e\prime}\}_{k \neq j})$$
$$\text{s.t. } \pi_j(\mu_j^e) \geq 0$$

Our equilibrium notion is steady-state Perfect Bayesian Equilibrium. A formal definition is below:

Definition 2. *Given $\mu_{1t-1}^e, \mu_{2t-1}^e$, a steady-state equilibrium consists of firms' equilibrium strategies, M_j^* and $(\mathbf{y}_j^*, p_j^{m*})$, and consumers' consumption decisions and firms' equilibrium data μ_j^{d*} such that:*

1. **(Utility Maximization)** *consumers maximize their utility given the set of goods and prices $(\mathbf{y}_j^*, p_j^{m*})$;*
2. **(Profit Maximization)** *each firm j chooses a set of payment vehicles to accept, M_j^*, and a good type and prices $(\mathbf{y}^*, p_j^{m*})$ to maximize profits.*
3. **(Stationarity)** *firms' historical market shares by payment vehicle μ_{jt-1}^m are equal to its current shares: $\mu_{jt-1}^m = \mu_{jt}^m = \mu_j^{m*}$ for $m \in \{c, e, d\}$.*

[12]One can imagine that the firm with larger μ_{t-1} offers larger non-pecuniary benefits, such as faster payment processing.

4 Payment-Driven Data Monopolies

We start by analyzing the market in the absence of digital cash. This reflects today's payment landscape, where consumers face one of two purchase options: cash or electronic payments. The trade-off between the two are convenience and privacy. Physical cash purchases offer consumers privacy, which consumers value to different degrees. Electronic payments offer convenience.

The payment method choice of consumers is important to firms, who collect exclusive customer data when purchases are made using electronic payments. This data offers firms a competitive edge in producing attractive goods in the future. Firms can influence consumers' choice of payment method through discriminatory price-setting. Every period, each firm posts a menu of price-payment vehicle pairs for their good. Amidst competition, firms must trade off offering attractive prices to maximize profits, and also inducing consumers to make electronic purchases to enhance their stock of data and future competitiveness. Based on their stock of data, firms engage in price competition, subject to a static non-zero profit condition.[13]

We first posit the existence of a steady-state equilibrium whereby one firm maintains a larger electronic market share than the other, i.e. an asymmetric steady-state. At some initial period, the two data-driven firms each begin with some randomly determined initial stock of data μ_i, for $i = 1, 2$. Let subscript J be used to represent the dominant firm in the steady state, and $-J$ to denote the dominated firm. We begin with the following observation:

Lemma 1. *Let firm J be the dominant firm in an asymmetric steady state. There does not exist an asymmetric steady state with $\mu^{e*}_{-J} \neq 0$.*

Lemma 1 states that there does not exist an asymmetric steady state in which the dominated firm maintains a positive electronic market share. Intuitively, firms' product-price pairs are ordinal within a given payment vehicle m, which implies that

[13] In reality, firms may benefit from focusing on extending market share (and hence their data) by incurring losses in the short run, thereby increasing their long-term profits. For instance, technology companies such as Amazon, Facebook, and Google had incurred negative profits for extended periods before eventually generating positive profits, and largely attributes this to a deliberate focus on expanding market coverage in the short-run.

a firm cannot retain a positive electronic share and also be dominated. Establishing Lemma 1 simplifies the characterization of firms' equilibrium pricing strategies, outlined below:

Lemma 2. *Consider an asymmetric steady-state with $\mu_J^{e*} > \mu_{-J}^{e*} = 0$, where firm J is the dominant firm. Equilibrium prices are:*

- $p_J^{m*} = \begin{cases} \gamma\rho(\mu_J^{e*}) + c - \mathbb{1}_{m=e}\left(\mu_J^{e*}\alpha - \kappa\right) & \text{if } \mu_J^{e*} \geq \frac{\kappa}{\alpha} \\ \gamma\rho(\mu_J^{e*}) + c + \mathbb{1}_{m=c}\left(\mu_J^{e*}\alpha - \kappa\right) & \text{if } \mu_J^{e*} < \frac{\kappa}{\alpha} \end{cases}$ *for $m = c, e$.*
- *and $p_{-J}^{m*} = c$ for $m = c, e$.*

In an asymmetric steady state, two key factors determine the pricing strategy of the dominant firm. First, non-dominant firms offer goods competitively for both cash and electronic goods, and at the marginal cost c. Given this, the dominant firm uses preemptive prices to dominate other firms' good-price pairs for each payment vehicle, effectively capturing the entire market share for both electronic and cash purchases. Second, the dominant firm must select a menu of prices for cash and electronic money purchases such that a steady state μ_J^{e*} is maintained. Specifically, in order to maintain an electronic market share of $\mu_J^{e*} \geq \frac{\kappa}{\alpha}$ ($< \frac{\kappa}{\alpha}$), firm J must offer a discount (premium) electronic price $\mu_J^{e*}\alpha - \kappa$ to induce the marginal consumer to forgo privacy.

Given the pricing strategies, we can characterize the steady state equilibrium. We show that this equilibrium exists and is unique:

Proposition 1 (Data Monopoly Equilibrium)**.** *Without digital cash, there exists a unique steady-state equilibrium, where*

- *electronic market shares are $\mu_J^{e*} = 1 > \mu_{-J}^{e*} = 0$;*
- *cash market shares are $\mu_j^{c*} = 0$ for $j = 1, 2$;*
- *and prices are $p_J^{m*} = \gamma + c - \mathbb{1}_{m=e}(\alpha - \kappa)$ for $m = c, e$ and $p_{-J}^{m*} = c$ for $m = c, e$.*

We refer to this equilibrium as a data monopoly equilibrium.

Payment data facilitates the formation of a *data monopoly* by allowing a firm to leverage small advantages in product quality to accumulate more data and to solidify control over the entire market. In the context of our model, small differences in the initial stock of data perpetuates into the long-run steady-state with a winner-takes-all market. This phenomenon, in which payment data acts as a catalyst for market concentration, does not rely on the random initial stock of data. Any source of volatility with respect to period-by-period product quality that creates an asymmetry in product quality would be enough to sow the seeds of a monopoly.

In the data monopoly equilibrium, a dominant firm retains the entire market share and induces all consumers to purchase goods using electronic money. As implied by Lemma 2, in order to do so, the dominant firm must set electronic prices sufficiently low such that even consumers with the strongest preference for privacy (i.e. $\alpha_i = \alpha$) forgo the cash option. Under Assumption 1, the value of data γ is sufficiently large relative to privacy α and the dominant firm finds it incentive compatible to maintain its stock of data at 1.[14]

In the model, data is essential to firms' productivity, expressed through the utility value of goods they produce. In our setting, firms compete in a market where data exhibits positive and increasing returns to scale, i.e. $\rho' > 0$, $\rho'' > 0$. As a result, market concentration enables the greatest concentration of data, and unambiguously increases the total surplus accrued from the use of data. The total equilibrium surplus is:

$$v - c + \gamma \tag{2}$$

where $v - c$ represents the reservation gains from transactions, and γ corresponds to the additional surplus generated from data. As the data monopolist sets prices to acquire an electronic market share of 1, it also attains the maximum possible surplus from data, γ.

As outlined in Proposition 1, all consumers in equilibrium make an electronic purchase of the dominant firm's good, which yields a payoff of:

$$v + \gamma \underbrace{\nu(\mathbf{y}_j, \mathbf{x})}_{=1} - p_J^{e*} \tag{3}$$

[14]Note, Assumption 1 is a simplifying assumption; the data monopoly result generally holds.

The extent to which consumers reap the benefits of their data depends on the equilibrium price p_J^{e*}. Substituting the equilibrium price p_J^{e*} from Lemma 2 into Equation 3 yields the consumers' equilibrium surplus:

$$v - c + \underbrace{(\alpha - \kappa)}_{\text{compensation for privacy}} \tag{4}$$

The existence of physical cash enables consumers to reap benefits of the surplus derived from data. Consumers that highly value privacy are compensated by the firm through discounted prices that make electronic purchases attractive for consumers of all types.

Compensation for privacy is the dominant firm's acquisition cost associated with inducing even the consumer with the highest privacy preference $\alpha_i = \alpha$ to provide data. This value decreases in the convenience disutility κ associated with cash. This also implies that as physical cash becomes prohibitively costly to use, consumer welfare diminishes.

5 The Welfare Implications of Data Sharing

A key policy concern is that the nature of data leads to natural monopolies. This holds true in our environment, where data exhibits positive and increasing returns to scale, and consumers value product improvements generated from data. While data monopolies increase surplus in our framework, these gains come at the cost of consumers' privacy, who themselves only marginally benefit from their data.

Are there actions that a regulator can take to improve consumer welfare? One practical solution is to level the playing field and promote competition through the implementation of data-sharing policies. In this subsection, we explore how such a policy can be implemented and its implications on welfare and efficiency. We consider a policy that requires firms to share any and all exclusive data derived from past activities. We begin by showing that this policy does in fact restore competition between firms.

The intuition is as follows. The primary asset through which a firm exercises monopoly power is through its stock of data. A dominant firm retains its monopoly status by using payment data to produce superior goods, and setting prices just low enough to retain its advantage over competitors. By "democratizing" information derived from payment data, a competing firm can produce goods of comparable quality to the dominant firm. This forces both firms to engage in price competition, driving down prices and letting consumers benefit from the utilization of their personal data.

Through the implementation of a data-sharing policy, the market transitions into into a steady-state equilibrium where data-driven firms retain equal share of the market. We summarize the above below:

Proposition 2 (Data-Sharing Levels Competition)**.** *Suppose that a regulator requires all firms to share data at the end of each period. In equilibrium, firms* $i = 1, 2$ *retain equal share of the market.*

There exists a clear, implementable policy to restore competition. However, we find that in general such a policy is not desirable, from a standpoint of both efficiency and consumer welfare:

Proposition 3 (Effect of a Data-Sharing Policy)**.** *Suppose that a regulator imposes a data-sharing policy. Then, total surplus is strictly reduced. Consumer welfare is lower if data exhibits strong network effects, e.g.* ρ'' *is large.*

The reduction in total surplus is straightforward. Under a data sharing policy, the total surplus (per period) in the steady-state is given by:

$$v - c + \gamma\rho\left(\frac{\kappa}{\alpha}\right) + \underbrace{\frac{1}{2\alpha}(\alpha - \kappa)^2}_{\text{aggregate gains from privacy}} \tag{5}$$

Note that $\rho\left(\frac{\kappa}{\alpha}\right) \leq \frac{\kappa}{\alpha}$ since if ρ is continuous, monotonic, and $\rho(0) = 0$ and $\rho(1) = 1$, then ρ' is bounded above by 1. By comparison, total surplus (per period) under a data-monopoly is $v + \gamma - c$, which is always greater.

Comparing consumer surplus for the two equilibria reveals a trade-off. A consumer i who purchases firm j's good obtains

$$v + \gamma \cdot \nu(\mathbf{y}, \mathbf{x}) - c + \max\{\alpha_i - \kappa, 0\} \tag{6}$$

Aggregating over all consumers yields:

$$v - c + \underbrace{\gamma\rho\left(\frac{\kappa}{\alpha}\right)}_{\text{surplus from data}} + \underbrace{\frac{1}{2\alpha}(\alpha - \kappa)^2}_{\text{gains from privacy}} \tag{7}$$

A data-sharing policy eradicates the source of monopoly, and effectively allows consumers to extract the entire surplus of data accumulated through electronic purchases. However, competitive pressures also make the price of physical cash purchases competitive. In equilibrium, consumers that highly value privacy use physical cash, with aggregate gains from privacy amounting to $\frac{1}{2\alpha}(\alpha - \kappa)^2$. This, however comes at a cost of data. Competition also impedes the ability of any firm to acquire more data through price discrimination. As a result, the aggregate stock of data falls when firms share data.

Under data sharing, both firms obtain limited data, and neither are able to produce goods that maximize consumption value. In contrast, under a data monopoly, the dominant firm price discriminates between payment vehicles in order to acquire the maximum amount of data. Consumer welfare is greater under a data monopoly when data exhibits sufficiently strong network effects, i.e. large ρ''. Even though a data monopolist marks up prices to reflect the superior quality of its good, consumer welfare is improved through discounts that the data monopolist provides for digital purchases used to induce the marginal consumer to reveal her private information.

6 Digital Cash and Monetizing Privacy

In this section, we consider what happens when a low-cost privacy-preserving payment vehicle, i.e. digital cash, is introduced as a third payment option. Purchasing a good using digital cash allows consumers to acquire goods without revealing their identity, and yields an additional utility of α_i associated with privacy.

Digital cash allows consumers to enjoy utility derived from privacy α_i. Moreover, like electronic money, it offers the convenience associated with digital transactions. Hence, it is immediate that cash is strictly dominated by digital cash.

With this simple observation, we can extend the equilibrium uniqueness result of Proposition 1 to a world where digital cash exists, and consider the transition dynamics after the introduction of digital cash:

Proposition 4 (Equilibrium with Digital Cash)**.** *The introduction of digital cash leads to a transition into a data monopoly equilibrium with adjustments in equilibrium prices.*

Proposition 4 shows that introducing digital cash shifts equilibrium prices, but preserves equilibrium *market structure*. This allows us to draw direct inferences on welfare implications following the introduction of digital cash.

We start by observing that total surplus is unaffected by the introduction of digital cash. When data is sufficiently valuable, the monopolist maximizes profits by maintaining the maximum stock of data. Proposition 4 shows that even with the introduction of digital cash, the dominant firm's share of electronic purchases is unchanged at $\mu_J^e = 1$. This implies that the total surplus is equal to $v - c + \gamma$, as in Equation 2.

Since the overall market structure remains a data monopoly, changes in the price solely reflect a redistribution between the monopolist and consumers. We show that prices monotonically drop in equilibrium to reflect the improvements in the consumers' privacy preserving payment option. Consumer welfare unambiguously increases since $v - c + \alpha > v - c + \alpha - \kappa$. Putting this together we have:

Proposition 5 (Digital Cash Improves Consumer Welfare)**.** *The introduction of digital cash always improves consumer welfare in a data monopoly equilibrium. Furthermore, consumer welfare is strictly greater in the data monopoly equilibrium with the introduction of digital cash.*

This result arises from the pricing of privacy. The monopoly firm J sets the compensation of privacy equal to the amount required by the marginal consumer. Since the equilibrium stock of data is $\mu_J^e = 1$, the marginal consumer's privacy preference is α.

In contrast with the data-sharing policy, digital cash largely preserves the underlying market structure, but endows individual consumers with an additional payment option. Even though not all consumers value privacy highly, when the cost of using digital cash is low (e.g. zero in our case), digital cash becomes an attractive alternative to electronic payments. In a data monopoly equilibrium, where a dominant firm price-discriminates between various payment vehicles, digital cash effectively improves the bargaining power of individual consumers. In order to maintain its stock of data, the dominant firm lowers the price of electronic purchases. Consumers who make electronic purchases are compensated by the maximum value of privacy, α, and effectively "monetize" privacy.

7 Relevance to Central Bank Digital Currency Initiatives

Currently, many central banks are contemplating, and some are even experimenting with, the issuance of a CBDC that would be accessible to the public.[15] In support of these efforts, researchers have examined potential impacts of CBDCs on monetary policy implementation (Bordo and Levin (2017)), deposit and lending markets (Andolfatto (2018), Keister and Sanches (2018) and Chiu et al. (2018)), price stability (Fernández-Villaverde et al. (2020b)) and financial stability (Keister and Monnet (2020), Fernández-Villaverde et al. (2020a), Monnet et al. (2019) and Williamson (2018)). For the most part, these works focus on the impact of an interest-bearing CBDC. Generally these models do not provide a mechanism for a non-interest bearing, cash-like CBDC to do anything but substitute for physical cash. As such, they suggest that a cash-like CBDC would not affect market outcomes or welfare.

Our work contributes to an emerging literature that argues privacy considerations matter in the design of CBDC.[16] We demonstrate a novel channel through which the provision of a privacy-preserving CBDC increases consumer welfare. Interestingly,

[15] Boar et al. (2020) report that 80 percent of central banks (sample size of 66 covering 90 percent of world economic output) that responded to their Bank for International Settlements survey are engaged in some sort of work related to central bank digital currency.

[16] See the aformentioned paper by Garratt and van Oordt (2019). See also ECB (2019) and Darbha and Arora (2020) for examples of central bank investigations into the feasibility and trade-offs of privacy-preserving CBDC.

as in Chiu et al. (2018), a CBDC impacts market outcomes in our framework despite not being used in equilibrium. In both frameworks, the CBDC acts as a threat point that changes the terms negotiated in another market. In Chiu et al. (2018) the interest rate on CBDC becomes a lower bound on the deposit interest rate. Banks respond to the introduction of an interest-bearing CBDC by raising the rate on deposits and the CBDC is not used. In our case, the threat is that consumers will use the CBDC and hence not provide their data to firms. To prevent this, consumers are compensated by receiving lower prices when they use the payment method that reveals their data, and thus the CBDC is not used.

Our conclusions do not depend on the CBDC not being used. We could extend our environment to have a subset of consumers with a very high preference for privacy, or that seek privacy to protect themselves from issues such as targeted advertising or price discrimination discussed above. Such consumers could use the CBDC, if it were available, and this would not diminish the CBDCs role as a threat point.

8 Conclusion

We develop a framework in which market structure is endogenously determined by historical data acquisition, which requires consumers to purchase goods through electronic payments. Data acquisition increases surplus, by allowing firms to make products that better match consumer preferences.

Our model generates a stark result. Payment data catalyzes the formation of data monopolies. Under a monopoly, consumers only marginally benefit from their data. A natural policy reaction is to democratize data, and thereby improve competition. We show that a data-sharing policy does in fact improve competition, and also find that the entire surplus from data accrues to consumers. And yet, consumer welfare can be lower as competition also impedes on the collection of data.

We consider the introduction of a privacy-preserving means of payment, i.e. digital cash. Digital cash alters the economic environment by providing consumers with a costless way to protect their private information. Firms must pay consumers to use privacy revealing payment instruments, thus allowing consumers to monetize their data. We show that digital cash preserves the underlying market structure while redistributing the gains from payment data to consumers. A privacy-preserving CBDC could be an effective implementation of digital cash.

References

Acquisti, Alessandro and Hal R. Varian, "Conditioning Prices on Purchase History," *Marketing Science*, August 2005, *24* (3), 367–381.

_ **, Curtis Taylor, and Liad Wagman**, "The Economics of Privacy," *Journal of Economic Literature*, 2016, *54* (2), 442–492.

Agarwal, Sumit, Souphala Chomsisengphet, Chunlin Liu, Changcheng Song, and Nicholas S Souleles, "Benefits of Relationship Banking: Evidence from Consumer Credit Markets," *Journal of Monetary Economics*, 2018, *96*, 16–32.

Andolfatto, David, "Assessing the Impact of Central Bank Digital Currency on Private Banks," 2018.

Bech, Morten L and Rodney J Garratt, "Central Bank Cryptocurrencies," *BIS Quarterly Review*, September 2017.

Bergemann, Dirk, Alessandro Bonatti, and Tan Gan, "The Economics of Social Data," 2020, *Cowles Foundation Discussion Paper NO. 2203R.*

Boar, Codruta, Henry Holden, and Amber Wadsworth, "Impending Arrival a Sequel to the Survey on Central Bank Digital Currency," *BIS Papers No 107*, 2020.

Bordo, Michael D. and Andrew T. Levin, "Central Bank Digital Currency and the Future of Monetary Policy," 2017.

Bourreau, Marc, Alexandre de Streel, and Inge Graef, "Big Data and Competition Policy: Market Power, Personalised Pricing and Advertising," 2017, *Project Report* (Centre on Regulation in Europe).

Chiu, Jonathan, Mohammadreza Davoodalhosseini, Janet Jiang, and Yu Zhu, "Central Bank Digital Currency and Banking," 2018.

Choi, J.P., D.-S. Jeon, and B.-C. Kim, "Privacy and Personal Data Collection with Information Externalities," *Journal of Public Economics*, 2019, *173*, 113–124.

Darbha, Sriram and Rakesh Arora, "Privacy in CBDC Technology," June 2020.

ECB, "Exploring Anonymity in Central Bank Digital Currencies," *In Focus*, December 2019, (4).

Farboodi, Maryam, Roxana Mihet, Thomas Philippon, and Laura Veldkamp, "Big Data and Firm Dynamics," in "AEA Papers and Proceedings," Vol. 109 2019, pp. 38–42.

Fernández-Villaverde, Jesús, Daniel Sanches, Linda Schilling, and Harald Uhlig, "Central Bank Digital Currency: Central Banking For All?," 2020.

_ , _ , _ , and _ , "Central Bank Digital Currency: When Price and Bank Stability Collide," 2020.

Furman, Jason, Dianne Coyle, Amelia Fletcher, Derek McAuley, and Philip Marsden, "Unlocking Digital Competition," 2019, *Report of the Digital Competition Expert Panel* (HM Government).

Gambaro, Marco, "Big Data, Competition and Market Power," *Departmental Working Papers 2017-15*, Department of Economics, Management and Quantitative Methods at Universit degli Studi di Milano 2017.

Garratt, Rodney and Maarten R.C. van Oordt, "Privacy as a Public Good: A Case for Electronic Cash," 2019.

Ibarra, Imanol Arrieta, Leonard Goff, Diego Jimnez Hernndez, Jaron Lanier, and Eric Glen Weyl, "Should We Treat Data as Labor? Moving Beyond 'Free'," *American Economic Association Papers Proceedings*, 2018, *108*, 38–42.

Johnson, Justin P., "Targeted Advertising and Advertising Avoidance," *The Rand Journal of Economics*, 2013, *44* (1), 128–144.

Judd, Kenneth, "The Law of Large Numbers with a Continuum of IID Random Variables," *Journal of Economic Theory*, 1985, *35* (1), 19–25.

Keister, Todd and Cyril Monnet, "Central Bank Digital Currency: Stability and Information," 2020.

_ **and Daniel Sanches**, "Should Central Banks Issue Digital Currency?," 2018.

Lagarde, Christine, "Winds of Change: The Case for New Digital Currency," *Speech*, 2018.

Lagos, Ricardo and Randall Wright, "A Unifed Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 2005, *113*, 463–484.

Lane, Timothy, "Money and Payments in the Digital Age," *CFA Montral FinTech RDV2020*, February 2020, *Montral, QC.*

Libra Association, "Libra white paper," 2019.

Liu, John Zhuang, Michael Sockin, and Wei Xiong, "Data Privacy and Temptation," 2020.

Monnet, Cyril, Asgerdur Petursdottir, and Mariana Rojas-Breu, "Central Bank Accounts For All: Efficiency and Stability," 2019.

Odlyzko, Andrew, "Privacy, Economics, and Price Discrimination on the Internet," in L.J. Camp and S. Lewis, eds., *Economics of Information Security. Advances in Information Security*, Boston, MA: Springer, 2004, pp. 187–211.

Parlour, Christine A, Uday Rajan, and Haoxiang Zhu, "FinTech Disruption, Payment Data, and Bank Information," 2019.

Rayna, T., J. Darlington, and L. Striukova, "Pricing Music Using Personal Data: Mutually Advantageous First-degree Price Discrimination," *Electronic Markets*, 2015, *25*, 39–154.

Riksbank, "E-krona Project, Report 1," 2017.

_ , "E-krona Project, Report 2," 2018.

Stavins, Joanna, "How Do Consumers Make Their Payment Choices?," 2017.

Taschereau-Dumouchel, Mathieu, "The Union Threat," *The Review of Economic Studies*, 2020, *87* (6), 2859–2892.

Uhlig, Harald, "A Law of Large Numbers for Large Economies," *Economic Theory*, 1996, *8* (2), 41–50.

Wasastjerna, Maria C., "The Role of Big Data and Digital Privacy in merger review," *European Competition Journal*, 2018, *14* (2–3), 417–444.

Williamson, Stephen, "Central Bank Digital Currency: Welfare and Policy Implications," 2018.

A Proofs

Proof of Lemma 1. Without loss of generality, let $J = 1$. Suppose by contradiction that $\mu_2^{e*} > 0$. Since $\mu_1^{e*} > \mu_2^{e*}$, firm 1's good strictly dominates firm 2. This implies that for any (y_2^*, p_2^{e*}), firm 2's market share is strictly positive only if a positive measure of consumers strictly prefer it to (y_1^*, p_1^{e*}). However, since electronic purchases are perfect substitutes (up to quality-price pair), this implies that all consumers must strictly prefer (y_2^*, p_2^{e*}) to (y_1^*, p_1^{e*}). This implies that $\mu_2^{e*} > \mu_1^{e*}$. We reach a contradiction. □

Proof of Lemma 2. Take as given that there exists some asymmetric steady-state equilibrium where $\mu_1^{e*} > \mu_2^{e*} = 0$ are the equilibrium stocks of data for firms 1 and 2. First, note that for any electronic price p_1^e, a necessary condition for firm 1 to retain electronic share μ_1^{e*} is $p_1^e \leq p_1^c - \left(\mu_1^{e*}\alpha - \kappa\right)$. Second, note that firm 1 must offer a strictly dominating product-price pair such that $p_1^e \geq p_2^e + \gamma\rho(\mu_1^{e*})$, as otherwise $\mu_2^{e*} > 0$. For any p_1^e, firm 2 gains market share (and positive profits) by setting price $p_2^e < \max\{p_1^e - \gamma\rho(\mu_1^{e*}), c\}$. Together this implies that $p_1^e \leq c + \gamma\rho(\mu_1^{e*})$. Hence, $p_1^e \leq \min\{c + \gamma\rho(\mu_1^{e*}), p_1^c - \left(\mu_1^{e*}\alpha - \kappa\right)\}$. Recall that since $\mu_2^{e*} = 0$, firm 2 and traditional firm(s) offer perfect substitutes. Following the argument for Lemma 4, firm 2 offers $p_2^e = p_2^c = c$. Then, p_1^e must also satisfy $p_1^e \leq c + \gamma\rho(\mu_1^{e*}) - \left(\mu_1^{e*}\alpha - \kappa\right)$. First consider the case where $\mu_1^{e*} \geq \frac{\kappa}{\alpha}$ so that $\mu_1^{e*}\alpha - \kappa > 0$. Note that $p_1^c = c + \gamma\rho(\mu_1^{e*})$ is optimal under this case, since it is the maximum p_1^c such that $p_1^e \leq \min\{c + \gamma\rho(\mu_1^{e*}), p_1^c -$

$\left(\mu_1^{e*}\alpha-\kappa\right), c+\gamma\rho(\mu_1^{e*})-\left(\mu_1^{e*}\alpha-\kappa\right)\}$ holds. Second consider the case where $\mu_1^{e*}<\frac{\kappa}{\alpha}$ so that $\mu_1^{e*}\alpha-\kappa<0$. Then $p_1^e\leq c+\gamma\rho(\mu_1^{e*})$ is more binding. For $p_1^e=c+\gamma\rho(\mu_1^{e*})$, then $\mu_1^{e*}<\frac{\kappa}{\alpha}$ is obtained only if $p_1^c=c+\gamma\rho(\mu_1^{e*})+(\mu_1^{e*}\alpha-\kappa)$. □

Proof of Proposition 1. We show existence and uniqueness by showing that (1) per-period payoff is greatest at steady state with $\mu_1^e=1$, and (2) deviations to any $\mu_1^{e\prime}<1$ conditional on initial state $\mu_1^e=1$ are not profitable. Consider a candidate equilibrium with $\mu_1^{e*}=1$ and $\mu_2^{e*}=0$. The value function of firm 1 using prices specified under Lemma 2 is $\gamma+\beta V(1,0)$. Since such prices lead to a steady state in market shares, it suffices to show that there does not exist any deviation in prices from Lemma 2 that is profitable. From Lemma 2, note that $\mu_1^{e*}<\frac{\kappa}{\alpha}$ cannot be an equilibrium as setting $p_1^c=\gamma\rho(\mu_1^{e*})+c$ strictly increases the expected payoff of firm 1. It suffices to consider any deviation in prices that result in some ${\mu^{e\prime}}_1\geq\frac{\kappa}{\alpha}$. We show this in two steps: (1) per-period payoff is greatest at steady state with $\mu_1^e=1$, and (2) deviations to any $\mu_1^{e\prime}<1$ conditional on initial state $\mu_1^e=1$ are not profitable.

Note that per-period payoff from $\mu_1^e\in[\frac{\kappa}{\alpha},1]$ is given by

$$\mu_1^e\left(\gamma\rho(\mu_1^e)-(\mu_1^e-\frac{\kappa}{\alpha})\alpha\right)+(1-\mu_1^e)\gamma\rho(\mu_1^e)=\gamma\rho(\mu_1^e)-\mu_1^e(\mu_1^e\alpha-\kappa) \tag{8}$$

Per-period payoff is greater at $\mu_1^e=1$ relative to ${\mu^{e\prime}}_1<1$ if:

$$\gamma(1-\rho({\mu^{e\prime}}_1))-(1-{\mu^{e\prime}}_1^{2})\alpha+(1-{\mu^{e\prime}}_1)\kappa>(1-{\mu^{e\prime}}_1)(\gamma-(1+{\mu^{e\prime}}_1)\alpha+\kappa), \tag{9}$$

which is always greater than 0 when $\gamma>2\alpha-\kappa$. This implies that expected payoff is when $\mu_1^e=1$ is greater than any other ${\mu^{e\prime}}_1\geq\frac{\kappa}{\alpha}$.

It remains to check that conditional on μ_1^e, deviations are not profitable. Take as given an initial stock of data $\mu_1^e=1$ and $\mu_2^e=0$. Since per-period payoff is greater with $\mu_1^e=1$, deviation is only profitable if a one-period profit from Deviating is not profitable if $-(\alpha-\kappa)+{\mu^{e\prime}}_1({\mu^{e\prime}}_1\alpha-\kappa)+\beta\left(\frac{\gamma-(\alpha-\kappa)}{1-\beta}-\frac{\gamma\rho({\mu^{e\prime}}_1)-{\mu^{e\prime}}_1(\alpha{\mu^{e\prime}}_1-\kappa)}{1-\beta}\right)\geq0$. Reorganizing the inequality $\frac{\beta\gamma}{1-\beta}\left(1-\rho({\mu^{e\prime}}_1)\right)\geq(\alpha-\kappa)-{\mu^{e\prime}}_1({\mu^{e\prime}}_1\alpha-\kappa)$. Since $1-\rho({\mu^{e\prime}}_1)>1-\mu$, a sufficient condition is $\gamma>\frac{(1+\mu)\alpha-\kappa}{\beta}$. □

Proof of Proposition 2. Under a data-sharing policy, this implies that both firms $i=$

1, 2 share the same data stock at each period. We establish equilibrium existence by taking as given an equilibrium, and first by characterizing the optimal pricing strategy given any arbitrary μ^e, and then pinning down the equilibrium stock of data μ^e in a symmetric steady-state equilibrium. Observe that a symmetric steady-state equilibrium requires that firms employ symmetric pricing strategies in equilibrium.

Lemma 3. *In any steady state equilibrium with equal market share, firms must employ symmetric pricing strategies in equilibrium.*

Therefore, we can restrict our attention to symmetric pricing strategies. Firms independently decide on which payment vehicles to accept, and prices for each option. For now, take as given that all firms offer prices for both electronic money and cash. Given some stock of data $\rho(\mu^e)$, firms compete with prices per payment vehicle, $m = e, c$. When firms compete using the same quantity of data, their goods are perfect substitutes. Take as given μ^e, and consider the pricing strategy for a given payment vehicle e. Given p^e_{-j}, firm j's single-period payoff from its electronic market share is given by:

$$\begin{cases} 0 & \text{if } p^m_j > p^m_{-j} \\ \mu^e(p^m_j - c) & \text{if } p^m_j = p^m_{-j} \\ 2\mu^e(p^m_j - c) & \text{if } p^m_j < p^m_{-j} \end{cases} \tag{10}$$

It follows directly, that the only symmetric equilibrium strategy is when $p^e_j = c$ for $j = 1, 2$. A similar argument can be applied to the equilibrium pricing strategy for cash. Taking as given a market share of $\mu^c = 1 - \mu^e$, both firms' best responses drive prices down to the marginal cost c. This implies that firms make zero profits in equilibrium. This shows:

Lemma 4. *In a symmetric steady state equilibrium, all prices are equal to c.*

Consider firms' decisions on the set of accepted payment vehicles. Taking as given some stock of data $\rho(\mu^e)$ for each firm, where $\mu^e \in (0, \frac{1}{2})$, consider first the decision of whether to accept cash or not. The subset of agents with privacy preference $\alpha_i > \kappa$ strictly prefer using cash to electronic money. Conditional on no firm offering cash purchases, firm j can make a profitable deviation to offering cash at price $p^c_j = c + \epsilon$

for some $\epsilon > 0$, and capture all purchases made by these agents. Oppositely, agents with privacy preference $\alpha_i < \kappa$ strictly prefer electronic money. Applying the same argument, we reach the following observation:

Lemma 5. *In a equilibrium with a symmetric steady state, all firms opt to offer goods using all payment vehicles.*

Finally, consider some candidate level $\mu^{e'} \in (0,1)$. In a symmetric steady state, both firms produce goods with equal utility value. The marginal agent i with privacy preference α_i is indifferent between using cash and electronic money to purchase firm j's good if:

$$p_j^e = p_j^m - (\alpha_i - \kappa) \tag{11}$$

If $\mu^{e\prime}$ is greater than $\frac{\kappa}{\alpha}$, this implies that firms are setting prices such that the marginal buyer i using electronic money has a privacy preference α_i is greater than κ. This implies, however, that prices must be offered below the marginal cost, leading to negative profits in equilibrium. In contrast, if $\mu^{e\prime}$ is less than $\frac{\kappa}{\alpha}$, this implies that firms are setting prices such that the marginal buyer i's privacy preference α_i is less than κ. This also cannot arise in equilibrium, as such a buyer would strictly prefer to purchase the good at the competitive cash price instead.

Together, we obtain a unique steady state equilibrium exactly at the boundary where the marginal agent is exactly indifferent between the two forms of payment vehicles, where the electronic market shares are $\mu_j^{e*} = \frac{\kappa}{2\alpha}$ $j = 1, 2$; cash market shares are $\mu_j^{c*} = \frac{\alpha - \kappa}{2\alpha}$ for $j = 1, 2$; and price strategies are $p_j^{m*} = c$ for $m = c, e$ and $j = 1, 2$. In equilibrium, firms' collective data is given by $\frac{\kappa}{\alpha}$. Results on surplus and welfare presented in text. □

Proof of Proposition 3. Follows from text. □

Proof of Proposition 4. Consider when $\mu_{1t-1}^e = 1$ and $\mu_{2t-1}^e = 0$. The lowest possible price offered by either firm 0 and 1 is c. Suppose that $p_j^e = c$ for firm 2. A consumer of type α_i prefers to purchase firm 1's good using electronic if $p_1^e = \gamma + c - \alpha_i$ and firm 1′

Firms offer prices specified by Lemma 1 with $\kappa \to 0$. This establishes that given each steady state, introducing digital cash leads to a single-period transition into the new steady state equilibrium. $\square$

Proof of Proposition 5. Lemma 2, combined with the fact that $\mu_J^{e*} = 1$ implies that when dominant firm J lowers prices, consumer welfare directly increases. Note that total consumer surplus is equal to $v - c + \alpha$. Since $v - c + \alpha > v - c + \frac{\alpha}{2}$, consumer surplus is strictly greater under a data monopoly equilibrium. $\square$