STAFF REPORTS

NO. 961
MARCH 2021

Inflation Expectations and Risk Premia in Emerging Bond Markets: Evidence from Mexico

Remy Beauregard | Jens H. E. Christensen | Eric Fischer
Simon Zhu

FEDERAL RESERVE BANK *of* NEW YORK

Inflation Expectations and Risk Premia in Emerging Bond Markets: Evidence from Mexico
Remy Beauregard, Jens H. E. Christensen, Eric Fischer, and Simon Zhu
Federal Reserve Bank of New York Staff Reports, no. 961
March 2021
JEL classification: D84, E31, E43, E44, E47, E52, E58, G12

Abstract

To study inflation expectations and associated risk premia in emerging bond markets, this paper provides estimates for Mexico based on an arbitrage-free dynamic term structure model of nominal and real bond prices that accounts for their liquidity risk. In addition to documenting the existence of large and time-varying liquidity premia in nominal and real bond prices that are only weakly correlated, the results indicate that long-term inflation expectations in Mexico are well anchored close to the inflation target of the Bank of Mexico. Furthermore, Mexican inflation risk premia are larger and more volatile than those in Canada and the United States.

Key words: term structure modeling, liquidity risk, financial market frictions, central bank credibility

---

Fischer: Federal Reserve Bank of New York. Beauregard, Christensen (corresponding author), Zhu: Federal Reserve Bank of San Francisco (email: jens.christensen@sf.frb.org). This paper has also been circulated as Federal Reserve Bank of San Francisco Working Paper 2021-08.

To view the authors' disclosure statements, visit
https://www.newyorkfed.org/research/staff_reports/sr961.html.

1 Introduction

Breakeven Inflation (BEI)—the difference between yields on comparable-maturity nominal and real debt—is a widely used indicator of inflation expectations. In particular, long-term BEI is frequently used to measure the credibility of the central bank's inflation objective.[1] However, BEI is a noisy measure of expected inflation because it contains both an inflation risk premium and differential liquidity premia. As a consequence, better measures of investors' underlying inflation expectations could be obtained by subtracting both inflation risk premia and the differential liquidity premia in nominal and real yields from BEI rates.

The literature on inflation expectations and associated inflation risk premia extracted from nominal and real yields in advanced economies is burgeoning.[2] However, these topics have received far less attention for emerging economies, although they arguably matter more there for policymakers and bond investors due to higher macroeconomic uncertainty, in general, and larger inflation variability, specifically. The main contribution of this paper is to fill the gap by building on recent advances in fixed-income analysis.

The challenge in accounting for the differential liquidity premia in nominal and real bond prices is to distinguish them from more fundamental factors such as inflation risk premia that would affect asset prices even in a world without any frictions to trading. To achieve this separation, we augment a flexible dynamic term structure model of nominal and real bond prices studied in Carriero et al. (2018) with separate liquidity risk factors for nominal and real bonds using the approach described in Andreasen et al. (2020, henceforth ACR). For each class of bonds, the identification of the liquidity risk factor comes from its unique loading, which mimics the idea that, over time, an increasing amount of the outstanding notional of individual securities gets locked up in buy-and-hold investors' portfolios. This increases their sensitivity to variation in the market-wide liquidity risk captured by the corresponding liquidity risk factor. By observing prices for balanced panels of nominal and real bonds, their respective liquidity risk factors can be separately identified. This separation is particularly salient in emerging bond markets as they tend to be much less deep and liquid than the well-established major international bond markets in the United States and elsewhere.

To better understand the properties and dynamics of inflation expectations and associated risk premia in emerging bond markets, we choose to focus on a country where they are likely to play a first-order role, namely Mexico. Several motivations underlie this choice. First, Mexico has a long history of high and fairly volatile inflation.[3] Second and equally important

[1]Provided the objective is credible, it should be reflected in inflation expectations for the distant future as any current inflation shocks should be considered temporary and not affect long-run inflation expectations.

[2]For Canada, see Christensen et al. (2020); for the euro area, see Hördahl and Tristani (2012, 2014); for the United Kingdom, see Joyce et al. (2010) and Carriero et al. (2018); for the United States, see Christensen et al. (2010), Abrahams et al. (2016), D'Amico et al. (2018), among many others.

[3]For the 2010-2019 period, the year-over-year inflation in Mexico as measured by the consumer price index averaged 3.96 percent with a standard deviation of 1.02 percent. For comparison, the corresponding statistics in the United States were 1.75 percent and 0.85 percent, respectively, while the matching statistics for Canada were 1.74 percent and 0.65 percent, respectively.

for our analysis, it has well-functioning markets for both standard nominal fixed-coupon government bonds, so-called bonos, and real inflation-indexed government bonds, known as udibonos. Finally, we consider Mexico and its government bond market to be representative of the wider set of large emerging bond markets; for example, it typically makes up between 10 percent and 13 percent of most emerging market bond indices.

In terms of our empirical findings, we make a number of observations. First, we find that the model delivers estimates of investors' inflation expectations that are robust to a range of different model implementations.

Second, our results indicate that the average liquidity premia embedded in both nominal and real Mexican bond yields exhibit notable time variation. For nominal yields, our benchmark sample covers the period from January 2007 through December 2019, and their estimated liquidity premia average 46 basis points with a standard deviation of 23 basis points. For real yields, our benchmark sample contains data from May 2009 through December 2019, and their estimated liquidity premia average 72 basis points with a standard deviation of 63 basis points. Thus, the liquidity premia of Mexican inflation-indexed government bonds are larger and more variable than those of standard Mexican nominal government bonds. These results are consistent with the findings of ACR, who report that the average liquidity premium in U.S. Treasury Inflation-Protected Securities (TIPS) is estimated at 34 basis points for the 1997-2013 period, which is well above measures of liquidity premia in regular U.S. Treasury bonds. The difference in liquidity premium levels across the U.S. TIPS and the Mexican udibonos markets is likely to be due to the much greater relative liquidity of U.S. Treasury securities. Importantly, the nominal and real bond liquidity risk premia we estimate are practically uncorrelated in levels and their monthly changes are only mildly positively correlated. These results suggest that inflation-indexed Mexican bonds are less liquid and less desirable than nominal Mexican bonds. However, a full judgment of the tradeoff between nominal and inflation-indexed debt on the part of the Mexican government requires an estimate of the inflation risk premium, which is the excess return investors demand to hold nominal bonds. Given that our estimated inflation risk premia average 91 basis points during the 2009-2019 period, our results imply that the Mexican government likely would benefit from increasing its issuance of udibonos.

Third, the model's decomposition of the liquidity-adjusted or frictionless BEI rates indicates that investors' long-term inflation expectations have been stable at a level close to the inflation target of the Bank of Mexico with some mild fluctuations. This finding implies that most of the variation in the liquidity-adjusted BEI rates is driven by fluctuations in the inflation risk premium, which has trended lower since 2009 and fallen on net slightly more than 200 basis points during our benchmark sample period. Furthermore, we compare our estimated inflation risk premium series to estimates from Canada and the United States and find them to be positively correlated and of fairly similar magnitude, although the estimated

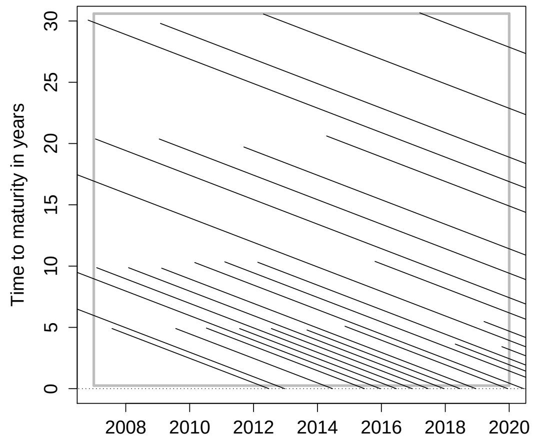
Mexican inflation risk premium is clearly larger and more volatile, as anticipated. Still, for extended periods, inflation risk in Mexico only commands a premium slightly above those seen in Canada and the United States, which is somewhat surprising. These findings support the claim that long-term inflation expectations in Mexico are well anchored, as also noted by De Pooter et al. (2014), and they underscore that the inflation target of the Bank of Mexico is viewed as being highly credible by financial market participants.[4]

As for the determinants of Mexican inflation risk premia, we perform regression analysis with a large battery of explanatory variables. The regressions have large explanatory power with adjusted R^2s of 0.80 and above. Furthermore, focusing on our preferred regression specification, we note that higher CPI inflation tends to depress the inflation risk premia. This somewhat counterintuitive result comes about because the positive effects of higher CPI inflation on the real term premia dominate its positive effects on the nominal term premia and hence squeezes both BEI and the inflation risk premia. Also, a higher debt-to-GDP ratio tends to put downward pressure on the Mexican inflation risk premium as it tends to depress nominal term premia less than real term premia, resulting in lower inflation risk premia. Lastly, increases in the ten-year U.S. Treasury yield tend to push up Mexican inflation risk premia, but only by about one-third of its nearly one-for-one impact on Mexican nominal and real term premia. Overall, we take these findings to be sensible in light of the results we obtain in supporting regressions using the model-implied nominal and real term premia as dependent variables.

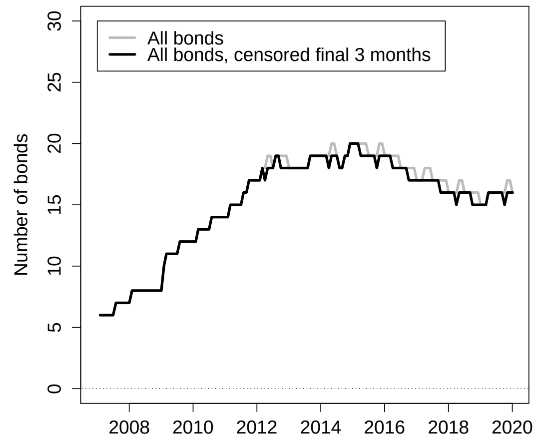
In a final exercise, we provide an update during the early stages of the coronavirus pandemic with data through December 2020, along with model-based projections of the outlook for long-term inflation expectations in Mexico for the following three-year period. The results suggest that bond investors' long-term inflation expectations in Mexico were temporarily depressed and fell below the 3 percent inflation target of the Bank of Mexico. However, the model projections indicate that they are likely to return to pre-pandemic levels slightly above 3 percent within less than three years, which we take as another sign that long-term inflation expectations are indeed well anchored in Mexico.

The remainder of the paper is structured as follows. Section 2 contains the data description. Section 3 provides evidence that parts of nominal and real yields in the Mexican government bond market are not spanned by either expected inflation or inflation risk premia and likely reflect their respective illiquidity premia. Section 4 details the model, the empirical results, and some sensitivity analysis. Section 5 analyzes the liquidity premia and identifies their determinants, while Section 6 contains a similar analysis of the estimated term premia. Section 7 describes the BEI decomposition and scrutinizes the estimated inflation risk premia, while Section 8 contains the updated analysis during the COVID-19 crisis. Section 9 concludes.

[4]The ability of the Bank of Mexico to affect asset prices, the exchange rate, and portfolio flows through its conduct of monetary policy is documented in Solís (2020).



(a) Distribution of bonos



(b) Number of bonos

Figure 1: **Overview of the Mexican Government Bonos Data**
Panel (a) shows the maturity distribution of the Mexican government fixed-coupon bonos considered in the paper. The solid gray rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on January 31, 2007, and limited to bonos prices with more than three months to maturity after issuance. Panel (b) reports the number of outstanding bonos at a given point in time.

2 Mexican Government Bond Data

This section first describes the Mexican government bond data we use in the model estimation before we proceed to a discussion of the credit risk, bond holdings, and bid-ask spreads in the markets for these bonds.

2.1 Bonos

The available universe of individual Mexican government fixed-coupon bonds, known as bonos, is illustrated in Figure 1(a). Each bond is represented by a solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. These bonds are all marketable non-callable bonds denominated in Mexican pesos that pay a fixed rate of interest semiannually. We note that we track the entire universe of bonos issued since January 2007. In addition, we include a set of bonos outstanding at the start of our sample period identical to those analyzed by Christensen et al. (2019, henceforth CFS). As a consequence, our sample entirely encompasses theirs. In general, the Mexican government has been issuing five-, ten-, twenty- and thirty-year bonos on a fairly regular basis during the period shown. As a result, there is a wide variety of bonds with different maturities and coupon rates in the data throughout our sample. It is this variation that provides the foundation for the econometric identification of the factors in the yield curve models we use.

Fixed-coupon bonos	No. obs.	Issuance Date	Issuance Amount	Number of auctions	Total notional amount
(1) 9% 12/20/2012†	68	1/9/2003	1,500	40	85,033
(2) 8% 12/7/2023+	156	10/30/2003	1,000	46	256,860
(3) 8% 12/17/2015†	104	1/5/2006	3,100	33	102,797
(4) 10% 11/20/2036×	156	10/26/2006	2,000	36	88,945
(5) 7.5% 6/3/2027+	156	1/18/2007	4,650	41	298,760
(6) 7.25% 12/15/2016†	116	2/1/2007	4,800	26	129,746
(7) 7.5% 6/21/2012*	56	7/26/2007	4,500	30	100,825
(8) 7.75% 12/14/2017†	116	1/31/2008	7,650	25	85,454
(9) 8.5% 5/31/2029+	132	1/15/2009	2,000	35	253,419
(10) 8.5% 11/18/2038×	132	1/29/2009	2,000	51	167,443
(11) 8.5% 12/13/2018†	115	2/12/2009	2,500	29	91,276
(12) 6.25% 6/19/2014*	56	7/23/2009	5,000	15	75,837
(13) 8% 6/11/2020†	119	2/25/2010	25,000	21	179,356
(14) 6% 6/18/2015*	54	7/8/2010	108,010	15	94,361
(15) 6.5% 6/10/2021†	107	2/3/2011	25,000	30	241,630
(16) 6.25% 6/16/2016*	56	7/22/2011	25,000	17	142,425
(17) 7.75% 5/29/2031+	100	9/9/2011	60,500	21	144,780
(18) 6.5% 6/9/2022†	95	2/15/2012	74,500	22	290,969
(19) 7.75% 11/13/2042×	93	4/20/2012	33,000	37	196,466
(20) 5% 6/15/2017*	56	7/19/2012	30,000	11	114,713
(21) 4.75% 6/14/2018*	55	8/30/2013	211,054	19	125,477
(22) 7.75% 11/23/2034+	69	4/11/2014	15,000	25	96,462
(23) 5% 12/11/2019*	58	11/7/2014	15,000	20	203,968
(24) 5.75% 3/5/2026†	51	10/16/2015	17,000	11	187,736
(25) 8% 11/7/2047×	34	3/10/2017	3,000	24	145,266
(26) 7.25% 12/9/2021	21	4/20/2018	25,000	18	236,936
(27) 8% 9/5/2024*	10	3/15/2019	9,700	10	191,289
(28) 6.75% 3/9/2023	3	10/4/2019	10,500	6	76,689

Table 1: **Sample of Mexican Government Bonos**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in millions of Mexican pesos for the available universe of Mexican government fixed-coupon bonos in the sample. Also reported are the number of monthly observation dates for each bond during the sample period from January 31, 2007, to December 31, 2019. Asterisk * indicates five-year bonds, dagger † indicates ten-year bonds, plus + indicates twenty-year bonds, and cross × indicates thirty-year bonds.

The contractual characteristics of all 28 bonos securities in our sample are reported in Table 1. The number of monthly observations for each bond using three-month censoring before maturity is also reported in the table.

Figure 1(b) shows the distribution across time of the number of bonds included in the sample. We note a gradual increase from six bonds at the start of the sample to 16 at its end. Combined with the cross sectional dispersion in the maturity dimension observed in Figure 1(a), this implies that we have a very well-balanced panel of Mexican bonos prices.

Figure 2 shows the time series of the yields to maturity implied by the observed Mexican

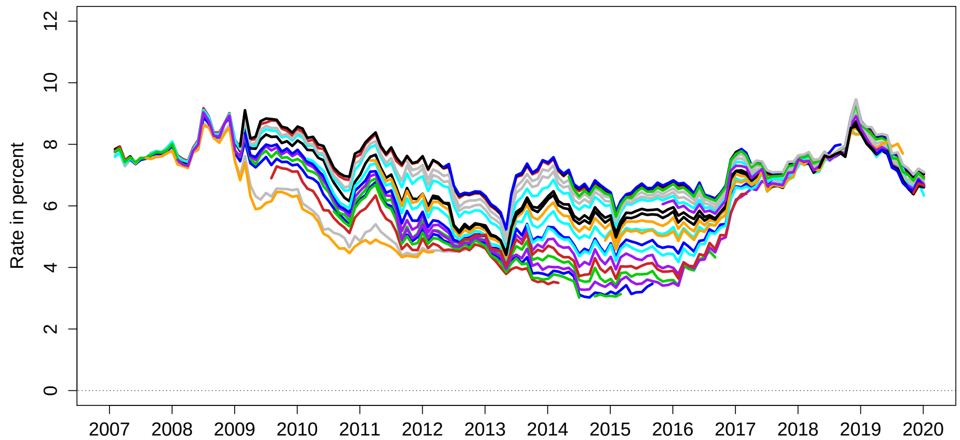


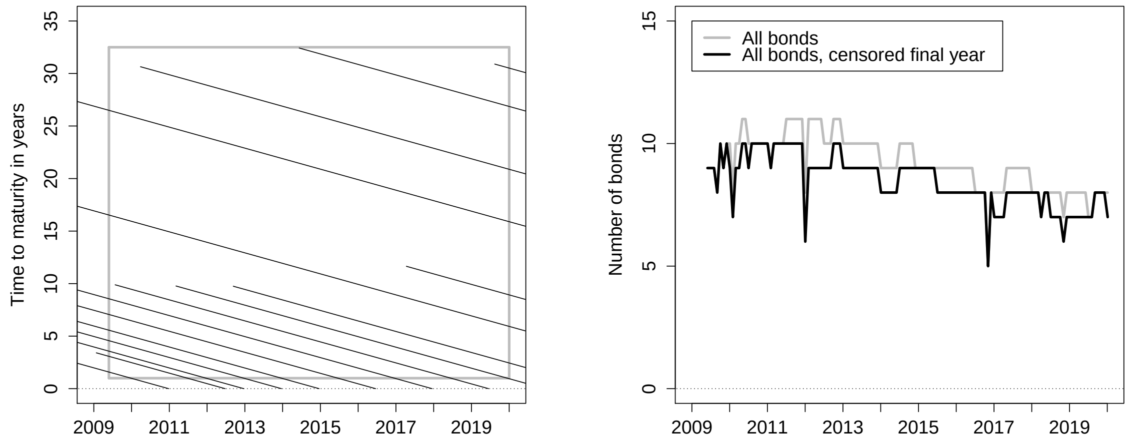
Figure 2: **Yield to Maturity of Mexican Government Bonos**
Illustration of the yields to maturity implied by the Mexican government fixed-coupon bonos prices downloaded from Bloomberg. The data is monthly covering the period from January 31, 2007, to December 31, 2019, and censors the last three months for each maturing bond.

bonos prices downloaded from Bloomberg. First, we note that the general yield level in Mexico has been fairly stable since 2007, unlike government bond yields in advanced economies, which have declined significantly during this period, see Holston et al. (2017) and Christensen and Rudebusch (2019), among others. Second, as in U.S. Treasury yield data, there is notable variation in the shape of the yield curve. At times like in mid-2018, yields across maturities are relatively compressed. At other times, the yield curve is steep with long-term bonos trading at yields that are 300-400 basis points above those of shorter-term securities like in 2015.

Finally, regarding the important question of a lower bound, the Bank of Mexico has never been forced to lower its conventional policy rate even close to zero, and the bond yields in the data have remained well above zero throughout the sample period. Thus, there is no need to account for any lower bounds to model these fixed-coupon bond prices, which motivates our focus on Gaussian models.

2.2 Udibonos

The Mexican government also issues inflation-indexed bonds, known as udibonos. These bonds pay semiannual interest based upon a real interest rate. Unlike standard fixed-coupon marketable bonds, interest payments on udibonos are adjusted for changes in the general price level. Technically, their payoff is measured in a unit called Unidad de Inversión (UDI), which is calculated and published daily by the Bank of Mexico. UDI changes with the biweekly release of the National Consumer Price Index, abbreviated INPC in Spanish, according to



(a) Distribution of udibonos

(b) Number of udibonos

Figure 3: **Overview of the Mexican Government Udibonos Data**
Panel (a) shows the maturity distribution of the Mexican government inflation-indexed udibonos considered in the paper. The solid gray rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on May 29, 2009, and limited to udibonos prices with more than one year to maturity after issuance. Panel (b) reports the number of outstanding bonos at a given point in time.

the procedure determined by the Bank of Mexico as originally laid out in Mexico's Federal Official Gazette on April 4, 1995. UDI represents the accumulated inflation in Mexico since April 1, 1995, denominated in Mexican pesos, and it is the factor used to convert the real return of udibonos into the corresponding value measured in current pesos at any given point in time.

The Mexican government launched its udibonos program in 1996. However, due to the quality of the available data from Bloomberg, we are limited to starting our sample in May 2009. The available universe of udibonos and their maturity distribution across time is shown in Figure 3(a). It includes the entire universe of udibonos issued since May 2009 combined with the outstanding stock of udibonos at the start of our sample. We note that the issuance is concentrated in ten-year udibonos with occasional issuance of twenty- and thirty-year udibonos.

The contractual details of each udibonos in our sample are reported in Table 2. It also contains the number of monthly observations for each bond in our sample with the last year before maturity censored to avoid erratic variation in their prices arising from seasonality in the inflation adjustment of their payoffs.

The total number of udibonos in our sample across time is shown in Figure 3(b). As with the nominal bonos, we stress that the sample of udibonos we use is very well-balanced across maturities at all times, which underpins the econometric identification of the state variables

Indexed udibonos	No. obs.	Issuance Date	Issuance amount	Number of auctions	Total notional amount
(1) 5.5% 12/20/2012†	116	1/2/2003	300	23	8,950
(2) 3.5% 12/19/2013†	116	1/15/2004	500	24	10,230
(3) 4.5% 12/18/2014†	116	1/13/2005	500	35	22,501
(4) 4.5% 12/4/2025^{+}	116	1/5/2006	1,700	65	42,793
(5) 4.5% 11/22/2035$^{\times}$	116	1/5/2006	21,533	63	25,266
(6) 5% 6/16/2016†	116	7/27/2006	600	41	30,467
(7) 3.25% 12/23/2010	35	10/4/2007	300	22	7,519
(8) 3.5% 12/14/2017†	116	1/10/2008	550	41	14,571
(9) 3.25% 6/21/2012	38	1/22/2009	450	27	7,211
(10) 4% 6/13/2019†	116	7/23/2009	450	43	29,674
(11) 4% 11/15/2040$^{\times}$	116	3/25/2010	3,500	55	46,632
(12) 2.5% 12/10/2020†	106	3/3/2011	550	32	26,475
(13) 2% 6/9/2022†	88	9/7/2012	12,118	39	47,054
(14) 4% 11/8/2046$^{\times}$	67	6/6/2014	3,000	58	42,598
(15) 4% 11/30/2028†	33	4/7/2017	3,000	32	36,633
(16) 4% 11/3/2050$^{\times}$	5	8/9/2019	600	8	9,216

Table 2: **Sample of Mexican Government Udibonos**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in millions of Mexican pesos for the available sample of Mexican government inflation-indexed udibonos. Also reported are the number of monthly observation dates for each bond during the sample period from May 29, 2009, to December 31, 2019. Dagger † indicates ten-year bonds, plus + indicates twenty-year bonds, and cross × indicates thirty-year bonds.

in the term structure models we use.

Figure 4 shows the yields to maturity implied by the udibonos prices downloaded from Bloomberg. Similar to what we observe for the nominal bonos yields, the yields of udibonos have fluctuated around a fairly stable level during the shown period, but with some variation in the steepness of the udibonos yield curve. Our models are intended to exploit this variation to deliver estimates of their liquidity premia, as explained in Section 4. Also, the greater dispersion in the udibonos yields across maturities in the early part of the sample is a tangible sign that bond-specific premia in this market are likely to play a particularly large role during this period.

2.3 The Credit Risk of Mexican Government Bonds

To gauge whether there are any material credit risk issues to consider in modeling Mexican government bond prices, we use rates on credit default swap (CDS) contracts. They reflect the annual rate investors are willing to pay to buy protection against default-related losses on these bonds over a fixed period of time stipulated in the contract. Such contracts have been used to price the credit risk of many countries, including Mexico, since the early 2000s.

In Figure 5, we plot the series for the one- and five-year Mexican CDS rate since 2007 with

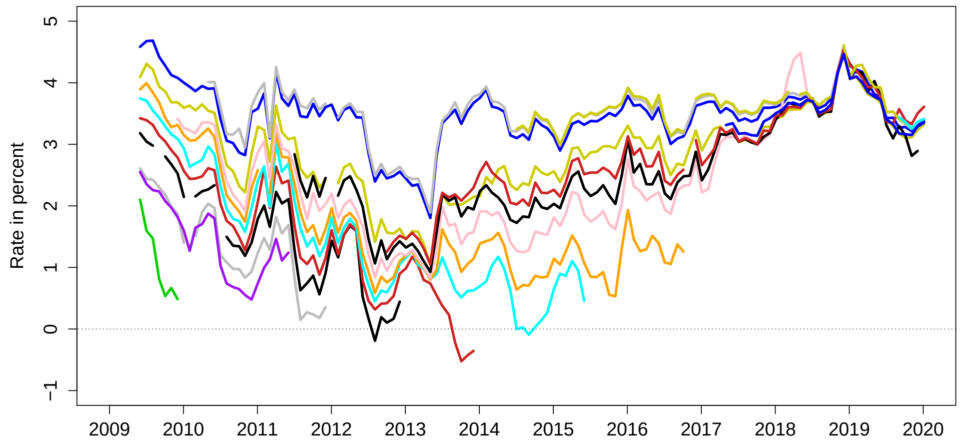


Figure 4: **Yield to Maturity of Mexican Government Udibonos**
Illustration of the yield to maturity implied by the Mexican government inflation-indexed udibonos prices considered in this paper, which are subject to two sample choices: (1) sample limited to the period from May 29, 2009, to December 31, 2019; (2) censoring of a bond's price when it has less than one year to maturity.

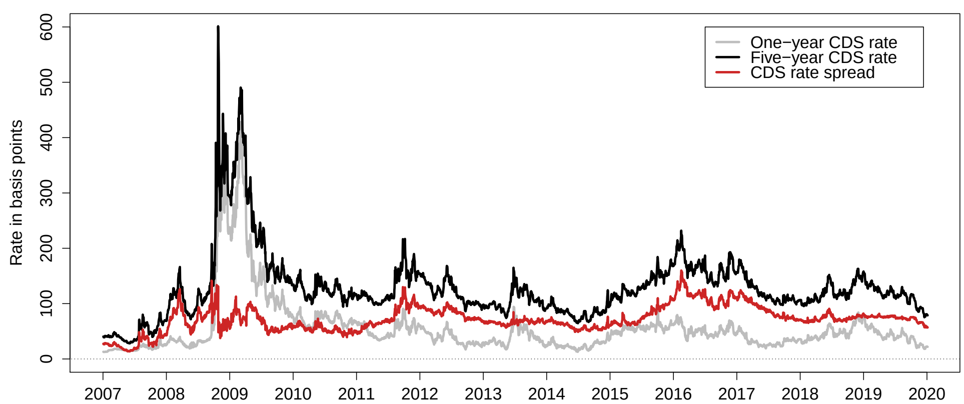


Figure 5: **Mexican CDS Rates**

solid gray and black lines, respectively. Also shown with a solid red line is the spread between these two CDS rates. We note that the five-year CDS rate has fluctuated in a fairly narrow range between 100 and 200 basis points, except for a brief period during the global financial crisis when Mexican CDS rates temporarily spiked above 300 basis points. This is a level of credit risk on par with most investment-grade firms in the United States, and its variation is mostly very gradual. This suggests that credit risk-related components are unlikely to be the driver of the results we present later on. To further support this view, we note that our measure of the Mexican government debt relative to GDP never goes above 52 percent, which is not a high value by international standards. Furthermore, the slope of the CDS rate curve measured as the difference between the five-year and one-year CDS rates is always positive and fairly stable and fluctuates in a narrow range, and it is almost uncorrelated (11%) with the one-year CDS rate. Thus, the steepness of the CDS rate curve for Mexican government debt has little connection to the near-term level of the priced credit risk of the Mexican government. We take this as a sign that the bulk of the variation in Mexican CDS rates reflect investor sentiment and risk aversion rather than actual credit risk.[5] Overall, we take this evidence to imply that credit risk is not likely to materially affect our results and we therefore feel comfortable not accounting for credit risk premia in our analysis.

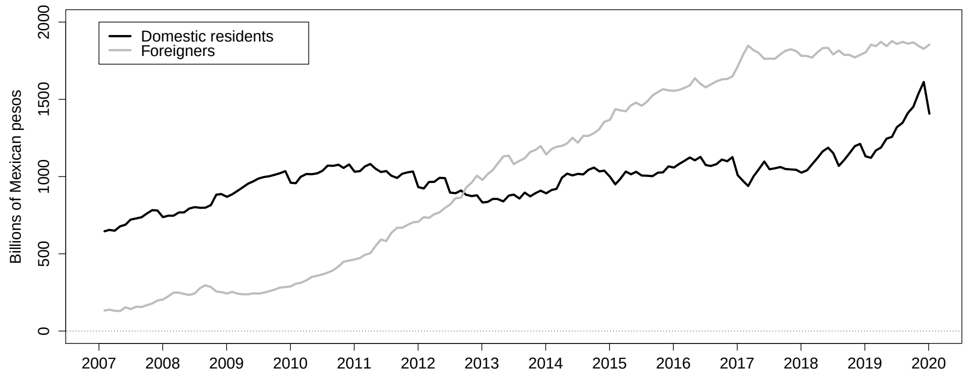
More importantly, on a practical note, there are no differences in the credit risk of bonos and udibonos in the sense that they will receive the same treatment in case the Mexican government stops servicing its debt. Thus, using arguments similar to those made by Fleckenstein et al. (2014) for U.S. Treasuries and TIPS, there is no reason to believe that there are any differentials in the pricing of bonos and udibonos tied to credit risk. By implication, our measures and decompositions of Mexican BEI are unaffected by variation in the credit risk premia of Mexican government debt.

2.4 Domestic and Foreign Mexican Government Bond Holdings

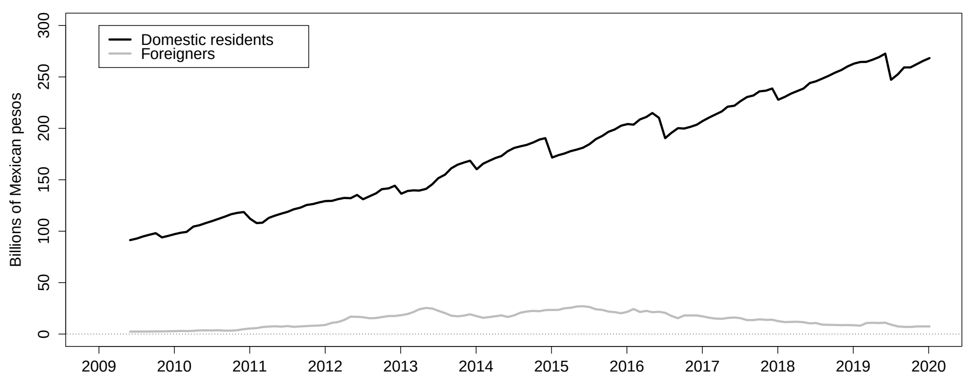
In addition to the bond price data described above, our regression analysis later on utilizes data on domestic and foreign holdings of Mexican government debt securities that the Bank of Mexico requires financial intermediaries to report as a way to track market activity in the Mexican sovereign bond markets. These data have been collected since 1978 and are available at a daily frequency up to the present. A key strength of the data set is that it covers any change in Mexican government debt holdings by either domestic or foreign investors. For each transaction, the reporting forms also identify the type of Mexican government security. Therefore, we are able to exploit the data reported for holdings of both bonos and udibonos. Although the data are available at a daily frequency, we use only the observations at the end of each month to align them with our bond price data.

Figure 6(a) shows the monthly level of bonos holdings by domestic residents and foreigners over the period from January 2007 through December 2019. We note that foreigners overtook domestic residents in total holdings by late 2012 and have continued to increase their share quite notably such that they now exceed those of domestic residents by a wide margin. In contrast, for the udibonos holdings shown in Figure 6(b) for the period from May 2009 through December 2019, we note that only a very small share of this market is held by foreigners. We note that the difference in the foreign participation across the two markets could lead to

[5] This is a phenomenon also seen in the pricing of corporate bonds and frequently referred to as the credit spread puzzle; see Christensen (2008) and references therein.



(a) Bonos



(b) Udibonos

Figure 6: **Holdings of Mexican Bonos and Udibonos**

significant differences in their trading dynamics and perceived liquidity risks.

To provide a sense of the relative size of the market for udibonos, we note that, as of December 31, 2019, the total outstanding amount of bonos was 3,261 billion pesos, or about USD160 billion, while the total amount of udibonos outstanding was 276 billion pesos, or about USD14 billion.[6] Hence, udibonos represent about 8.5 percent of the government's long-term debt denominated in Mexican pesos. Finally, we add that the total outstanding domestic debt of the Mexican federal government was 7,586 billion pesos, or almost USD380 billion, while its gross long-term foreign debt is reported as USD102 billion as of September 30, 2019.[7] Thus, the vast majority of the government's debt is issued in local currency, which underscores the importance of its domestic government bond markets analyzed here.

[6]See data from the Bank of Mexico at: https://www.banxico.org.mx/SieInternet/consultarDirectorioInternetAction.do?sector=7&accion=consultarDirectorioCuadros&locale=en

[7]See CEIC data at: https://www.ceicdata.com/en/mexico/gross-external-debt/gross-external-debt-federal-government-long-term

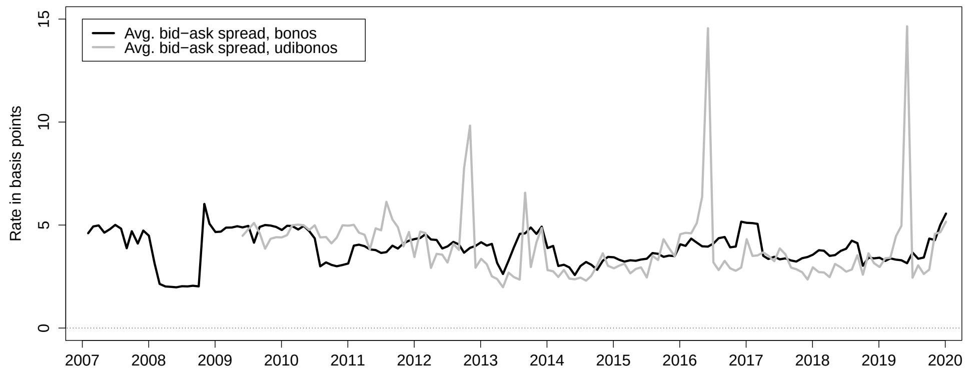


Figure 7: **Bid-Ask Spreads of Mexican Bonos and Udibonos**

2.5 Bid-Ask Spreads of Mexican Government Bonds

In this section, to shed light on the trading frictions in the markets for bonos and udibonos, we compare the average bid-ask spread of the udibonos in our sample to the average bid-ask spread of the bonos in our sample. These series are four-week moving averages and shown in Figure 7. Note that the two series tend to be close to each other. Thus, most of the time, there is no discernible difference in this measure of current liquidity across the two markets. Still, there are occasional large spikes in the average bid-ask spread of udibonos, which are driven by erratic pricing of individual udibonos as they approach maturity. This phenomenon is much less pronounced in the bonos market and further mitigated by the much larger number of bonos in our sample.

The key takeaway from this evidence is twofold. First, with a level of bid-ask spreads around 5 basis points, the trading of these securities is indeed associated with some amount of liquidity risk. Second, the occasional large spikes in the bid-ask spreads of udibonos would be another sign that the liquidity risk of these securities is somewhat greater than that of bonos.

3 Analysis of Nominal and Real Bond-Specific Factors

Motivated by the evidence in the previous section, our model assumes that both nominal and real bond prices contain liquidity premia that investors demand to assume their liquidity risk. In this section, we aim to build further support for that assumption. In doing so, we follow D'Amico et al. (2018, henceforth DKW), who note that, in a world without any financial market frictions, nominal yields y_t^N must be the sum of the matching real yield y_t^R, expected inflation π_t^e, and the inflation risk premium ϕ_t:

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau). \tag{1}$$

Furthermore, the frictionless real yield is the sum of the neutral or natural real rate r_t^* and a real term premium:

$$y_t^R(\tau) = r_t^* + TP_t^R(\tau).$$

In turn, the frictionless BEI that would prevail in a world without any financial frictions is then given by

$$BEI \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau). \tag{2}$$

Comparing equations (1) and (2) motivates DKW to regress BEI on the first three principal components (PC) of nominal yields, which normally explain more than 99 percent of the nominal yield variation. If there are no priced frictions or other deviations from the law of one price in the data, this could be expected to yield high R^2s, in particular if the frictionless real yields y_t^R have stationary dynamics (this is a big if, as we will explain below).

As for the observed nominal and real yields, denoted $\overline{y}_t^N$ and $\overline{y}_t^R$, respectively, they may each contain unobserved liquidity premia, denoted η_t^N and η_t^R, respectively. Hence, we have the following relationships to the frictionless yields discussed above:

$$\begin{aligned}\overline{y}_t^N(\tau) &= y_t^N(\tau) + \eta_t^N(\tau),\\ \overline{y}_t^R(\tau) &= y_t^R(\tau) + \eta_t^R(\tau).\end{aligned}$$

This implies that the observed nominal yield can be written as

$$\overline{y}_t^N(\tau) = r_t^* + TP_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau) + \eta_t^N(\tau),$$

while the observed BEI becomes

$$\overline{BEI}_t(\tau) \equiv \overline{y}_t^N(\tau) - \overline{y}_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau) + \eta_t^N(\tau) - \eta_t^R(\tau).$$

Returning to DKW's proposed regression, it is now clear that, provided $\eta_t^N(\tau) - \eta_t^R(\tau) \approx 0$ and r_t^* and $TP_t^R(\tau)$ are stationary, it will be the case that observed $\overline{BEI}$ regressed on the first three PCs of nominal yields should generate fairly large R^2s. On the other hand, if either of these two assumptions are not met, we are likely to see fairly low R^2, which then suggests that either (a) the frictionless real yields contain some trending component; or (b) nominal and/or real yields contain some sizable persistent priced frictions or liquidity premia that prevent the condition $\eta_t^N(\tau) - \eta_t^R(\tau) \approx 0$ from being satisfied.

DKW study U.S. Treasury and TIPS data, where it is reasonable to assume that, indeed, $\eta_t^N(\tau) \approx 0$. Also, they implicitly assume that there are no trends in TIPS yields, although

Coef.	BEI			
	$\tau = 2$	$\tau = 5$	$\tau = 7$	$\tau = 10$
α	0.01	0.01	0.00	0.00
	(0.01)	(0.00)	(0.00)	(0.00)
β_L	0.34**	0.40**	0.43**	0.46**
	(0.06)	(0.06)	(0.06)	(0.06)
β_S	0.13**	0.14**	0.13**	0.13**
	(0.03)	(0.02)	(0.02)	(0.02)
β_C	0.03	0.06**	0.07**	0.07**
	(0.02)	(0.02)	(0.02)	(0.02)
R^2	0.40	0.52	0.58	0.66

Table 3: **Breakeven Inflation Regressions**
The table reports the results of regressions with breakeven inflation as the dependent variable and the estimated level, slope, and curvature factors from an AFNS model of nominal bonos prices. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

that may be a questionable assumption given that evidence provided in Laubach and Willimas (2016) and Christensen and Rudebusch (2019), among many others, points to a long-term secular decline in the natural real rate in the United States. As a consequence, when they obtain really low R^2s in their regressions, DKW conclude that TIPS yields contain a significant liquidity premium η_t^R.

For our Mexican data, we first estimate arbitrage-free Nelson-Siegel (AFNS) models from Christensen et al. (2011) for the Mexican bonos and udibonos prices separately. This gives us fitted nominal and real yield curves at all relevant maturities, which are then used to calculate the corresponding fitted BEI rates. We then regress those fitted BEI rates on the three filtered state variables from the AFNS model estimation based on our sample of Mexican bonos prices, which serve as our equivalent of the first three principal components of nominal yields in the analysis of DKW:

$$\widehat{BEI}_t(\tau) = \alpha^\tau + \beta_L^\tau \widehat{L}_t + \beta_S^\tau \widehat{S}_t + \beta_C^\tau \widehat{C}_t + \varepsilon_t^\tau.$$

The results at four maturities from two to ten years are reported in Table 3. The level and slope factors in the AFNS model are highly statistically significant across all considered maturities and have very stable coefficients. As for the curvature factor, its loading is insignificant at shorter maturities, but highly statistically significant at medium- and long-term maturities. This is consistent with its hump-shaped loading structure across maturities.

Despite this very stable and significant pattern in the regression coefficients of the three factors, the obtained R^2 values fall in the range from 0.40 to 0.66 and decline as the maturity shortens. DKW interpret this kind of pattern as evidence of the existence of liquidity premia in the underlying bond yields. In our case, we think a similar interpretation applies. First, in

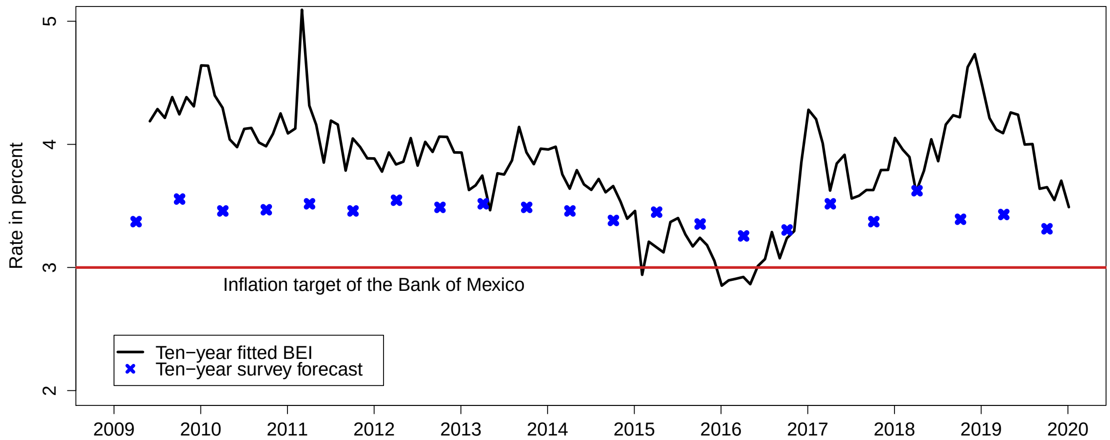


Figure 8: **Survey Inflation Forecasts and Fitted BEI**
Illustration of the ten-year fitted BEI obtained by fitting an AFNS model to Mexican bonos and udibonos prices separately. Also shown are the ten-year inflation forecasts from the semiannual Consensus Forecasts survey of professional forecasters tracking the Mexican economy.

our Mexican data, the udibonos real yields do not appear to have a trend. Hence, when we get R^2s ranging from 0.40 to 0.66, it is most likely caused by $\eta_t^N(\tau) - \eta_t^R(\tau) \approx 0$ not being satisfied. Second, the decline in R^2 as maturity is shortened is consistent with a liquidity premium interpretation given that yields at shorter maturities in our data primarily reflect the prices of seasoned bonds that have been outstanding for many years. As a consequence, shorter-term BEI rates are likely to be more biased by liquidity premia than the ten-year sector where a majority of the bond issuance has taken place historically. To summarize, building on the findings of DKW for the large U.S. TIPS market, where they find sizable liquidity premia, our regression results imply the existence of large and time-varying liquidity premia in the much smaller market for udibonos.

To provide evidence of the existence of important liquidity premia in Mexican bonos (beyond that provided in CFS), we again follow DKW. For U.S. data, they document that TIPS BEI tends to be *below* the inflation forecasts reported in surveys of both consumers and professional forecasters. Furthermore, as demonstrated by DKW, this is due to the existence of large positive liquidity premia in TIPS yields in combination with small and negligible liquidity premia in Treasury yields.

To repeat this exercise in our setting, Figure 8 compares the ten-year fitted BEI considered earlier with the ten-year CPI inflation forecasts that can be constructed from the long-term economic forecasts reported semiannually in the Consensus Forecasts surveys of professional forecasters tracking the Mexican economy.

In the Mexican data, we see the opposite pattern of DKW whereby BEI tends to be *above* the survey forecasts of inflation. This leaves the possibility that there could be large liquidity premia in nominal bonos yields that more than offset the negative effects from the liquidity premia in the udibonos prices. Alternatively, this could be a sign that there are large positive and time-varying inflation risk premia in bonos prices.

As in DKW, we explore this further by correlating the difference between the ten-year fitted BEI and the ten-year survey inflation forecasts with measures of the priced frictions in the bonos market. It turns out that the difference is weakly positively correlated with the average bonos bid-ask spread in our sample (17%) and with the mean absolute fitted errors of the bonos prices from the AFNS model estimation used in the construction of the fitted BEI (19%).[8] The small number of observations (21) prevents us from further substantiating this result. However, similar to DKW, we take this as weak evidence of the existence of liquidity premia in the bonos prices, even though we note that this variation could equally well reflect changes in inflation risk premia independent of the bonos and udibonos liquidity premia.[9]

Finally, we stress that it is the purpose of the remainder of the paper to quantify the relative magnitudes of these three different types of risk premia in the pricing of bonos and udibonos and what they imply about bond investors' underlying inflation expectations.

4 Model Estimation and Results

In this section, we first detail the model of the frictionless dynamics that would prevail in a world without any financial frictions. This model is fundamental to our empirical analysis. Second, we augment the frictionless model to adjust the observed yields for their liquidity bias before we proceed to a description of how BEI is decomposed within the model. We then detail the model estimation and its econometric identification. We end the section with a brief description of the main estimation results and related sensitivity analysis.

4.1 A Frictionless Arbitrage-Free Model of Nominal and Real Yields

To begin, we need an accurate model of the instantaneous nominal and real rate, r_t^N and r_t^R, in order to precisely measure nominal and real liquidity premia. With that goal in mind we choose to focus on the tractable affine dynamic term structure model of nominal and real yields briefly summarized below. We emphasize that, even though this model and others considered are not formulated using the canonical form of affine term structure models introduced by Dai and Singleton (2000), they can all be viewed as restricted versions of the corresponding canonical Gaussian models.

Our joint model of nominal and real yields was first used by Carriero et al. (2018) to analyze nominal and real U.K. gilt yields. In this model, the state vector is denoted by

[8]This is a noise measure of arbitrage capital frictions similar to the one developed in Hu et al. (2013).

[9]Hördahl and Tristani (2012) report a similar pattern for euro-area BEI rates and tie it to positive inflation risk premia.

$X_t = (L_t^N, S_t^N, C_t^N, L_t^R, S_t^R)$, where (L_t^N, S_t^N, C_t^N) represent level, slope, and curvature factors in the nominal yield curve,[10] while (L_t^R, S_t^R) represent separate level and slope factors in the real yield curve.[11] The instantaneous nominal and real risk-free rates are defined as

$$r_t^N = L_t^N + S_t^N, \tag{3}$$

$$r_t^R = L_t^R + S_t^R. \tag{4}$$

To preserve the Nelson and Siegel (1987) factor loading structure in the yield functions, the risk-neutral (or $\mathbb{Q}$-) dynamics of the state variables are given by the stochastic differential equations:

$$\begin{pmatrix} dL_t^N \\ dS_t^N \\ dC_t^N \\ dL_t^R \\ dS_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda^N & \lambda^N & 0 & 0 \\ 0 & 0 & -\lambda^N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda^R \end{pmatrix} \begin{pmatrix} L_t^N \\ S_t^N \\ C_t^N \\ L_t^R \\ S_t^R \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L^N,\mathbb{Q}} \\ dW_t^{S^N,\mathbb{Q}} \\ dW_t^{C^N,\mathbb{Q}} \\ dW_t^{L^R,\mathbb{Q}} \\ dW_t^{S^R,\mathbb{Q}} \end{pmatrix}, \tag{5}$$

where Σ is the constant covariance (or volatility) matrix.

Based on this specification of the $\mathbb{Q}$-dynamics, zero-coupon nominal bond yields preserve the Nelson and Siegel (1987) factor loading structure as

$$y_t^N(\tau) = L_t^N + \left(\frac{1-e^{-\lambda^N\tau}}{\lambda^N\tau}\right) S_t^N + \left(\frac{1-e^{-\lambda^N\tau}}{\lambda^N\tau} - e^{-\lambda^N\tau}\right) C_t^N - \frac{A^N(\tau)}{\tau}, \tag{6}$$

where the nominal yield-adjustment term is given by

$$\begin{aligned} \frac{A^N(\tau)}{\tau} &= \frac{\sigma_{11}^2}{6}\tau^2 + \sigma_{22}^2\Big[\frac{1}{2(\lambda^N)^2} - \frac{1}{(\lambda^N)^3}\frac{1-e^{-\lambda^N\tau}}{\tau} + \frac{1}{4(\lambda^N)^3}\frac{1-e^{-2\lambda^N\tau}}{\tau}\Big] \\ &\quad + \sigma_{33}^2\Big[\frac{1}{2(\lambda^N)^2} + \frac{1}{(\lambda^N)^2}e^{-\lambda^N\tau} - \frac{1}{4\lambda^N}\tau e^{-2\lambda^N\tau} - \frac{3}{4(\lambda^N)^2}e^{-2\lambda^N\tau} \\ &\qquad + \frac{5}{8(\lambda^N)^3}\frac{1-e^{-2\lambda^N\tau}}{\tau} - \frac{2}{(\lambda^N)^3}\frac{1-e^{-\lambda^N\tau}}{\tau}\Big], \end{aligned}$$

while zero-coupon real bond yields preserve a simplified Nelson and Siegel (1987) factor loading structure given by

$$y_t^R(\tau) = L_t^R + \left(\frac{1-e^{-\lambda^R\tau}}{\lambda^R\tau}\right) S_t^R - \frac{A^R(\tau)}{\tau}, \tag{7}$$

[10] To motivate this choice, we note that Espada et al. (2008) show that the first three principal components in their sample of Mexican government bond yields have a level, slope, and curvature pattern in the style of Nelson and Siegel (1987) and account for more than 99 percent of the yield variation.

[11] Chernov and Mueller (2012) provide evidence of a hidden factor in the nominal yield curve that is observable from real yields and inflation expectations. Our model accommodates this stylized fact via the (L_t^R, S_t^R) factors.

where the real yield-adjustment term is given by

$$\frac{A^R(\tau)}{\tau} = \frac{\sigma_{44}^2}{6}\tau^2 + \sigma_{55}^2\Big[\frac{1}{2(\lambda^R)^2} - \frac{1}{(\lambda^R)^3}\frac{1-e^{-\lambda^R\tau}}{\tau} + \frac{1}{4(\lambda^R)^3}\frac{1-e^{-2\lambda^R\tau}}{\tau}\Big].$$

To complete the description of the model and implement it empirically, we continue to use the essentially affine risk premium specification

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^5$ and $\gamma^1 \in \mathbf{R}^{5\times 5}$ contain unrestricted parameters.

Thus, the resulting unrestricted five-factor joint model of nominal and real yields has $\mathbb{P}$-dynamics given by

$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t) + \Sigma dW_t^{\mathbb{P}},$$

where $K^{\mathbb{P}}$ is an unrestricted 5×5 mean-reversion matrix, $\theta^{\mathbb{P}}$ is a 5×1 vector of mean levels, and Σ is a 5×5 lower triangular volatility matrix.

This is the transition equation in the Kalman filter estimation. Going forward, we refer to this joint five-factor Gaussian model of nominal and real yields as the $G(5)$ model.

4.2 An Arbitrage-Free Model of Nominal and Real Yields with Liquidity Risk

In this section, we augment the frictionless model of nominal and real yields described in the previous section with two liquidity risk factors, (X_t^N, X_t^R), to account for the liquidity risk in the pricing of nominal and real bonds, respectively. In this case, we extend the terminology of ACR and refer to this seven-factor Gaussian model as the $G^{X^N,X^R}(7)$ model.

To begin, let $X_t = (L_t^N, S_t^N, C_t^N, X_t^N, L_t^R, S_t^R, X_t^R)$ denote the state vector of this model. As before, (L_t^N, S_t^N, C_t^N) represent level, slope, and curvature factors in the nominal yield curve, while (L_t^R, S_t^R) represent separate level and slope factors in the real yield curve. Finally, (X_t^N, X_t^R) represent the added nominal and real liquidity risk factors.

As in the $G(5)$ model, we let the frictionless instantaneous nominal and real risk-free rates be defined by equations (3) and (4), respectively, while the risk-neutral dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t^N \\ dS_t^N \\ dC_t^N \\ dX_t^N \\ dL_t^R \\ dS_t^R \\ dX_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda^N & -\lambda^N & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda^N & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa_N^Q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda^R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_R^Q \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_N^Q \\ 0 \\ 0 \\ \theta_R^Q \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t^N \\ C_t^N \\ X_t^N \\ L_t^R \\ S_t^R \\ X_t^R \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,\mathbb{Q}} \\ dW_t^{S^N,\mathbb{Q}} \\ dW_t^{C^N,\mathbb{Q}} \\ dW_t^{X^N,\mathbb{Q}} \\ dW_t^{L^R,\mathbb{Q}} \\ dW_t^{S^R,\mathbb{Q}} \\ dW_t^{X^R,\mathbb{Q}} \end{pmatrix},$$

where Σ continues to be a diagonal matrix. This structure implies that X_t^N and X_t^R are assumed to be independent Ornstein-Uhlenbeck processes under the pricing measure with means $\theta_N^{\mathbb{Q}}$ and $\theta_R^{\mathbb{Q}}$ and mean-reversion rates $\kappa_N^{\mathbb{Q}}$ and $\kappa_R^{\mathbb{Q}}$.

Due to the liquidity risk in the markets for nominal and real bonds, their yields are sensitive to liquidity pressures. As a consequence, the pricing of nominal and real bonds is not performed with the frictionless discount functions in equations (6) and (7), but rather with discount functions that account for the liquidity risk:

$$\overline{r}_t^{N,i} = r_t^N + \beta^{N,i}(1 - e^{-\delta^{N,i}(t-t_0^i)})X_t^N = L_t^N + S_t^N + \beta^{N,i}(1 - e^{-\delta^{N,i}(t-t_0^i)})X_t^N, \qquad (8)$$

$$\overline{r}_t^{R,j} = r_t^R + \beta^{R,j}(1 - e^{-\delta^{R,j}(t-t_0^j)})X_t^R = L_t^R + S_t^R + \beta^{R,j}(1 - e^{-\delta^{R,j}(t-t_0^j)})X_t^R, \qquad (9)$$

where t_0^i and t_0^j denote the dates of issuance of the specific nominal and real bonds, respectively, and $\beta^{N,i}$ and $\beta^{R,j}$ are their sensitivities to the variation in their respective liquidity risk factors. Furthermore, the decay parameters $\delta^{N,i}$ and $\delta^{R,j}$ are assumed to vary across securities.

Christensen and Rudebusch (2019) show that the net present value of one unit of currency paid by nominal bond i at time $t + \tau^i$ has the following exponential-affine form

$$\begin{aligned} P_t^N(t_0^i, \tau^i) &= E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i} \overline{r}^{N,i}(s,t_0^i)ds}\Big] \\ &= \exp\Big(B_1^N(\tau^i)L_t^N + B_2^N(\tau^i)S_t^N + B_3^N(\tau^i)C_t^N + B_4^N(t, t_0^i, \tau^i)X_t^N + A(t, t_0^i, \tau^i)\Big). \end{aligned}$$

By similar arguments, the net present value of one unit of the consumption basket paid by real bond j at time $t + \tau^j$ has the following exponential-affine form

$$\begin{aligned} P_t^R(t_0^j, \tau^j) &= E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^j} \overline{r}^{R,j}(s,t_0^j)ds}\Big] \\ &= \exp\Big(B_1^R(\tau^j)L_t^R + B_2^R(\tau^j)S_t^R + B_3^R(t, t_0^j, \tau^j)X_t^R + A(t, t_0^j, \tau^j)\Big). \end{aligned}$$

These formulas imply that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing $\beta^{N,i} = 0$ for all i and $\beta^{R,j} = 0$ for all j, we recover the $G(5)$ model.

Now, consider the whole value of the nominal bond i issued at time t_0^i with maturity at $t + \tau^i$ that pays an annual coupon C^i semiannually. Its price is given by[12]

$$\begin{aligned} \overline{P}_t^{N,i}(t_0^i, \tau^i, C^i) &= C^i(t_1 - t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1} \overline{r}^{N,i}(s,t_0^i)ds}\Big] + \sum_{k=2}^{n} \frac{C^i}{2} E^{\mathbb{Q}}\Big[e^{-\int_t^{t_k} \overline{r}^{N,i}(s,t_0^i)ds}\Big] \\ &\quad + E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i} \overline{r}^{N,i}(s,t_0^i)ds}\Big]. \end{aligned}$$

[12]This is the clean price that does not account for any accrued interest and maps to our observed bond prices.

Next, consider the whole value of the real bond j issued at time t_0^j with maturity at $t+\tau^j$ that pays an annual coupon C^j semiannually. Its clean price is given by[13]

$$\begin{aligned}
\overline{P}_t^{R,j}(t_0^j,\tau^j,C^j) &= C^j(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^{R,j}(s,t_0^j)ds}\Big]+\sum_{k=2}^{n}\frac{C^j}{2}E^{\mathbb{Q}}\Big[e^{-\int_t^{t_k}\overline{r}^{R,j}(s,t_0^j)ds}\Big] \\
&\quad +E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^j}\overline{r}^{R,j}(s,t_0^j)ds}\Big].
\end{aligned}$$

The only minor omission in the bond price formula above is that we do not account for the lag in the inflation indexation of the real bond payoff, but the potential error should be modest in most cases; see Grishchenko and Huang (2013) and DKW for evidence in the case of the U.S. TIPS market.

To complete the model description, we need to specify the risk premia that connect the factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the objective $\mathbb{P}$-measure, where we continue to use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^7$ and $\gamma^1 \in \mathbf{R}^{7\times 7}$ contain unrestricted parameters. Thus, the resulting unrestricted $G^{X^N,X^R}(7)$ model has $\mathbb{P}$-dynamics given by

$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t) + \Sigma dW_t^{\mathbb{P}},$$

where $K^{\mathbb{P}}$ is an unrestricted 7×7 mean-reversion matrix, $\theta^{\mathbb{P}}$ is a 7×1 vector of mean levels, and Σ is a 7×7 lower triangular volatility matrix. This is the transition equation in the extended Kalman filter estimation of this model.

4.3 Decomposing Bond Yields

Christensen et al. (2010) show that the price of a nominal zero-coupon bond with maturity in τ years can be written as

$$P_t^N(\tau) = P_t^R(\tau)\times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]\times\left(1+\frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R},\frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right]\times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right),$$

where $P_t^R(\tau)$ is the price of a real zero-coupon bond that pays one consumption unit in τ years, M_t^R is the real stochastic discount factor, and Π_t is the price level.

[13]Unlike U.S. TIPS, Mexican udibonos have no embedded deflation protection option, which makes their pricing straightforward.

By taking logs, this can be converted into

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where $y_t^N(\tau)$ and $y_t^R(\tau)$ are nominal and real zero-coupon yields as described in the previous section, while the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right] \tag{10}$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation demonstrates that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation and is ultimately determined by investor preferences, as in, for example, Rudebusch and Swanson (2011).

Now, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

that is, the difference between nominal and real yields of the same maturity. Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

Finally, we define the nominal and real term premia as

$$TP_t^j(\tau) = y_t^j(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^j]ds, \quad j = N, R.$$

That is, the nominal term premium is the difference in expected nominal return between a buy and hold strategy for a τ-year nominal bond and an instantaneous rollover strategy at the risk-free nominal rate r_t^N. The interpretation for the real term premium is similar. The model thus allows us to decompose nominal and real yields into their respective term premia and short-rate expectations components.

4.4 Model Estimation and Econometric Identification

Due to the nonlinearity of the bond pricing formulas, the models cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012); see Christensen and Rudebusch (2019) for details. To make the fitted errors comparable across bonds of various maturities, we follow ACR and scale each bond price by its duration. Thus, the measurement equation for the nominal bond prices takes the following form:

$$\frac{\overline{P}_t^N(\tau^i)}{D_t^N(\tau^i)} = \frac{\widehat{P}_t^N(\tau^i)}{D_t^N(\tau^i)} + \varepsilon_t^{N,i},$$

where $\widehat{P}_t^N(\tau^i)$ is the model-implied price of nominal bond i and $D_t^N(\tau^i)$ is its duration, which is fixed and calculated before estimation. Similarly, the measurement equation for the real bond prices takes the following form:

$$\frac{\overline{P}_t^R(t_0^j, \tau^j)}{D_t^R(\tau^j)} = \frac{\widehat{P}_t^R(t_0^j, \tau^j)}{D_t^R(\tau^j)} + \varepsilon_t^{R,j},$$

where $\widehat{P}_t^R(\tau^j)$ is the model-implied price of real bond j and $D_t^R(\tau^j)$ is its duration, which is again fixed and calculated before estimation. See Andreasen et al. (2019) for evidence supporting this formulation of the measurement equations.

Since the liquidity factors are latent factors that we do not observe, their levels are not identified without additional restrictions. As a consequence, when we include the nominal liquidity factor X_t^N, we let the first thirty-year bonos issued after the start of our sample window have a unit loading on the liquidity factor, that is, bonos number (10) in our sample issued on January 29, 2009, with maturity on November 18, 2038, and a coupon rate of 8.5 percent has $\beta^{N,i} = 1$. When we include the real liquidity factor X_t^R, we let the first thirty-year udibonos in our sample have a unit loading on this factor, that is, udibonos number (5) issued on January 5, 2006, with maturity on November 22, 2035, and a coupon rate of 4.5 percent has $\beta^{R,j} = 1$.

Furthermore, we note that the liquidity decay parameters, $\delta^{N,i}$ and $\delta^{R,j}$, can be hard to identify if their values are too large or too small. As a consequence, we impose the restriction that they fall within the range from 0.0001 to 10, which is without practical consequences based on the evidence presented in ACR and Christensen and Rudebusch (2019). Also, for numerical stability during the model optimization, we impose the restrictions that the liquidity sensitivity parameters, $\beta^{N,i}$ and $\beta^{R,j}$, fall within the range from 0 to 250, which turns out not to be a binding constraint at the optimum.

In addition, we assume that all nominal bond price measurement equations have *i.i.d.* fitted errors with zero mean and standard deviation σ_ε^N. Similarly, all real bond price measurement equations have fitted errors that are assumed to be *i.i.d.* with zero mean and standard deviation σ_ε^R.

Finally, we assume that the state variables are stationary and therefore start the Kalman filter at the unconditional mean and covariance matrix. This assumption is supported by the analysis in Chiquiar et al. (2010), who find that Mexican inflation seems to have become stationary at some point in the early 2000s, while De Pooter et al. (2014) document that measures of long-term inflation expectations from both surveys and the Mexican government bond market have remained anchored close to the 3 percent inflation target of the Bank of Mexico at least since 2003. Assuming real rates and bond risk premia are stationary,[14] this evidence would imply that Mexican government bond yields should be stationary as well, as also suggested by visual inspection of the individual yield series depicted in Figures 2 and 4.

We also incorporate long-term forecasts of inflation from surveys of professional forecasters in our model estimation. These are the projected ten-year CPI inflation forecasts that can be constructed semiannually from the Consensus Forecasts survey for Latin America and shown in Figure 8.[15] As demonstrated by Kim and Orphanides (2012), the inclusion of long-term survey forecasts can help the model better capture the appropriate persistence of the factors under the objective $\mathbb{P}$-dynamics, which can otherwise suffer from significant finite-sample bias.[16]

The measurement equation for the survey expectations incorporating these long-term forecasts takes the form

$$\pi_t^{CF}(10) = \pi_t^e(10) + \varepsilon_t^{CF},$$

where $\pi_t^e(10)$ is the model-implied ten-year expected inflation calculated using equation (10), which is affine in the state variables, while the measurement error is $\varepsilon_t^{CF} \sim \mathcal{NID}\left(0, (\sigma_\varepsilon^{CF})^2\right)$. As for the value σ_ε^{CF}, we follow DKW and fix it at 75 basis points in order to not overly influence the estimation results by including the survey forecasts. Alternatively, this approach can be interpreted as treating the survey forecasts as relatively noisy and infrequent measures of bond investors' inflation expectations. We perform a comprehensive sensitivity analysis to assess the impact of this assumption on our results in a later section.

4.5 Results

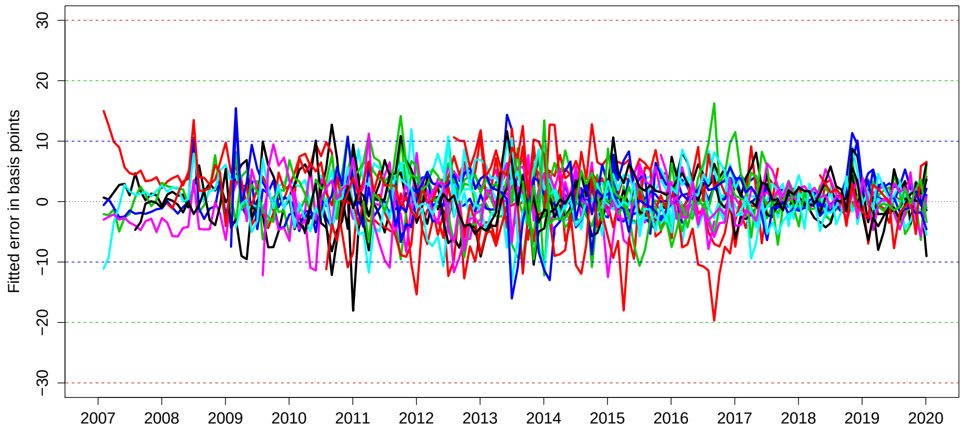
In this section, we briefly summarize the estimation results.

To examine the model fit, pricing errors are computed based on the implied yield on each coupon bond to make these errors comparable across securities. That is, for the price on the ith coupon bond $P_t^i(\tau, C^i)$, we find the value of $y_t^{i,c}$ that solves

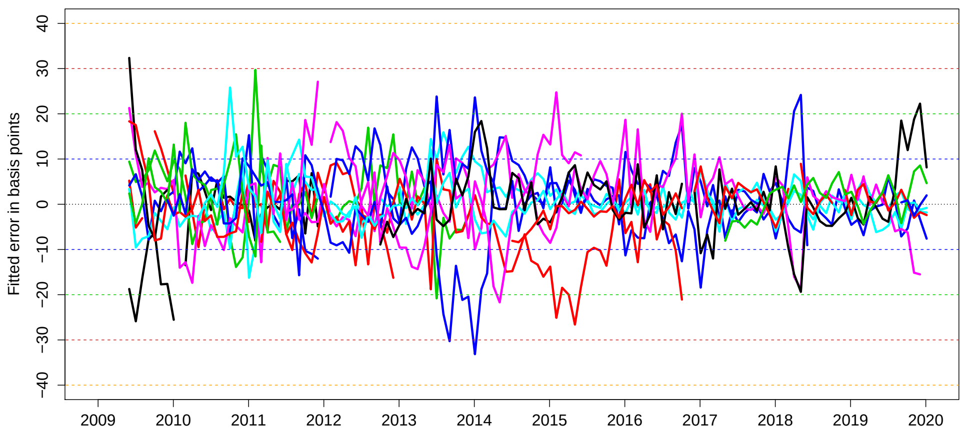
[14] We note that these might be strong assumptions. In the United States, there is evidence of a persistent downward trend in real yields the past two decades; see Christensen and Rudebusch (2019).

[15] Similar to Christensen et al. (2010) and Abrahams et al. (2016), we do not include inflation data in the model estimation. This omission is expected to, at most, have a small impact on our results due to the relatively long maturities of most of our real yield observations, see DKW for evidence.

[16] Also, see Bauer et al. (2012).



(a) Bonos



(b) Udibonos

Figure 9: **Fitted Errors of Mexican Bonos and Udibonos**

$$P_t^i(\tau^i, C^i) = C^i(t_1 - t)\exp\left\{-y_t^{i,c}(t_1 - t)\right\} + \sum_{j=2}^{N}\frac{C^i}{2}\exp\left\{-y_t^{i,c}(t_j - t)\right\} + \exp\left\{-y_t^{i,c}(t_N - t)\right\}. \quad (11)$$

For the model-implied estimate of this bond price, denoted $\hat{P}_t^i(\tau, C^i)$, we find the corresponding implied yield $\hat{y}_t^{i,c}$ and report the pricing error as $y_t^{i,c} - \hat{y}_t^{i,c}$.

Figure 9 shows the fitted error series for each bond price calculated this way. The top panel shows the results for the 28 bonos in our sample, while the bottom panel shows the results for the 16 udibonos in the sample. For the nominal bonos the root mean-squared error (RMSE) for all bonds combined is 4.16 basis points, while the corresponding statistics for the real udibonos is 7.47 basis points. Thus, the $G^{X^N,X^R}(7)$ model provides a very good fit to

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$K^{\mathbb{P}}_{\cdot,5}$	$K^{\mathbb{P}}_{\cdot,6}$	$K^{\mathbb{P}}_{\cdot,7}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	5.9163	2.5581	1.6649	0.4368	-7.5017	-2.1747	-1.2846	0.0984	σ_{11}	0.0122
	(0.8806)	(0.5646)	(0.4851)	(0.5088)	(1.0715)	(0.6215)	(0.4967)	(0.0307)		(0.0007)
$K^{\mathbb{P}}_{2,\cdot}$	8.8549	3.6741	3.0031	0.1987	-12.5581	-2.8339	-1.7168	-0.0105	σ_{22}	0.0191
	(1.0284)	(0.8020)	(0.6096)	(0.6848)	(1.1683)	(0.8280)	(0.6926)	(0.0463)		(0.0032)
$K^{\mathbb{P}}_{3,\cdot}$	-0.0261	1.2171	-0.7501	0.6680	4.5812	-0.9520	-0.4987	-0.0580	σ_{33}	0.0186
	(1.0325)	(0.7511)	(0.4944)	(0.5347)	(1.1659)	(0.7956)	(0.4988)	(0.0717)		(0.0040)
$K^{\mathbb{P}}_{4,\cdot}$	0.5218	3.2414	-1.6310	2.3381	4.2963	-2.9285	-1.3533	-0.0662	σ_{44}	0.0358
	(1.1766)	(0.9603)	(0.8948)	(0.9147)	(1.1990)	(0.9770)	(0.6977)	(0.0965)		(0.0104)
$K^{\mathbb{P}}_{5,\cdot}$	-2.8076	-0.7808	-1.5647	0.3684	5.2189	0.5198	0.3538	0.0271	σ_{55}	0.0091
	(0.8301)	(0.5530)	(0.4007)	(0.3506)	(1.0083)	(0.6562)	(0.3858)	(0.0122)		(0.0011)
$K^{\mathbb{P}}_{6,\cdot}$	-3.6630	-3.4080	-1.1220	-0.4870	6.6708	3.8295	2.1684	-0.1186	σ_{66}	0.0184
	(1.0454)	(0.8016)	(0.7337)	(0.5638)	(1.1632)	(0.8468)	(0.6715)	(0.2063)		(0.0039)
$K^{\mathbb{P}}_{7,\cdot}$	4.4426	0.3681	2.7566	-0.6943	-8.3530	0.1017	0.0086	0.2120	σ_{77}	0.0174
	(1.0370)	(0.8088)	(0.6756)	(0.6887)	(1.2174)	(0.8612)	(0.5777)	(0.3872)		(0.0090)

Table 4: **Estimated Dynamic Parameters of the $G^{X^N,X^R}(7)$ Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the $G^{X^N,X^R}(7)$. The estimated value of λ^N is 0.2675 (0.0139), while $\lambda^R = 0.4690$ (0.0439), $\kappa^{\mathbb{Q}}_N =$ 2.1291 (0.3350, $\theta^{\mathbb{Q}}_N = 0.0102$ (0.0015), $\kappa^{\mathbb{Q}}_R = 0.3782$ (0.0847), and $\theta^{\mathbb{Q}}_R = 0.0178$ (0.0037). The maximum log likelihood value is 19,121.94. The numbers in parentheses are the estimated parameter standard deviations.

both sets of bond prices.

Finally, as for the survey inflation forecasts, the mean error is 0.83 basis point, while the RMSE is 39.22 basis points, well below the 75 basis points assumed in the model estimation. Thus, the model is also able to simultaneously deliver an accurate fit to the survey forecasts.

The estimated dynamic parameters in the $G^{X^N,X^R}(7)$ model are reported in Table 4. We note that the estimated mean and volatility parameters for the four nominal factors $(L^N_t, S^N_t, C^N_t, X^N_t)$ are very similar to those reported by CFS for their shorter and smaller sample of 21 bonos price series. Thus, the nominal side of our joint model of bonos and udibonos prices fits the bonos data in much the same way as their nominal model.

4.6 Sensitivity Analysis

In this section, we analyze the sensitivity of our estimation results to two key assumptions in the model estimation. In the first set of robustness exercises, we change the assumption about the standard deviation of the measurement errors for the model-implied inflation forecasts, while in the other set of exercises we vary the horizon of the survey inflation forecasts used in the model estimation. To map the exercise to our later analysis, we focus on long-term inflation expectations covering a five-year period starting five years ahead, labeled 5yr5yr expected inflation.

Figure 10 shows the 5yr5yr model-implied expected inflation from changing the assumption made about the standard deviation of the measurement error of the ten-year survey inflation forecasts used in our benchmark model implementation. We vary this value from

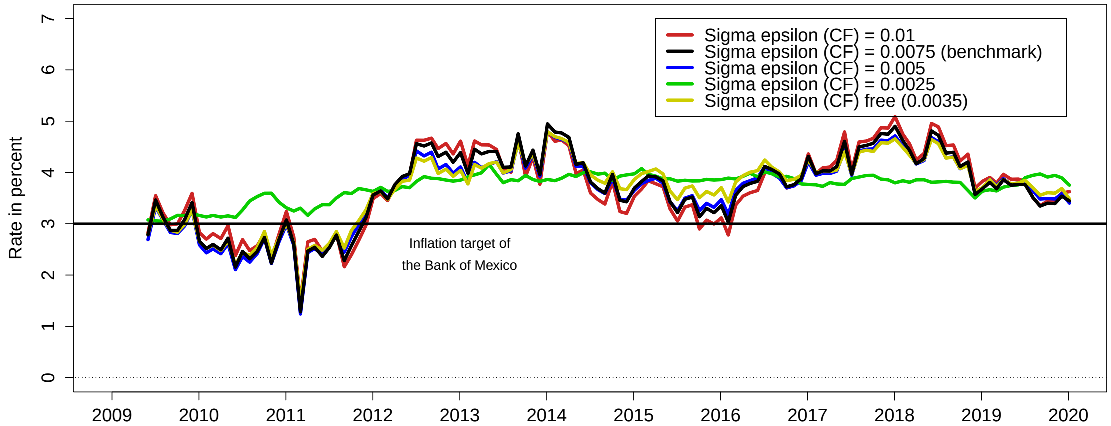


Figure 10: **Sensitivity of 5yr5yr Model-Implied Expected Inflation to $\sigma_{\varepsilon}^{CF}$**
Illustration of the 5yr5yr expected inflation implied by the $G^{X^N,X^R}(7)$ model estimated with five different assumptions about standard deviation of the measurement error of the ten-year survey inflation forecasts: (1) fixed 0.01; (2) fixed 0.0075, which is our benchmark model implementation; (3) fixed 0.005; (4) fixed 0.0025; (5) leave $\sigma_{\varepsilon}^{CF}$ as a free parameter to be determined by the model estimation, which produces a value of 0.0035. The shown data cover the period from May 31, 2009, to December 30, 2019.

0.0025 to 0.01 in 0.0025 increments, with 0.0075 being our benchmark as recommended by DKW. In addition, in a separate estimation, we leave it as a free parameter to be determined by the data, which yields a value of 0.0035. As can be seen from the figure, the model-implied 5yr5yr inflation expectations are practically insensitive to this choice unless the measurement error standard deviation is forced to be artificially low, in which case the model-implied long-term inflation expectations become almost as smooth as those reported in the surveys. Based on these observations we choose to proceed with our benchmark fixed value of 0.0075 for standard deviation of the survey forecast measurement errors throughout the remainder of the paper.

Figure 11 shows the 5yr5yr model-implied expected inflation from four different estimations. One is our benchmark estimation using the ten-year inflation survey forecasts, while the three other results use monthly data on inflation forecasts for the following full calendar, semiannual data on five-year inflation forecasts, and semiannual data on 5yr5yr inflation forecasts in the model estimation, respectively. We note that using shorter-term inflation forecasts in the model estimation gives rise to higher and less volatile model-implied inflation expectations, including long-term 5yr5yr inflation forecasts, while using 5yr5yr inflation forecasts produces model-implied inflation expectations that are very stable. Overall, we consider our benchmark implementation using ten-year survey inflation forecasts to be representative given that its estimate consistently falls in the range of estimates produced by these four

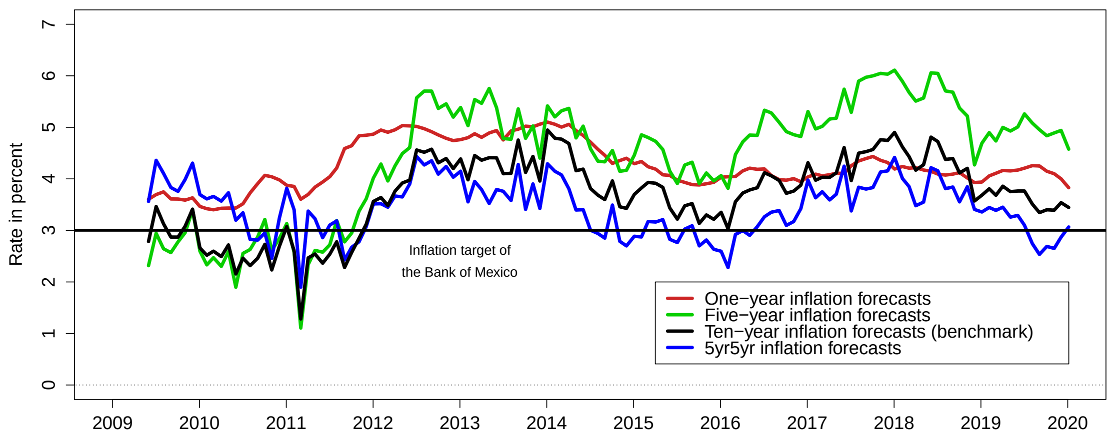


Figure 11: **Sensitivity of 5yr5yr Model-Implied Expected Inflation to Survey Inflation Forecasts**
Illustration of the 5yr5yr expected inflation implied by the $G^{X^N,X^R}(7)$ model estimated with four types of survey inflation forecasts: (1) the following calendar year; (2) the next five years; (3) five-year forecasts five years ahead; (4) the next ten years, which is our benchmark implementation. The shown data cover the period from May 31, 2009, to December 30, 2019.

alternative model implementations.

Furthermore, unreported results show that the estimated state variables are essentially indistinguishable across all four model estimations. In addition, the estimated $\mathbb{Q}$-dynamics are also practically identical. Thus, the source of the differences in the model-implied inflation expectations can be traced back to differences in the estimated real-world $\mathbb{P}$-dynamics.

In summary, this evidence suggests that the model-implied inflation expectations are not overly sensitive to changes in either the inflation forecasts used in the model implementation or the assumption made about the distribution of the fitted errors of these forecasts.

5 The Estimated Bonos and Udibonos Liquidity Premia

We now use the estimated $G^{X^N,X^R}(7)$ model to extract the liquidity premium in the bonos and udibonos prices. To compute these premia we first use the estimated parameters and the filtered states $\{X_{t|t}\}_{t=1}^{T}$ to calculate the fitted bond prices $\left\{\hat{P}_t^i\right\}_{t=1}^{T}$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ by solving the fixed-point problem

$$\begin{aligned} \hat{P}_t^i &= C(t_1-t)\exp\left\{-(t_1-t)\hat{y}_t^{c,i}\right\} + \sum_{k=2}^{n}\frac{C}{2}\exp\left\{-(t_k-t)\hat{y}_t^{c,i}\right\} \qquad (12) \\ &\quad + \exp\left\{-(T-t)\hat{y}_t^{c,i}\right\}, \end{aligned}$$

for $i = 1, 2, ..., n$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ is approximately the rate of return on the ith bond if held until maturity (see Sack and Elsasser 2004). To obtain the corresponding yields without correcting for liquidity risk, a new set of model-implied bond prices are computed from the estimated $G^{X^N,X^R}(7)$ model but using only its frictionless part, i.e., using the constraints that $X_{t|t}^N = 0$ for all t as well as $\sigma_{44} = 0$ and $\theta_N^Q = 0$ for the nominal bonos, and $X_{t|t}^R = 0$ for all t as well as $\sigma_{77} = 0$ and $\theta_R^Q = 0$ for the real udibonos. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^{T}$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using (12). They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity premium for the ith bond is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{13}$$

This can be calculated for bonos and udibonos separately.

Figure 16 shows the average bonos and udibonos liquidity premium series, denoted $\bar{\Psi}_t^N$ and $\bar{\Psi}_t^R$, across the outstanding set of each type of bond at each point in time. For comparison, we estimate a standard $G^{X^N}(4)$ model[17] of bonos prices alone and a standard $G^{X^R}(3)$ model[18] of udibonos prices alone, both with unrestricted mean-reversion matrix $K^{\mathbb{P}}$ and diagonal Σ. We note that the $G^{X^N,X^R}(7)$ model of bonos and udibonos prices jointly generates liquidity premium series that are very close to those one would get from a stand-alone analysis of each yield sample in isolation.

The bonos average liquidity premium series averages 45.60 basis points with a standard deviation of 22.71 basis points, while the average udibonos liquidity premium averages 71.88 basis points with a standard deviation of 63.33 basis points. Furthermore, their correlation in levels for the overlapping period is -7 percent, while it is 16 percent in first differences. Thus, the liquidity risk in the two markets is practically uncorrelated.

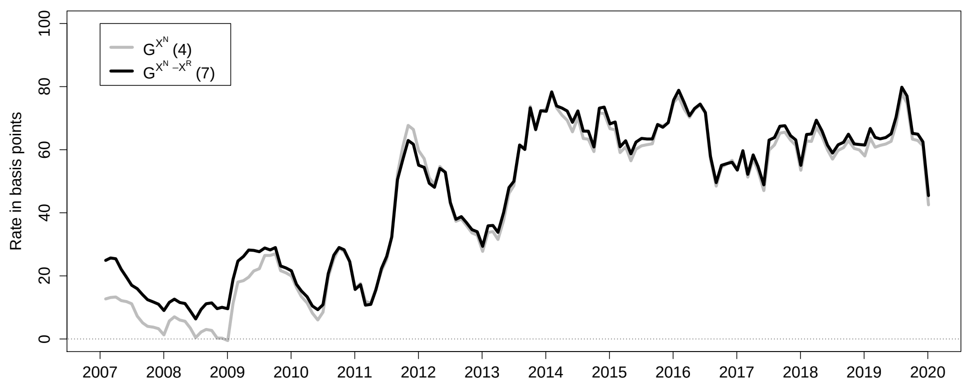
5.1 Determinants of Liquidity Risk Premia

To explain the variation of the bonos and udibonos liquidity premium series, we run standard regressions with each liquidity premium series as the dependent variable and a large set of explanatory variables that are thought to matter for bonos and udibonos market liquidity specifically or bond market liquidity more broadly as described in the following.

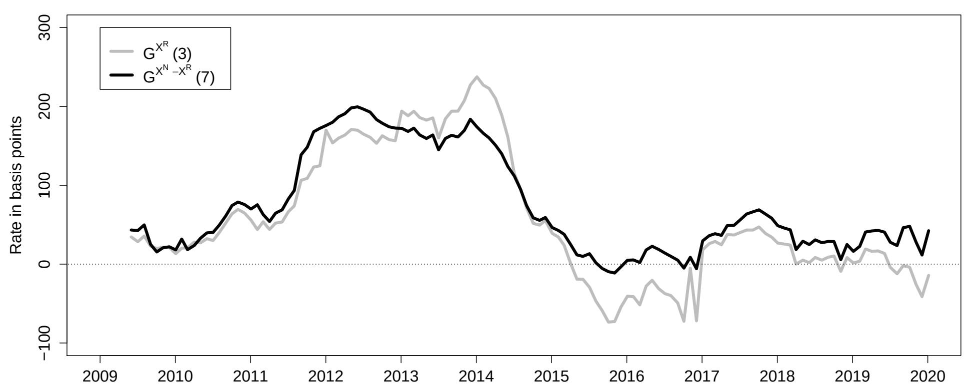
For the bonos liquidity premium regressions, building on the findings of CFS, we include the foreign-held share of the bonos market as a key explanatory variable. We then expand on the set of variables. In a core set of controls, we first include the Mexican peso-U.S. dollar exchange rate. Presumably foreign flows to and from the Mexican bonos and udibonos markets would be sensitive to exchange rate developments as discussed in Hördahl and Shim (2020). Second, to control for factors that affect emerging market sovereign bonds more broadly,

[17] This is identical to the AFNS-L model of bonos prices used in CFS.

[18] This is a three-factor model of udibonos prices with two frictionless factors representing level and slope components in addition to the liquidity risk factor X_t^R.



(a) Bonos



(b) Udibonos

Figure 12: **Average Estimated Liquidity Premia of Mexican Bonos and Udibonos** Illustration of the average estimated liquidity premium of Mexican bonos and udibonos for each observation date implied by the $G^{X^N,X^R}(7)$ model and compared to corresponding standard $G^{X^N}(4)$ and $G^{X^R}(3)$ models of nominal and real yields, respectively. The liquidity premiums are measured as the estimated yield difference between the fitted yield to maturity of individual bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The bonos data cover the period from January 31, 2007, to December 30, 2019, while the udibonos data cover the period from May 31, 2009, to December 30, 2019.

we consider the J.P. Morgan Emerging Market Bond Index (EMBI). The third variable is the West Texas Intermediate (WTI) Cushing crude oil price. Because Mexico is a major oil producing country, the revenue and bond issuance of the Mexican government are affected by changes in oil prices, which could play a role for the liquidity in the Mexican government bond market. Our next four core controls are specific to Mexico, namely the year-over-year change in the Mexican consumer price index (CPI), the public debt-to-GDP ratio as measured by the OECD, the average bid-ask spread in the bonos market, and the one-month cetes rate to proxy for the opportunity cost of holding money and the associated liquidity convenience premiums of bonos, as explained in Nagel (2016).[19] The final core variable is the ten-year U.S. Treasury yield from the Federal Reserve's H.15 database to control for reach-for-yield effects in advanced economies, which may be particularly relevant for our sample when U.S. short-term interest rates were constrained by the zero lower bound between December 2008 and December 2015. These eight variables represent our core set of control variables.

In an extended group of controls, we add the average bonos age and the one-month realized volatility of the ten-year bonos yield as additional proxies for bond liquidity following the work of Houweling et al. (2005). Inspired by the analysis of Hu et al. (2013), we also include a noise measure of bonos prices to control for variation in the amount of arbitrage capital available in this market. In addition, we use the five-year credit default swap (CDS) rate for Mexico and the monthly return of the MSCI Mexico stock index as two other measures of general developments in the Mexican economy of importance to investors in the bonos market.[20] We also add the VIX, which represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor's 500 stock price index and is widely used as a gauge of investor fear and risk aversion. Furthermore, we include the yield difference between seasoned (off-the-run) U.S. Treasury securities and the most recently issued (on-the-run) U.S. Treasury security of the same ten-year maturity mentioned earlier. This on-the-run (OTR) premium is a frequently used measure of financial frictions in the U.S. Treasury market. The final variable is the U.S. TED spread, which is calculated as the difference between the three-month U.S. LIBOR and the three-month U.S. T-bill interest rate. This spread represents a measure of the perceived general credit risk in global financial markets that could affect the pricing and trading of Mexican bonos.

To begin, we run regressions with each explanatory variable in isolation. The results are reported in the last two columns of Table 5. The foreign-held share of the bonos market and the debt-to-GDP ratio have the largest individual explanatory power, followed by the average bonos age series, the peso-U.S. dollar exchange rate, and the OTR premium, while the WTI oil price and the financial variables (the EMBI, the CDS rate, the return of the MSCI index, the VIX, and the TED spread) only have a weak link with the bonos liquidity premium. The same holds for our proxies of bonos market liquidity and frictions (bonos bid-ask spread, yield volatility, and noise measure). Finally, the one-month cetes rate and the associated opportunity cost of holding cash have a negative relationship with our estimated bonos liquidity premium series, which is consistent with the findings of Nagel (2016) as it increases the convenience yield of holding bonos.

The columns labeled (1) and (3) in Table 5 show the results of our preferred regression model with our core set of variables and the full joint regression with all explanatory variables included, respectively. Four things stand out. First, both regressions produce about the

[19]Cetes are short-term instruments issued by the Mexican government similar to U.S. Treasury bills.

[20]The MSCI index is a free-float weighted equity index designed to measure the performance of the large and mid cap segments of the Mexican stock market. The index is reported in U.S. dollars.

Explanatory variables	(1)	(2)	(3)	(4)	Individual regressions	
					$\widehat{\beta}$	adj. R^2
Foreign Share of Bonos	1.01**		0.77*		1.19**	0.77
	(0.24)		(0.37)		(0.08)	
Peso/USD exchange rate	-3.25	-2.95	-1.69	-2.26	4.93**	0.46
	(2.29)	(2.48)	(2.26)	(2.41)	(0.61)	
EMBI	0.03	0.02	0.14**	0.16**	0.04	0.02
	(0.02)	(0.02)	(0.05)	(0.05)	(0.05)	
WTI	-0.09	0.02	-0.05	-0.04	-0.39**	0.14
	(0.11)	(0.13)	(0.10)	(0.10)	(0.14)	
CPI Inflation	-2.44	-2.45*	-1.83	-0.36	-5.34	0.06
	(1.28)	(1.21)	(1.43)	(1.45)	(3.10)	
Debt-to-GDP ratio	1.56	3.07**	1.77	1.84	2.21**	0.70
	(0.93)	(0.96)	(1.40)	(1.56)	(0.14)	
Bonos bid-ask spread	0.52	-1.70	0.37	0.27	-4.24	0.02
	(1.44)	(2.16)	(1.44)	(1.73)	(5.05)	
One-month cetes rate	1.12	1.47	1.74	-0.15	-3.12	0.06
	(1.69)	(1.58)	(2.59)	(2.61)	(1.99)	
10yr US Treasury yield	3.65	-1.12	4.02	2.06	-17.94**	0.44
	(3.19)	(3.11)	(2.99)	(2.91)	(2.50)	
Avg. bonos age			-3.67	1.25	9.81**	0.64
			(5.85)	(5.47)	(0.85)	
One-month bonos yield vol.			0.12	0.19	-0.14	0.00
			(0.15)	(0.16)	(0.19)	
Bonos noise measure			-0.27	-0.58	0.38	-0.00
			(0.91)	(0.99)	(1.31)	
CDS rate			-0.14	-0.12	-0.03	-0.00
			(0.10)	(0.10)	(0.06)	
MSCI one-month return			0.08	-0.02	-0.05	-0.01
			(0.16)	(0.16)	(0.34)	
VIX			-0.10	-0.52	-1.28**	0.22
			(0.39)	(0.41)	(0.23)	
OTR premium			-0.29	-0.62**	-1.16**	0.34
			(0.28)	(0.22)	(0.23)	
TED spread			-0.05	-0.01	-0.24**	0.22
			(0.03)	(0.03)	(0.05)	
Intercept	-26.53	-29.48	-53.12	-26.34		
	(25.58)	(33.95)	(43.80)	(44.07)		
Adjusted R^2	0.80	0.73	0.81	0.80		

Table 5: **Regression Results for the Average Bonos Liquidity Premium**
The table reports the results of regressions with the average estimated bonos liquidity premium as the dependent variable and 17 explanatory variables. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

same adjusted R^2 (roughly 80 percent). Thus, the preferred regression yields about as much explanatory power as possible given our 17 variables. Second, the foreign-held share maintains a statistically significant coefficient close to 1 in the benchmark core regression, that is, we find a positive relationship whereby a 1 percentage point increase in the foreign-held share of Mexican bonos tends to raise their liquidity premium by about 1 basis point, which is slightly stronger than the results reported by CFS. Third, the core set of control variables all have a stable relationship with the bonos liquidity premium series in the two regressions. Finally, when we exclude the foreign share as an explanatory variable, we get a drop in the adjusted R^2 as reported in columns (2) and (4). This underscores the importance of the foreign share for liquidity premia in the bonos market, as also emphasized by CFS.

To summarize, the regression results reveal that the increase in the share of foreign holdings of Mexican bonos is significantly positively correlated with the change in the bonos liquidity premiums, both on its own and after including our control variables. In terms of magnitudes, the results imply that a 1 percentage point increase in the foreign share raises the liquidity premium by 0.01 percent or 1 basis point. Given that the foreign market share has increased by more than 40 percentage points between 2010 and 2017 as implied by Figure 6(a), our results suggest that the large increase in foreign holdings during our sample period has played a significant role for the upward trend in the liquidity premiums in the Mexican bonos market since then and raised them by as much as 0.4 percent.

A potential explanation for our findings is tied to the fact that our measure of liquidity premia is forward-looking and not determined by the current trading conditions in the market, which have varied around a stable level as shown in Figure 7. Hence, investors may expect that, as foreign holdings of Mexican government debt increase, the probability of a large sell-off has increased as well. In response, investors appear to demand higher liquidity premia to assume this risk as suggested by the analysis in CFS.

Now, we repeat the regression exercise using the average estimated udibonos liquidity premium series as the dependent variable. Furthermore, a number of variables are replaced with the appropriate substitutes. The foreign share is now the foreign-held share of the udibonos market. The bid-ask is the average bid-ask spread of the udibonos in our sample, while the average age is the average age of the udibonos in the sample. Also, the one-month yield volatility is the one-month realized volatility of daily changes in the fitted ten-year udibonos yield. Finally, the noise measure is constructed from the fitted error of a standard AFNS model of our sample of daily udibonos prices.

Based on the regression results reported in Table 6, we make the following observations. First, the foreign-held share is a significant determinant of the liquidity premia of udibonos in our preferred regression (1), similar to what we find for bonos. However, now the effect is two to three times larger so that, for each percentage point increase in the foreign share, the udibonos liquidity premia increase between 2 and 3 basis points. Second, increases in both the WTI oil price and CPI inflation are associated with increases in the udibonos liquidity premia. These results are unusual as they suggest that udibonos become less desirable in

Explanatory variables	(1)	(2)	(3)	(4)	Individual regressions	
					$\widehat{\beta}$	adj. R^2
Foreign Share of Udibonos	3.52*		2.33		7.66**	0.17
	(1.43)		(1.49)		(2.66)	
Peso/USD exchange rate	1.69	0.98	2.51	1.43	-12.53**	0.34
	(4.05)	(4.33)	(6.00)	(5.68)	(2.46)	
EMBI	0.18**	0.18**	0.21*	0.23*	-0.33*	0.06
	(0.05)	(0.06)	(0.09)	(0.09)	(0.16)	
WTI	2.49**	2.70**	2.20**	2.19**	2.30**	0.59
	(0.33)	(0.30)	(0.34)	(0.33)	(0.31)	
CPI Inflation	15.04**	17.47**	17.73**	19.28**	3.45	-0.00
	(3.64)	(3.08)	(3.55)	(3.29)	(7.48)	
Debt-to-GDP ratio	-3.04	-1.78	-3.82	-3.70	-6.13**	0.32
	(2.21)	(2.08)	(2.70)	(2.62)	(1.55)	
Udibonos bid-ask spread	0.80	0.44	0.63	0.32	-1.22	-0.01
	(0.70)	(0.67)	(0.67)	(0.67)	(3.62)	
One-month cetes rate	5.13	-0.30	3.25	-0.08	-11.04*	0.08
	(4.18)	(3.64)	(5.29)	(5.03)	(4.44)	
10yr US Treasury yield	-51.96**	-61.70**	-46.60**	-50.37**	-24.17	0.04
	(6.25)	(4.97)	(6.92)	(6.64)	(16.17)	
Avg. udibonos age			-2.10	-1.21	-16.20*	0.13
			(6.34)	(6.36)	(6.30)	
One-month udibonos yield vol.			-0.61*	-0.58*	0.72	0.00
			(0.29)	(0.28)	(0.74)	
Udibonos noise measure			1.88	1.94	3.82*	0.08
			(1.04)	(1.03)	(1.65)	
CDS rate			0.04	0.08	-0.71**	0.10
			(0.19)	(0.19)	(0.26)	
MSCI one-month return			0.40	0.27	0.61	-0.00
			(0.38)	(0.36)	(0.80)	
VIX			-0.05	-0.46	0.36	-0.01
			(0.84)	(0.77)	(1.43)	
OTR premium			-1.83**	-2.20**	-2.50*	0.07
			(0.63)	(0.63)	(1.02)	
TED spread			-0.10	-0.09	-0.76	0.01
			(0.25)	(0.26)	(0.87)	
Intercept	-53.24	-47.17	-18.25	23.43		
	(82.08)	(83.66)	(90.33)	(90.35)		
Adjusted R^2	0.88	0.87	0.89	0.88		

Table 6: **Regression Results for the Average Udibonos Liquidity Premium**
The table reports the results of regressions with the average estimated udibonos liquidity premium as the dependent variable and 17 explanatory variables. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

more inflationary environments, which is when they are most valuable for investors as a hedge against inflation risk. Third, increases in the debt-to-GDP ratio tend to lower udibonos liquidity premia, although not significantly so. Provided the extra debt is mainly issued in the bonos market, which is a reasonable assumption given the small size of the udibonos market, this finding could reflect the resulting change in the relative desirability of udibonos vis-á-vis bonos. Finally, we note that the ten-year U.S. Treasury yield has a statistically significant negative coefficient in all these regressions. Thus, higher U.S. yields tend to lower the liquidity premia in the udibonos market. In addition, we note that we also get satisfactorily high R^2s in these regressions, with 0.88 produced by our preferred benchmark regression (1).

6 The Estimated Bonos and Udibonos Term Premia

As an additional model validation exercise, we examine the properties of the nominal bonos and real udibonos term premia implied by the estimated $G^{X^N,X^R}(7)$ model.

As already noted in Section 4.3, we define the nominal and real term premia in the standard way as

$$TP_t^j(\tau) = y_t^j(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^j]ds, \quad j = N, R.$$

Here, $y_t^j(\tau)$ refers to the liquidity-adjusted frictionless zero-coupon yields, while $E_t^{\mathbb{P}}[r_s^j]$ represents the corresponding expected future short rate, which is determined by the estimated model dynamics.

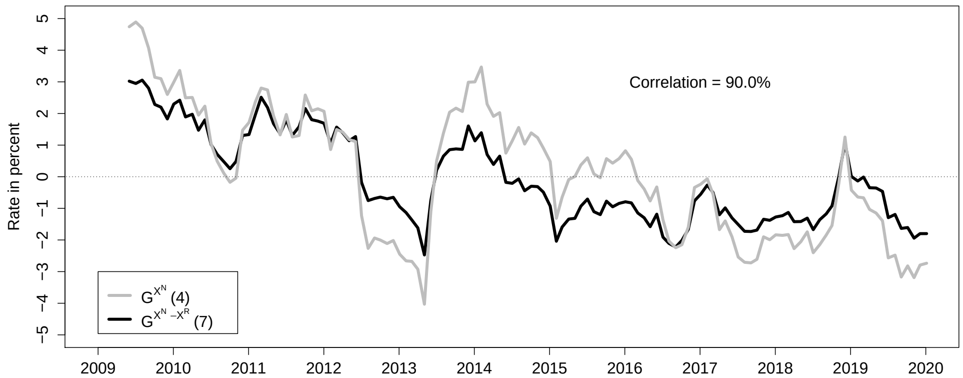
For comparison, we estimate a standard $G^{X^N}(4)$ model of bonos prices alone and a standard $G^{X^R}(3)$ model of udibonos prices alone, both with unrestricted mean-reversion matrix $K^{\mathbb{P}}$ and diagonal Σ. The results are shown in Figure 13.

The top panel shows the estimated ten-year nominal bonos term premium from the $G^{X^N,X^R}(7)$ model and compares it to the corresponding result from the $G^{X^N}(4)$ model using bonos prices only. The two series are highly positively correlated (90%) and clearly reflect the same shocks. The main difference is that the estimated series from the $G^{X^N,X^R}(7)$ model is less volatile.

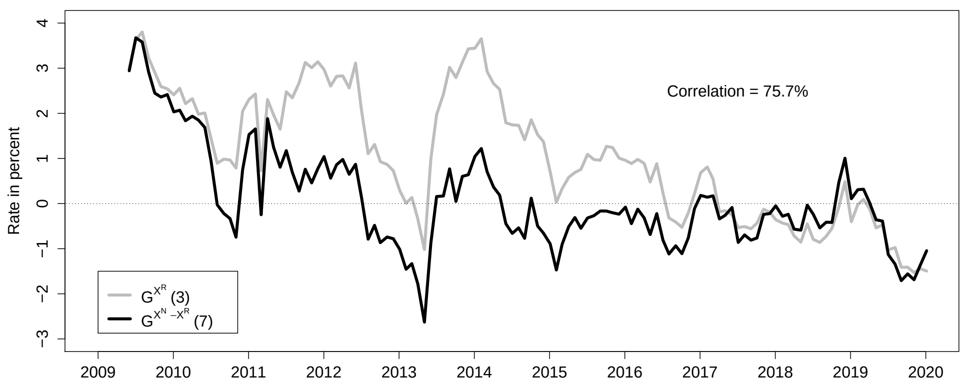
The bottom panel in the figure shows the estimated ten-year real term premia embedded in the udibonos prices. Again, there is a high positive correlation with the results from the udibonos-only model, and again the term premium series implied by the $G^{X^N,X^R}(7)$ model is less noisy and volatile.

Comparing the ten-year bonos and udibonos term premium series for the overlapping period, we note a high correlation and very similar level for the two series. Furthermore, they tend to flip their signs at the same time. This shows that, once we account for the liquidity premia in the two markets, the slopes of the frictionless bonos and udibonos yield curves behave very similarly, in particular they invert at roughly the same points in time.

The bonos ten-year term premium series averages -8.30 basis points with a standard deviation of 141.75 basis points, while the ten-year udibonos term premium averages 7.59 basis points with a standard deviation of 111.75 basis points. Furthermore, their correlation



(a) Bonos



(b) Udibonos

Figure 13: **Estimated Ten-Year Term Premia of Mexican Bonos and Udibonos**
Illustration of the estimated ten-year term premium of Mexican bonos and udibonos for each observation date implied by the $G^{X^N,X^R}(7)$ model. The Mexican bonos liquidity premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Mexican bonos and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data cover the period from May 31, 2009, to December 30, 2019, although we stress that both the $G^{X^N}(4)$ model and the $G^{X^N,X^R}(7)$ model estimations are based on bonos price data covering the period from January 31, 2007, to December 30, 2019.

in levels for the overlapping period is as high as 88 percent, while it is 56 percent in first differences. Thus, nominal and real term premia in the Mexican bonos and udibonos markets are strongly positively correlated.

We stress that this high correlation reflects the data and is not driven by our model structure, which is flexible enough that the nominal and real term premia could have been independent of each other, similar to what we observed for the bonos and udibonos liquidity premia.

6.1 Determinants of Term Risk Premia

We proceed to perform a set of regression exercises similar to those in Section 5.1. Specifically, we perform one set of regressions using the ten-year bonos term premium as the dependent variable and another set of regressions using the ten-year udibonos term premium as the dependent variable. Furthermore, in the bonos term premium regressions, we use the same set of explanatory variables as in the bonos liquidity premium regressions reported in Table 5, while in the udibonos term premium regressions we use the same set of explanatory variables as those used in the udibonos liquidity premium regressions in Table 6.

The results from the bonos term premium regressions are reported in Table 7. First, the foreign share is a very important determinant of the term premia of Mexican bonos yields, but now with a negative sign. This means that foreign bond fund outflows are associated with spikes in Mexican term premia. Second, both the peso-U.S. dollar exchange rate and the EMBI are positively correlated with the bonos term premia. Thus, a depreciation of the peso against the U.S. dollar tends to coincide with upticks in Mexican term premia. Upticks in the EMBI have similar effects. Third, increases in oil prices or Mexican CPI inflation also tend to push up Mexican bonos term premia. Fourth and unexpectedly, an increase in the Mexican debt-to-GDP ratio is associated with declines in the Mexican bonos term premium. Fifth, increases in bonos market frictions as captured by bonos bid-ask spreads tend to push up bonos term premia, while the one-month cetes rate has a negative coefficient. Hence, a tightening of monetary policy as reflected in higher cetes rates tends to flatten the bonos yield curve through a reduction of the bonos term premia. Lastly, U.S. Treasury securities represent an important substitute to Mexican bonos for foreign investors in this market. Therefore, it is not surprising that increases in U.S. Treasury yields tend to push up Mexican bonos term premia essentially one-for-one.[21] In addition, we note that the expanded set of explanatory variables in regression (3) hardly affect the explanatory power as measured by the adjusted R^2. This provides further support for our choice of preferred benchmark variables included in regression (1) in the table.

The results for the regressions using the ten-year udibonos term premium as the dependent variable are reported in Table 8. We note that the results are qualitatively very similar to those reported for the bonos term premium, which is reasonable in light of the high correlation of and great similarity between the two term premium series. In particular, it remains the case that a tightening of monetary policy flattens the udibonos real yield curve through a reduction in the real term premium and that variation in U.S. Treasury yields affects both bonos and udibonos term premia essentially one-for-one. Finally, the foreign share also has a negative coefficient in these regressions, but it is now insignificant.

[21]In the regressions, the bonos term premium is measured in basis points, while the ten-year U.S. Treasury yield is measured in percent. Thus, a coefficient of 100 on the U.S. Treasury yield implies a one-for-one relationship.

Explanatory variables	(1)	(2)	(3)	(4)	Individual regressions	
					$\widehat{\beta}$	adj. R^2
Foreign Share of Bonos	-3.35 *		-3.84		-8.52 **	0.66
	(1.40)		(2.04)		(0.80)	
Peso/USD exchange rate	15.40	19.23*	11.76	16.31	-30.66**	0.41
	(9.87)	(8.80)	(13.28)	(13.03)	(5.65)	
EMBI	0.89**	0.84**	0.66	0.58	-0.08	-0.01
	(0.19)	(0.19)	(0.34)	(0.33)	(0.43)	
WTI	2.58**	1.53*	2.80**	2.39**	3.74**	0.31
	(0.83)	(0.73)	(0.88)	(0.85)	(0.66)	
CPI Inflation	4.42	-2.98	12.97	0.54	7.13	-0.00
	(6.84)	(6.24)	(8.45)	(6.81)	(25.05)	
Debt-to-GDP ratio	-9.75	-20.05**	-12.81	-17.72*	-18.56**	0.58
	(5.65)	(3.83)	(8.14)	(7.48)	(2.50)	
Bonos bid-ask spread	16.98	20.03*	21.51	16.06	78.00*	0.12
	(8.81)	(8.19)	(12.25)	(10.19)	(32.68)	
One-month cetes rate	-7.16	-5.02	-19.07	-8.15	-22.24*	0.06
	(9.39)	(8.44)	(13.29)	(12.86)	(9.82)	
10yr US Treasury yield	91.74**	115.98**	83.40**	94.03**	161.42**	0.45
	(15.35)	(16.10)	(19.18)	(20.20)	(25.78)	
Avg. bonos age			37.38	17.98	-64.33**	0.53
			(28.26)	(30.01)	(9.98)	
One-month bonos yield vol.			-0.05	-0.34	1.45	0.00
			(0.57)	(0.64)	(1.75)	
Bonos noise measure			1.98	2.34	24.34**	0.13
			(3.12)	(3.37)	(6.75)	
CDS rate			0.66	0.49	1.18	0.05
			(0.55)	(0.62)	(0.79)	
MSCI one-month return			0.34	0.25	2.08	-0.00
			(1.01)	(1.01)	(2.30)	
VIX			1.27	1.93	14.78**	0.36
			(2.02)	(2.16)	(2.41)	
OTR premium			-0.08	2.59	13.53**	0.42
			(2.04)	(1.53)	(1.52)	
TED spread			-0.66	-0.49	-2.77	0.04
			(0.74)	(0.78)	(1.85)	
Intercept	-431.42	-174.93	-408.84	-322.63		
	(271.75)	(243.49)	(354.52)	(348.78)		
Adjusted R^2	0.88	0.86	0.87	0.87		

Table 7: **Regression Results for the Bonos Ten-Year Term Premium**
The table reports the results of regressions with the estimated ten-year bonos term premium as the dependent variable and 17 explanatory variables. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

7 Empirical BEI Decomposition

In this section, we explore the properties of the BEI decomposition implied by the $G^{X^N,X^R}(7)$ model with a particular emphasis on both the model-implied expected inflation and the associated inflation risk premium that investors in bonos demand to assume their inflation risk. First, we examine the BEI decomposition. We then analyze the inflation risk premia

Explanatory variables	(1)	(2)	(3)	(4)	Individual regressions	
					$\widehat{\beta}$	adj. R^2
Foreign Share of Udibonos	-8.07		-5.68		-13.15*	0.16
	(4.36)		(5.20)		(5.34)	
Peso/USD exchange rate	37.78**	39.41**	17.89	20.51	-15.03**	0.15
	(6.60)	(7.64)	(9.72)	(11.85)	(5.16)	
EMBI	0.69**	0.67**	0.44*	0.40	0.22	0.00
	(0.15)	(0.15)	(0.22)	(0.22)	(0.34)	
WTI	1.14*	0.64	1.50*	1.51*	1.50**	0.07
	(0.57)	(0.64)	(0.64)	(0.64)	(0.55)	
CPI Inflation	21.38**	15.82**	14.84*	11.05	20.15	0.03
	(5.35)	(6.04)	(6.50)	(5.90)	(17.86)	
Debt-to-GDP ratio	-17.05**	-19.91**	-11.85*	-12.14*	-10.27**	0.28
	(3.18)	(3.50)	(4.96)	(4.97)	(2.74)	
Udibonos bid-ask spread	3.54*	4.37*	3.28	4.06	12.04	0.03
	(1.58)	(2.17)	(1.92)	(2.38)	(8.29)	
One-month cetes rate	-40.07**	-27.64**	-25.47	-17.36	-9.56	0.01
	(9.42)	(5.63)	(15.47)	(10.44)	(7.34)	
10yr US Treasury yield	105.35**	127.63**	90.71**	99.88**	140.91**	0.56
	(14.13)	(13.52)	(12.21)	(14.15)	(22.19)	
Avg. udibonos age			13.05	10.88	-45.34**	0.33
			(15.06)	(15.18)	(11.94)	
One-month udibonos yield vol.			-0.51	-0.59	1.63	0.01
			(0.59)	(0.64)	(1.00)	
Udibonos noise measure			-1.07	-1.22	6.01**	0.06
			(2.01)	(1.99)	(2.26)	
CDS rate			0.74*	0.66	1.56*	0.17
			(0.34)	(0.40)	(0.64)	
MSCI one-month return			-0.42	-0.11	1.68	-0.00
			(0.65)	(0.80)	(2.05)	
VIX			-2.44 *	-1.43	9.98**	0.26
			(1.00)	(1.37)	(2.92)	
OTR premium			4.11*	5.00**	12.49**	0.58
			(1.79)	(1.51)	(0.99)	
TED spread			0.63	0.61	-0.73	-0.00
			(0.54)	(0.54)	(1.31)	
Intercept	-272.64	-286.53	-303.43	-404.80		
	(203.40)	(197.85)	(223.20)	(208.46)		
Adjusted R^2	0.82	0.81	0.84	0.84		

Table 8: **Regression Results for the Udibonos Ten-Year Term Premium**
The table reports the results of regressions with the estimated ten-year udibonos term premium as the dependent variable and 17 explanatory variables. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

and their determinants, including an international comparison. Finally, we end the section with an update of our analysis through the COVID-19 crisis in the first half of 2020, along with projections of the long-term inflation expectations in Mexico.

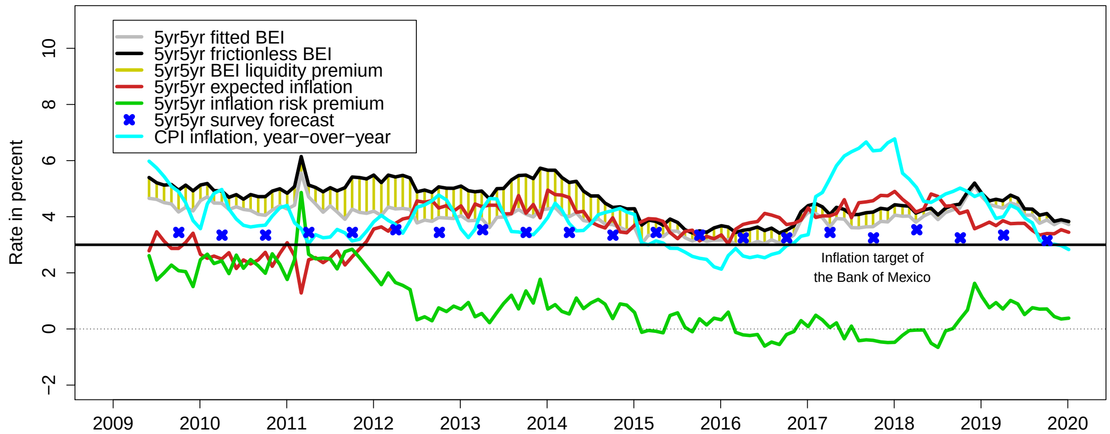


Figure 14: **Decomposition of 5yr5yr BEI**
Illustration of the fitted 5yr5yr BEI obtained by fitting an AFNS model to Mexican bonos and udibonos prices separately and its decomposition based on the $G^{X^N,X^R}(7)$ model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ into: (1) the estimated frictionless BEI, (2) expected inflation, and (3) the residual inflation risk premium. The difference between the fitted and frictionless 5yr5yr BEI is highlighted in yellow and represents the net liquidity premium of the observed 5yr5yr BEI. The shown data cover the period from May 31, 2009, to December 30, 2019.

7.1 BEI Decomposition

In this section, we examine the BEI decompostion implied by the estimated $G^{X^N,X^R}(7)$ model. To be consistent with the existing literature, we focus on a horizon long enough into the future that most transitory shocks to the economy can be expected to have vanished. At the same time, the horizon must be practically relevant and covered by the available maturities in the underlying bond data. Balancing these considerations, we limit our analysis to the five-year forward BEI rate that starts five years ahead, denoted 5yr5yr BEI.

The result of decomposing 5yr5yr BEI as described in Section 4.3 is shown in Figure 14. The solid gray line shows the fitted 5yr5yr BEI obtained by estimating a three-factor arbitrage-free Nelson-Siegel model to nominal bonos and real udibonos prices separately as described in Section 3. This can be compared to the estimated 5yr5yr frictionless BEI implied by the $G^{X^N,X^R}(7)$ model and shown with a solid black line in the figure. The difference between these two measures of 5yr5yr BEI represents the net liquidity premium or distortion of the observed BEI series due to bond-specific liquidity risk premia in both bonos and udibonos prices. The fact that the 5yr5yr frictionless BEI is entirely above the 5yr5yr fitted BEI implies that the distortions due to liquidity risk are systematically larger in the real yields compared to those in the nominal yields at the 5yr5yr horizon.

Due to its theoretical consistency, the $G^{X^N,X^R}(7)$ model allows us to break down the

5yr5yr frictionless BEI into an expected inflation component, shown with a solid red line in Figure 14, and the residual inflation risk premium, shown with a solid green line. Also shown in the figure with a solid black horizontal line is the 3 percent inflation target of the Bank of Mexico introduced in 1995. For comparison, the figure also shows the 5yr5yr expected CPI inflation in Mexico reported semiannually in the Consensus Forecasts surveys. We stress that these particular survey forecasts are *not* included in the model estimation. Thus, the closeness of the model's expected inflation to the survey forecasts reflects its ability to appropriately capture the term structure of inflation expectations among investors in the Mexican bonos and udibonos market. Furthermore, the figure shows the year-over-year change in the Mexican CPI with a solid cyan line to provide a sense of the actual inflation outcomes during this ten-year period.

Note that annual CPI inflation has averaged 4.02 percent during the shown period, somewhat above the Bank of Mexico's target, but mostly within the acceptable $\pm$ 1 percentage point tolerance band around the target. As a consequence, it seems reasonable that both the survey inflation forecasts and the model-implied expected inflation are generally somewhat above the announced inflation target. However, given that the Bank of Mexico in implementing monetary policy works with a $\pm$ 1 percentage point tolerance band around its 3 percent target, both the survey inflation forecasts and the model-implied inflation expectations can be viewed as anchored at a level consistent with the central bank's inflation target. Furthermore, the 5yr5yr expected inflation from the model is positively correlated with the year-over-year change in the CPI, as expected, but only weakly so with a correlation of 36 percent. Overall, we take our findings to suggest that long-term inflation expectations in Mexico are well anchored near the central bank's official target, as also argued by De Pooter et al. (2014).

7.2 Analysis of Inflation Risk Premia

In this section, we first explore what determines the size of and variation in Mexican inflation risk premia using regression analysis similar to what we used for the other types of bond risk premia in the previous sections. This is followed by an international comparison to Canadian and U.S. inflation risk premia.

7.2.1 Determinants of Inflation Risk Premia

While the long-term inflation expectations in Mexico are largely determined by the inflation target of the Bank of Mexico, it is less clear what determines Mexican long-term inflation risk premia. Specifically, to explain the variation of the 5yr5yr Mexican inflation risk premium series, we again run standard regressions with it as the dependent variable and the combined set of explanatory variables used in the previous analysis of the other bond risk premium series.

Thanks to the merger of the set of explanatory variables we now have a total of 22

Explanatory variables	(1)	(2)	(3)	(4)	Individual regressions $\widehat{\beta}$	adj. R^2
Foreign Share of Bonos	1.06		-1.26		-6.35**	0.69
	(1.54)		(2.15)		(0.70)	
Foreign Share of Udibonos	-11.19**		-11.38**		-10.08**	0.11
	(4.03)		(3.85)		(3.43)	
Peso/USD exchange rate	6.51	6.85	3.98	10.16	-23.02**	0.43
	(10.52)	(10.18)	(12.85)	(13.03)	(4.17)	
EMBI	0.14	0.13	0.03	-0.06	-0.34	0.02
	(0.13)	(0.15)	(0.21)	(0.23)	(0.29)	
WTI	0.01	-0.35	-0.22	-0.27	2.67**	0.30
	(0.59)	(0.52)	(0.63)	(0.74)	(0.54)	
CPI Inflation	-28.58**	-33.83**	-11.41	-24.44**	-8.91	0.00
	(5.98)	(5.85)	(8.05)	(7.07)	(17.24)	
Debt-to-GDP ratio	-18.12**	-18.58**	-21.84**	-22.15**	-13.95**	0.62
	(5.55)	(3.46)	(7.24)	(7.53)	(1.71)	
Bonos bid-ask spread	-10.54	-10.17	-3.69	-8.48	37.88	0.05
	(5.58)	(5.75)	(7.55)	(8.77)	(20.69)	
Udibonos bid-ask spread	0.83	1.84	0.93	2.36	12.60	0.04
	(1.80)	(2.02)	(1.70)	(1.91)	(9.13)	
One-month cetes rate	-0.88	16.07	-23.20	1.08	-13.85	0.04
	(9.57)	(7.99)	(14.94)	(15.45)	(7.19)	
10yr US Treasury yield	37.99*	60.89*	30.98*	52.58**	95.72**	0.30
	(15.01)	(13.47)	(14.33)	(14.85)	(19.22)	
Avg. bonos age			32.95	21.15	-45.30**	0.50
			(18.48)	(21.52)	(7.76)	
Avg. udibonos age			16.48	9.03	-49.79**	0.47
			(17.30)	(17.06)	(8.71)	
One-month bonos yield vol.			0.02	-0.27	1.02	-0.00
			(0.76)	(0.89)	(1.33)	
One-month udibonos yield vol.			1.28	1.09	3.90**	0.09
			(1.14)	(1.06)	(1.45)	
Bonos noise measure			-5.70*	-2.96	16.46*	0.11
			(2.86)	(3.20)	(6.34)	
Udibonos noise measure			2.87	2.12	9.13**	0.17
			(2.52)	(2.87)	(3.01)	
CDS rate			0.19	-0.05	0.38	0.00
			(0.34)	(0.40)	(0.47)	
MSCI one-month return			0.08	0.54	1.66	0.00
			(0.78)	(0.82)	(1.49)	
VIX			0.71	2.99	11.06**	0.38
			(1.37)	(1.75)	(1.19)	
OTR premium			-1.33	1.18	7.82**	0.26
			(1.67)	(1.62)	(1.74)	
TED spread			-0.12	-0.05	-2.71*	0.08
			(0.61)	(0.72)	(1.29)	
Intercept	812.15**	705.37**	925.25**	697.91**		
	(198.21)	(149.62)	(232.28)	(233.33)		
Adjusted R^2	0.81	0.79	0.83	0.80		

Table 9: **Regression Results for the 5yr5yr Inflation Risk Premium**
The table reports the results of regressions with the estimated 5yr5yr inflation risk premium as the dependent variable and 22 explanatory variables. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

variables, in addition to the dependent variable with the 5yr5yr inflation risk premium series. The results of the regression analysis are reported in Table 9. First, we stress that our benchmark regression (1) remains important as indicated by the high adjusted R^2 of 0.81. Second, we note that neither the exchange rate, the EMBI, the WTI, the one-month cetes rate nor bonos and udibonos bid-ask spreads have any significant effects on the estimated 5yr5yr inflation risk premium. The same holds for the 11 variables included in our expanded set of control variables, with the possible exception of the bonos noise measure, which has a statistically significant negative coefficient in regression (3). Furthermore, adding the 11 extra control variables in regression (3) only delivers a marginally better adjusted R^2 of 0.83. Third, the foreign share of udibonos is highly statistically significant with a negative sign, while the foreign share of bonos is insignificant and even flips its sign depending on the specification of the regressions. Still, including the foreign share variables materially affects the adjusted R^2.

Finally, focusing on our preferred regression (1), we note the following. Higher CPI inflation tends to depress the inflation risk premia. This reveals that the positive effects of higher CPI inflation on the real term premia dominates its positive effects on the nominal term premia and hence squeezes both BEI and the inflation risk premia. A higher debt-to-GDP ratio tends to put downward pressure on the Mexican inflation risk premium similar to its effect on the nominal and real term premia. Increases in the ten-year U.S. Treasury yield tend to push up Mexican inflation risk premia, but less than its impact on the nominal and real term premia.

In summary, we take these findings to be sensible in light of the results we obtained in the term premium regressions in the previous section.

7.2.2 International Comparison of Inflation Risk Premia

To go beyond the regression analysis above, we compare the estimated 5yr5yr inflation risk premium for Mexico with matching estimates from Canadian and U.S. nominal and real yields.[22] Figure 15 shows all three series for the available overlapping sample period.

The Canadian and U.S. inflation risk premia are highly positively correlated (85%). The Mexican inflation risk premium series is also positively correlated with each, 64 percent and 55 percent, respectively. Thus, both in terms of size and time variation, Mexican inflation risk premia share similarities with those observed in Canadian and U.S. bond markets. However, as expected, Mexican inflation risk premia are more volatile with a standard deviation of 103.23 basis points compared with 23.94 basis points and 34.91 basis points for the Canadian and U.S. series, respectively.

The mostly positive and small inflation risk premia in Canada and the United States are consistent with the findings from simple macro-finance representative agent models; see

[22] The Canadian estimate is taken from Christensen et al. (2020), while the U.S. estimate comes from an update of the model described in ACR using all available TIPS.

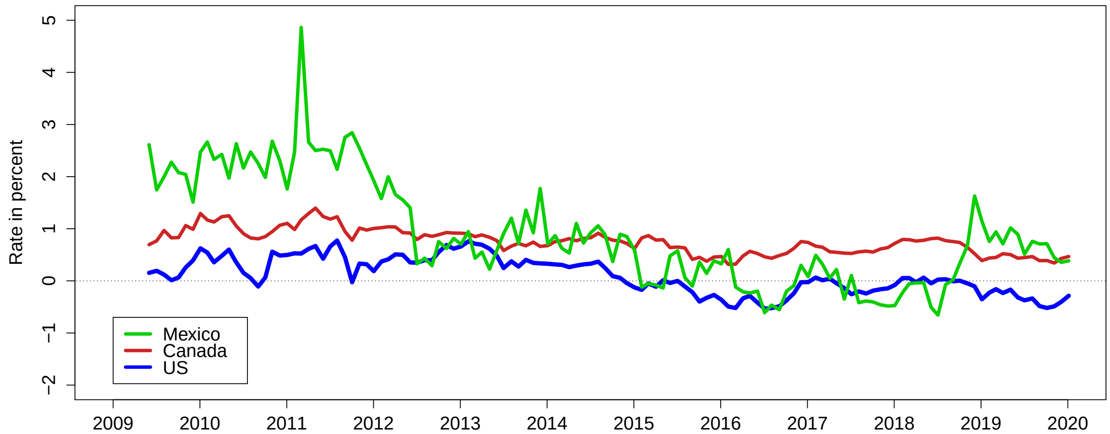


Figure 15: **International Panel of 5yr5yr Inflation Risk Premia**
Illustration of the estimated 5yr5yr inflation risk premium series from Mexican, Canadian, and U.S. nominal and real bond prices as described in the text. The shown data cover the period from May 31, 2009, to December 30, 2019.

Hördahl and Tristani (2012). For the United States, DKW also report empirical estimates of inflation risk premia, which are mostly positive and relatively small. In turn, to observe larger and more volatile inflation risk premia in an emerging market economy such as Mexico would seem like a reasonable result given the higher and more volatile CPI inflation in Mexico compared with Canada and the United States.

8 The COVID-19 Crisis

In light of the unprecedented global economic shock caused by the spread of the coronavirus pandemic in the spring of 2020, we update the analysis through the end of December 2020 to examine whether the pandemic has had any impact on the long-term inflation expectations of investors in the Mexican government bond market. We also use the update to assess the outlook for long-term inflation expectations in Mexico based on the updated $G^{X^N,X^R}(7)$ model.

Figure 16 shows the updated estimates of the bonos and udibonos liquidity premia, where we note a significant uptick in both series since the start of 2020 that is larger for udibonos. This underscores the importance of correcting for both types of liquidity premia to be able to fully understand the variation in BEI during this challenging period. Furthermore, this is consistent with evidence from the United States, where the Treasury and TIPS markets experienced dislocations during the early stages of the coronavirus crisis; see ACR and He et al. (2020).

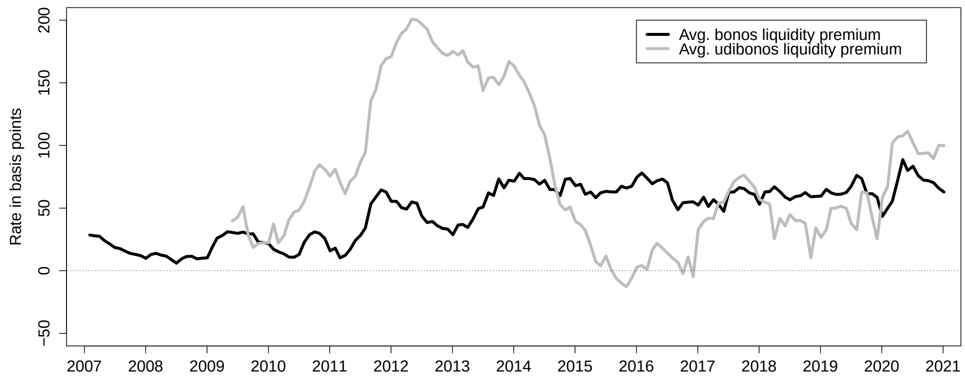


Figure 16: **Average Estimated Liquidity Premium of Mexican Bonos and Udibonos**
Illustration of the average estimated liquidity premium of Mexican bonos and udibonos for each observation date implied by the $G^{X^N,X^R}(7)$ model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. The Mexican bonos liquidity premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Mexican bonos and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. Their data cover the period from January 31, 2007, to December 31, 2020. The Mexican udibonos liquidity premiums are calculated using a similar approach. Their data cover the period from May 31, 2009, to December 31, 2020.

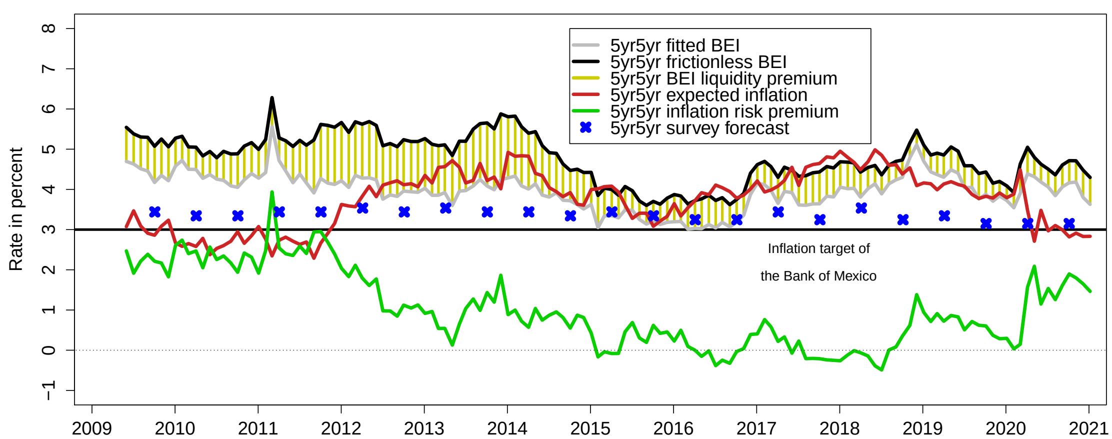


Figure 17: **Decomposition of 5yr5yr BEI**
Illustration of the fitted 5yr5yr BEI obtained by fitting an AFNS model to Mexican bonos and udibonos prices separately and its decomposition based on the $G^{X^N,X^R}(7)$ model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ into: (1) the estimated frictionless BEI, (2) the expected inflation, and (3) the residual inflation risk premium. The difference between the fitted and frictionless 5yr5yr BEI is highlighted in yellow and represents the net liquidity premium of the observed 5yr5yr BEI. The shown data cover the period from May 31, 2009, to December 31, 2020.

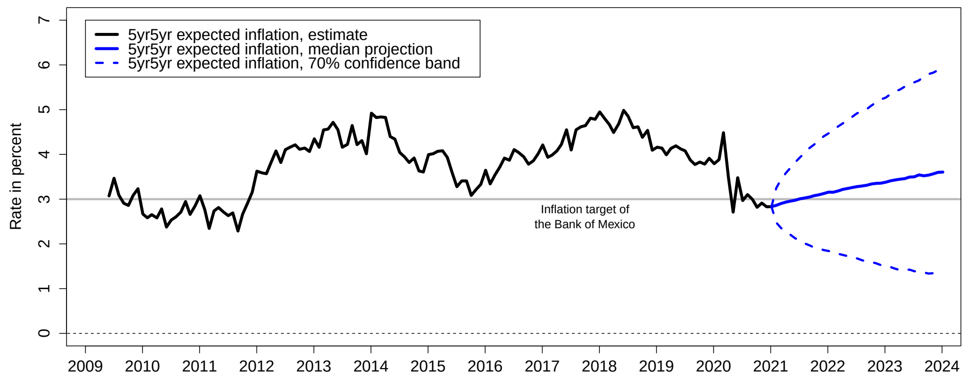


Figure 18: **Three-Year Projections of 5yr5yr Expected Inflation**

Figure 17 shows the 5yr5yr BEI decomposition implied by our updated model. First, we note that both fitted and frictionless 5yr5yr BEI have spiked up early in the coronavirus crisis before reversing most of the increase during the second half of 2020. Importantly, the model decomposition indicates that long-term expected inflation in Mexico has declined and dropped slightly below the 3 percent target of the Bank of Mexico, and that this has been more than offset by a sharp spike in the inflation risk premium, leaving it close to 150 basis points at the end of 2020. Thus, the premium on inflation uncertainty in Mexico has increased notably during the last year of our sample.

This raises the question of whether long-term inflation expectations are likely to remain low in Mexico going forward. To assess the outlook for long-term inflation expectations based on the updated $G^{X^N,X^R}(7)$ model, we follow the approach of Christensen et al. (2015) and simulate 10,000 factor paths over a three-year horizon, conditioned on the shapes of the nominal and real yield curves and investors' embedded forward-looking expectations as of the end of December 2020 (that is, using estimated state variables and factor dynamics as of December 31, 2020). The simulated factor paths are then converted into forecasts of 5yr5yr expected inflation. Figure 18 shows the median projection and the 15th and 85th percentile values for the simulated 5yr5yr expected inflation over a three-year forecast horizon.[23]

The model projections indicate that the long-term inflation expectations are likely to gradually reverse their recent declines and return to a level slightly above 3 percent by the end of the three-year forecast period. This represents evidence that long-term inflation expectations have remained well anchored in Mexico, despite the dramatic temporary downturn in domestic and global economic activity during the early stages of the COVID-19 pandemic.

[23]Note that the lines do not represent paths from a single simulation run over the forecast horizon; instead, they delineate the distribution of all simulation outcomes at a given point in time.

9 Conclusion

In this paper, we introduce a flexible joint model of nominal and real yields that accounts for liquidity risk premia in both nominal and real bond prices. We estimate the model on a representative sample of nominal and real bond prices from Mexico. This allows us to be the first to provide estimates of the liquidity-adjusted frictionless BEI in a major emerging market economy, along with its decomposition into investors' underlying inflation expectations and associated inflation risk premia.

Our results indicate that long-term inflation expectations in Mexico appear to have remained well anchored during our entire sample period at a level close to the 3 percent inflation target of the Bank of Mexico. Furthermore, inflation risk premia in Mexico are larger and more volatile than matching estimates from Canada and the United States.

An update through the early stages of the coronavirus pandemic show that long-term inflation expectations in Mexico were temporarily depressed and fell below the 3 percent inflation target of the Bank of Mexico. However, model projections indicate that they are likely to return to pre-pandemic levels slightly above 3 percent within less than three years. We take this as a further sign that long-term inflation expectations are indeed well anchored in Mexico.

Finally, we feel compelled to stress that our model framework can be applied to other emerging market economies with established nominal and real bond markets such as Brazil, Chile, and Colombia, among many others. We leave those applications for future research.

References

Abrahams, Michael, Tobias Adrian, Richard K. Crump, Emanuel Moench, and Rui Yu, 2016, "Decomposing Real and Nominal Yield Curves," *Journal of Monetary Economics*, Vol. 84, 182-200.

Andreasen, Martin M., Jens H. E. Christensen, and Simon Riddell, 2020, "The TIPS Liquidity Premium," Working Paper 2017-11, Federal Reserve Bank of San Francisco.

Andreasen, Martin M., Jens H. E. Christensen, and Glenn D. Rudebusch, 2019, "Term Structure Analysis with Big Data: One-Step Estimation Using Bond Prices," *Journal of Econometrics*, Vol. 212, 26-46.

Bauer, Michael D., Glenn D. Rudebusch, and Jing (Cynthia) Wu, 2012, "Correcting Estimation Bias in Dynamic Term Structure Models," *Journal of Business and Economic Statistics*, Vol. 30, No. 3, 454-467.

Carriero, Andrea, Sarah Mouabbi, and Elisabetta Vangelista, 2018, "U.K. Term Structure Decompositions at the Zero Lower Bound," *Journal of Applied Econometrics*, Vol. 33, 643-661.

Chernov, Mikhail and Philippe Mueller, 2012, "The Term Structure of Inflation Expectations," *Journal of Financial Economics*, Vol. 106, No. 2, 367-394.

Chiquiar, Daniel, Antonio E. Noriega, and Manuel Ramos-Francia, 2010, "A Time-Series Approach to Test a Change in Inflation Persistence: the Mexican Experience," *Applied Economics*, Vol. 42, 3067-3075.

Christensen, Jens H. E., 2008, "The Corporate Bond Credit Spread Puzzle," *FRBSF Economic Letter*, 2008-10 (March 14).

Christensen, Jens H. E., Francis X. Diebold, and Glenn D. Rudebusch, 2011, "The Affine Arbitrage-Free Class of Nelson-Siegel Term Structure Models," *Journal of Econometrics*, Vol. 164, No. 1, 4-20.

Christensen, Jens H. E., Eric Fischer, and Patrick J. Shultz, 2019, "Bond Flows and Liquidity: Do Foreigners Matter?," Working Paper 2019-08, Federal Reserve Bank of San Francisco.

Christensen, Jens H. E., Eric Fischer, and Patrick J. Shultz, 2020, "Emerging Bond Markets and COVID-19: Evidence from Mexico," *FRBSF Economic Letter*, 2016-11 (April 4).

Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, 2010, "Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields," *Journal of Money, Credit and Banking*, Supplement to Vol. 42, No. 6, 143-178.

Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, 2015, "A Probability-Based Stress Test of Federal Reserve Assets and Income," *Journal of Monetary Economics*, Vol. 73, 26-43.

Christensen, Jens H. E. and Glenn D. Rudebusch, 2019, "A New Normal for Interest Rates? Evidence from Inflation-Indexed Debt," *Review of Economics and Statistics*, Vol. 101, No. 5, 933-949.

Christensen, Jens H. E., Glenn D. Rudebusch, and Patrick J. Shultz, 2020, "Accounting for Low Long-Term Interest Rates: Evidence from Canada," Working Paper 2020-35, Federal Reserve Bank of San Francisco.

Dai, Qiang and Kenneth J. Singleton, 2000, "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, Vol. 55, No. 5, 1943-1978.

D'Amico, Stefania, Don H. Kim, and Min Wei, 2018, "Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices," *Journal of Financial and Quantitative Analysis*, Vol. 53, No. 1, 243-268.

De Pooter, Michiel, Patrice Robitaille, Ian Walker, and Michael Zdinak, 2014, "Are Long-Term Inflation Expectations Well Anchored in Brazil, Chile, and Mexico?," *International Journal of Central Banking*, Vol. 10, No. 2, 337-400.

Duffee, Gregory R., 2002, "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, Vol. 57, No. 1, 405-443.

Espada, Josué Fernando Cortés, Manuel Ramos-Francia, and Alberto Torres García, 2008, "An Empirical Analysis of the Mexican Term Structure of Interest Rates," Working paper 2008-07, Bank of Mexico.

Fleckenstein, Mathias, Francis A. Longstaff, and Hanno Lustig, 2014, "The TIPS-Treasury Bond Puzzle," *Journal of Finance*, Vol. 69, No. 5, 2151-2197.

Grishchenko, Olesya V. and Jing-Zhi Huang, 2013, "Inflation Risk Premium: Evidence from the TIPS Market," *Journal of Fixed Income*, Vol. 22, No. 4, 5-30.

He, Zhiguo, Stefan Nagel, and Zhaogang Song, 2020, "Treasury Inconvenience Yields during the COVID-19 Crisis," Working Paper 2020-79, Booth School of Business, University of Chicago.

Holston, Kathryn, Thomas Laubach, and John C. Williams, 2017, "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, Vol. 108, 559-575.

Hördahl, Peter and Ilhyock Shim, 2020, "EME Bond Portfolio Flows and Long-Term Interest Rates during the COVID-19 Pandemic," *BIS Bulletin*, No. 18, 1-9.

Hördahl, Peter and Oreste Tristani, 2012, "Inflation Risk Premia in the Euro Area and the United States," *Journal of the European Economic Association*, Vol. 10, No. 3, 634-657.

Hördahl, Peter and Oreste Tristani, 2014, "Inflation Risk Premia in the Euro Area and the United States," *International Journal of Central Banking*, Vol. 10, No. 3, 1-47.

Houweling, Patrick, Albert Mentink, and Ton Vorst, 2005, "Comparing Possible Proxies of Corporate Bond Liquidity," *Journal of Banking and Finance*, Vol. 29, 1331-1358.

Hu, Grace Xing, Jun Pan, and Jiang Wang, 2013, "Noise as Information for Illiquidity," *Journal of Finance*, Vol. 68, No. 6, 2341-2382.

Joyce, Michael, Peter Lildholdt, and S. Sörensen, 2010, "Extracting Inflation Expectations and Inflation Risk Premia from the Term Structure: A Joint Model of the UK Nominal and Real Yield Curves," *Journal of Banking and Finance*, Vol. 34, 281-294.

Kim, Don H. and Athanasios Orphanides, 2012, "Term Structure Estimation with Survey Data on Interest Rate Forecasts," *Journal of Financial and Quantitative Analysis*, Vol. 47, No. 1, 241-272.

Kim, Don H. and Kenneth J. Singleton, 2012, "Term Structure Models and the Zero Bound: An Empirical Investigation of Japanese Yields," *Journal of Econometrics*, Vol. 170, No. 1, 32-49.

Laubach, Thomas and John C. Williams, 2016, "Measuring the Natural Rate of Interest Redux," *Business Economics*, Vol. 51, No. 2, 57-67.

Nagel, Stefan, 2016, "The Liquidity Premium of Near-Money Assets," *Quarterly Journal of Economics*, Vol. 131, No. 4, 1927-1971.

Nelson, Charles R. and Andrew F. Siegel, 1987, "Parsimonious Modeling of Yield Curves," *Journal of Business*, Vol. 60, No. 4, 473-489.

Rudebusch, Glenn D. and Eric Swanson, 2011, "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journals: Macroeconomics*, Vol. 4, No. 1, 105-43.

Sack, Brian and Robert Elsasser, 2004, "Treasury Inflation-Indexed Debt: A Review of the U.S. Experience," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 10, No. 1, 47-63.

Solís, Pavel, 2020, "Monetary Policy in Mexico: The Effects of Actions and Statements on Asset Prices and Portfolio Flows," Manuscript, Johns Hopkins University.