The Term Structure of Expectations

Richard K. Crump | Stefano Eusepi | Emanuel Moench |
Bruce Preston

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Abstract

Economic theory predicts that intertemporal decisions depend critically on expectations about future outcomes. Using the universe of professional survey forecasts for the United States, we document the behavior of the entire term structure of expectations for output growth, inflation, and the policy rate. We show that a simple unobserved components model of the trend and cycle explains the joint behavior of both consensus measures of expectations and the observed disagreement among individual forecasters. Importantly, univariate models of each variable are outperformed by a multivariate model of the joint dynamics of these three variables, particularly for nominal interest rates. Consistent with the data, the model predicts a link between revisions in long-run expectations to short-term forecast errors. In structural models, learning about the long run has important empirical and theoretical implications for monetary and fiscal policy.

Key words: expectation formation, imperfect information, survey forecasts, shifting endpoint models, monetary policy, term premiums

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Crump: Federal Reserve Bank of New York (email: richard.crump@ny.frb.org). Eusepi: University of Texas at Austin (email: eusepi73@yahoo.com). Moench: Deutsche Bundesbank, Goethe University, and CEPR (email: emanuel.moench@bundesbank.de). Preston: University of Melbourne (email: bruce.preston@unimelb.edu.au). This paper is a draft chapter for the *Handbook of Economic Expectations*, Volume 1, edited by R. Bachmann, G. Topa, and W. van der Klaauw. Oliver Kim and Nick Ritter provided excellent research assistance.

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Contents

1 Introduction

Economic theory predicts that intertemporal decisions depend critically on expectations about future outcomes. Over the past two decades, a concerted research program measures household, firm and policymaker beliefs using numerous data sources, including surveys, asset prices and controlled experiments. By dint of this effort, we have invaluable data that can be used to evaluate alternative theories of expectations formation and their implications for macroeconomics and finance.

Yet the vast majority of this work has focused on expectations about *short-term* economic developments. This choice is partly driven by what data are available, as there is substantially less information on long-run forecasts. But it also reflects the common assumption in macroeconomic models that economic agents operate in a stationary environment and, consequently, that they can quickly and efficiently come to understand the long-run behavior of the economy. Any information frictions that might be relevant to the expectation formation process, are only relevant to short-run economic dynamics.

These assumptions, however, belie the considerable uncertainty that confronts decision makers in practice. Indeed, direct survey evidence clearly reveals that expectations about the *long-run* values of economic and financial variables vary over time. For example, the Survey of Professional Forecasters annually queries respondents on their value of the non-accelerating inflation rate of unemployment, the Federal Reserve Bank of New York's Survey of Primary Dealers includes questions on "longer-run" values of economic variables such as output, inflation and the target interest rate, and the FOMC members themselves report, in the Survey of Economic Projections, the value that key macroeconomic variables would be expected to converge to under appropriate monetary policy and in the absence of further shocks to the economy. All of these long-run forecasts display substantial variation over time.

Movements in long-term expectations are not without consequence. Prominent debates in macroeconomics and finance rest on the nature of the long-term behavior of the economy. The seminal contribution of Lucas (2003) argued that the economic costs of short-term fluctuations pale in comparison to the implications of long-run growth, underscoring the need to study long-term expectations and their impact on economic decisions. Among academics and policymakers there is widespread agreement that the ability of central banks and fiscal authorities to manage business cycles depends on the maintenance of long-term fiscal sustainability and stable long-run inflation expectations. And a growing literature in finance understands movements in asset prices by linking them to changes in perceived long-run risk, again highlighting the need to understand market participants' shifting views about the long run (e.g., Bansal and Yaron (2004)).

In this chapter we use survey measures of U.S. professional forecasters. One key advantage of using professional forecasts is the wealth of available data in the U.S. and other countries. Multiple surveys covering a wide range of forecast horizons spanning "nowcasts" to the very long run are available. And unlike the growing number of new surveys of households and firms that have become available to researchers only in recent years, data on professional forecasts have been collected since at least the mid-1950s. Using these data we document the evolution of the entire term

structure of expectations since the 1980s and propose a simple expectations formation mechanism that rationalizes their behavior. Armed with this framework, we evaluate some implications in a standard New Keynesian dynamics general equilibrium model.

We show professional forecast data display three important stylized facts. First, long-run expectations about economic variables such as output growth, inflation and short-term nominal interest rates fluctuate significantly over time, tracking perceived slow-moving changes in the economy such as the long-run mean of inflation or the natural rate of interest.

Second, the individual components of the term structure of expectations display a clear pattern of co-movement across different variables and forecast horizons. Changes in long-term expectations are tied to short-term forecast errors, consistent with an expectations formation mechanism where agents estimate unobserved trend and cycle components from available data. At the same time, agents appear to form expectations about macroeconomic variables jointly, so that, for example, policy rate forecasts are tightly linked to inflation and output growth forecasts.

Third, individual long-term forecasts show a high degree of dispersion for all variables considered and for all forecasting horizons. This is consistent with economic agents facing fundamental uncertainty about the long-run behavior of the economy. For example, market participants disagree more about the long-run determinants of the policy rate, while they display fairly uniform views about short-term policy expectations.

Throughout the paper we use a model of expectations formation that is consistent with these observations. While we discuss the literature on the term structure of expectations throughout the chapter, our primary aim is not to provide an exhaustive summary of existing work. Recent research covers a wide range of theories that can potentially account for some of the empirical regularities in survey data. Here we focus on a specific class of information friction, and, therefore, a specific modeling approach, and discuss its implications for different aspects of the data. On empirical grounds, we introduce novel data sources together with a number of new empirical results which shine light on the behavior of expectations across forecast horizons.

This chapter is structured in five broad sections. Section 2 introduces our workhorse model of the expectations formation mechanism and the information frictions at its foundations. This is an unobserved components model of the trend and cycle which agents estimate using standard filtering methods. We then discuss novel survey evidence in support of these assumptions. Using a cross-section of professional forecasters we document how individual long-term expectations are revised partly in response to recent forecast errors.

In Section 3 we present a parsimonious monthly vector autoregression model with a time-varying long-run mean. The model captures the key aspects of our theory and accounts for the joint term structure of *consensus* expectations of output growth, inflation and the policy rate. The forecast data are measured from the universe of professional forecasts for the United States in the post-war era. We establish that a drifting long-run mean is essential to capture the low-frequency adjustment in long-run beliefs. Moreover, the multivariate model provides a far superior fit when compared to a univariate model specification for each variable. This suggests that the dynamic behavior of

survey forecasts of different macroeconomic variables need to be modeled jointly. Existing studies often focus on expectations about an individual variable.

The empirical model delivers tightly estimated forecast paths for these variables at each point in time over the sample from 1983 through 2019. This novel measure of consensus expectations enables us to track the evolution of the term structure of expectations since the mid-1980s. For example, we document the evolution of nominal and real expected interest rates over the monetary cycle; the adjustment of interest rate expectations in the aftermath of the financial crisis; and the gradual decline of the perceived natural rate of interest over the past ten years.

Section 4 focuses specifically on the term structure of interest rates, the primary component of the monetary policy transmission mechanism. We use our measure of expectations to evaluate the expectations hypothesis, stating that yields on government bonds reflect the average short rate that investors expect to prevail over the life of the bond. We compare the behavior of the term structure of consensus expectations with that of the term structure of interest rates derived from the U.S. Treasury yield curve. Despite the observed volatility of expectations, there remains substantial unexplained variation at the long-end of the yield curve. We obtain the term premium as the residual between observed yields and average expected future short rates. The survey-based measure of the term premium does not co-move in any meaningful way with the term structure of expectations. This finding begs research on how term premia transmit changes in the stance of monetary policy.

While having a measure of consensus expectations is an important contribution in its own right and useful in many economic applications, it neglects the wide dispersion in individual forecasts documented for professional forecasters, households, firms and policymakers. The heterogeneity in information can play an important role in explaining the aggregate behavior of the macroeconomy and asset prices. Section 5 uses data on individual professional forecasters to measure the term structure of disagreement, or the average disagreement about output growth, inflation and the policy rate at different forecast horizons. We show that our modeling framework is broadly consistent with the behavior of both the consensus measure and the cross-section of professional forecasts.

Turning our attention back to the term structure of interest rates, we use our model of expectations formation to investigate the factors behind the observed dispersion in interest rate forecasts, especially in the long run. In addition, we show a connection between forecasters' disagreement about the path of the policy rate and our measure of the term premium and discuss the implications for asset pricing in a term structure model that allows explicitly for long-run forecast dispersion.

Most of the analysis conducted in this chapter is based on a reduced-form model of the expectations formation process. This has the advantage of sidestepping detailed assumptions about information frictions and, in particular, taking a stand on the rationality of expectations. There are notable advantages, however, to a more structural approach. Dynamic general equilibrium models incorporating specific deviations from rationality have greatly helped improving our understanding of business cycles, asset prices and inflation dynamics. Moreover, structural models are required to address key questions of monetary and fiscal policy design under different assumptions about how

expectations are formed.

Section 6 presents a dynamic structural general equilibrium model where agents are boundedly rational and have to learn about a possibly changing economic environment. Subjective beliefs of households and firms are consistent with our reduced-form forecasting model based on survey evidence. However, the structural model assumes a specific deviation from the full information rational expectations setup: subjective and objective (model consistent) forecasting models differ. In particular, subjective beliefs are more persistent than the true data generating process. Household and firm expectations exhibit extrapolation bias, consistent with empirical and laboratory evidence. Having a structural theory of long-term expectations permits analysis of important practical policy questions. For example, we argue that our framework provides a coherent definition of anchored expectations, and clear predictions of the economic conditions under which expectations will be anchored or unanchored. Finally we discuss the implications of this expectations formation mechanism for monetary and fiscal policy.

2 Short-Term and Long-Term Forecasts: New Stylized Facts

In this section, we present a simple model of expectation formation and introduce a novel dataset of individual professional forecasters which provides evidence in support of the proposed mechanism. The key insight of the model—which will resonate throughout the chapter—is that agents revise their beliefs about both the trend and the cyclical components of macroeconomic variables in response to short-term forecast errors. As a result, unanticipated short-term innovations may drive the entire term structure of macroeconomic expectations.

2.1 Motivation: A Simple Model of Long-term Drift

Market participants observe a wealth of data about the current state of the economy. These data provide signals both about short-term economic developments as well as longer-run trends. Forming expectations about economic variables at different horizons into the future therefore requires decomposing the data into transitory and persistent components. Such decompositions have a long tradition in theoretical and empirical macroeconomic research. For instance, the seminal real-business cycle model in Kydland and Prescott (1982) assumes agents cannot perfectly observe the short- and long-term components of technical progress. Stock and Watson (1989) and Stock and Watson (2007) model inflation as having a trend and a transitory component. This approach has also been incorporated in countless structural models of inflation dynamics of which Cogley et al. (2010) is a prominent example. Various studies apply trend-cycle decompositions to other macroeconomics variables, showing that models which embed slow-moving time-varying drifts capture the dynamics properties of real GDP growth (Stock and Watson (1989), Cogley and Sargent 2005 and Laubach and Williams 2003) and the federal funds rate (Kozicki and Tinsley 2001 and Gürkaynak et al. (2005)) well.

We follow this literature in our analysis of the term structure of economic expectations. Our

approach embeds a key information friction that determines how new information is incorporated into expectations at different horizons. We argue this model provides an impressive account of economic expectations which we measure using survey data from professional forecasts.

2.1.1 Modeling a Drift in the Long-run Mean

Consider forecasting the variable z_t using the model

$$z_t \;=\; \omega_t + x_t \tag{2.1}$$

where

$$\omega_t \;=\; \omega_{t-1} + \epsilon_t^\omega \tag{2.2}$$

$$x_t \;=\; \phi x_{t-1} + \epsilon_t^x \tag{2.3}$$

with $0 < \phi < 1$ and ϵ_t^x and ϵ_t^ω both *i.i.d.* Gaussian innovations. The variables x_t and ω_t are *unobserved* by the forecaster. While x_t captures a stationary cyclical or business-cycle component of z_t, ω_t represents a slow-moving trend or drift. This could be the underlying productivity trend of the economy, the implicit or explicit inflation target of the central bank, or the long-term drift in the natural rate of interest. This trend is assumed to be non-stationary but a sufficiently persistent process would deliver essentially the same dynamics. Kozicki and Tinsley (2001) labelled the non-stationary case a 'shifting endpoint' model.

Observing z_t at time t, agents estimate the trend and cycle components, $\omega_{t|t}$ and $x_{t|t}$, using the Kalman filter. The expected value of z_t for any horizon $T > t$ is then

$$E_t z_T \equiv z_{T|t} \;=\; \omega_{t|t} + \phi^{T-t} x_{t|t},$$

where the persistent and cyclical components satisfy

$$\omega_{T|t} = \omega_{t|t} \;=\; \omega_{t-1|t-1} + \nu_t, \tag{2.4}$$

$$x_{T|t} = \phi^{T-t} x_{t|t} \;=\; \phi^{T-t} \times \left[\phi x_{t-1|t-1} + \eta_t \right], \tag{2.5}$$

where $\eta_t = \kappa_\omega \left(z_t - z_{t|t-1} \right)$ and $\nu_t = \kappa_x \left(z_t - z_{t|t-1} \right)$ are innovations measuring the forecast 'surprises'. These surprises are given by the one-step ahead or short-term forecast error scaled by the Kalman gain coefficients κ_ω and κ_x. The size of the Kalman gains depends on the relative volatility of the innovations in the trend component and the persistence of the stationary process (e.g., Hamilton 1994). Given the slow-moving nature of the trend component, κ_ω is assumed to be relatively small.

We explore three implications of this model in the data. First, the model parameters forge a tight connection among forecasts at different horizons. For example, we show that the term structure of inflation forecasts is consistent with a random-walk behavior, so that $\phi = 0$. The

<div align="center">5</div>

entire term structure of inflation expectations shifts in response to revisions in the estimate $\omega_{t|t}$. In contrast, interest rate forecasts at short horizons largely reflect a persistent cyclical component, while long-term forecasts are tied to the drift component.

Second, the model implies a tight connection between long-run forecasts and short-term forecast errors. To see this, for a forecast horizon $T^* > t$ sufficiently large that the cyclical component becomes unimportant, that is $\phi^{T^*-t} \approx 0$, we have

$$E_t z_{T^*} \quad \approx \quad \omega_{t|t}.$$

Using the law of motion for the estimated trend component in (2.4), the change in longer-term forecasts is tied to short-term forecast errors or surprises

$$E_t z_{T^*} - E_{t-1} z_{T^*} \approx \omega_{t|t} - \omega_{t|t-1} \quad = \quad \kappa_\omega \left(z_t - z_{t|t-1} \right).$$

A forecaster that under-predicts z_t for few periods should thus revise upwards their long-term forecast, reflecting a perceived increase in the estimated unobserved drift component.

Third, the model predicts a strong correlation, in this simple example perfect, between the updates to the trend and cycle components, as they both depend on the same forecast error: $z_t - z_{t|t-1}$.

2.1.2   A More General Setup

In practice, survey data on economic expectations might reflect information which is not directly associated with news about transitory or trend components of the predicted variables. When confronting the model with survey data we therefore consider three generalizations of the simple model: (i) the forecaster additionally receives signals about low-frequency developments in the data; (ii) the forecaster observes data with noise; and (iii) the survey forecast data are measured with noise. While these generalizations all attenuate the tight link between long- and short-term forecasts implied by the simple model above, the key mechanism remains intact: revisions of short- and longer-term forecasts comove with short-term forecast errors.

Market participants use many macroeconomic releases to extract signals about the different components of z_t. These *signals* can arise from common data sources such as monthly economic indicators, but they can also result from policy announcements or other policy communications. Such signals are likely to impact expectations in ways that are not captured by our simple model. Importantly, they are not necessarily observed by an econometrician attempting to capture the expectations formation process and can obscure the relation between observed long-term expectations and short-term forecast errors.

Suppose the forecaster receives a noisy signal on the unobserved trend

$$s_t^\omega \quad = \quad \omega_t + o_t$$

where o_t denotes uninformative *i.i.d.* Gaussian noise. Forecast updating now depends on two observables, s_t^ω and z_t, weakening the link between updates in longer-term forecasts and observed short-term forecast errors. Revisions to long-dated expectations are

$$E_t z_{T^*} - E_{t-1} z_{T^*} \quad \approx \quad \kappa_{\omega,z} \left(z_t - z_{t|t-1} \right) + \kappa_{\omega,s} \left(s_t^\omega - s_{t|t-1}^\omega \right).$$

Furthermore, the presence of such signals weakens the correlation between "innovations" ν_t and η_t as they are now represented by different linear combinations of forecast errors and signals.

A common additional informational friction is that the current state z_t is not fully observed by forecasters. Instead, agents may have access to a noisy signal $s_t^z = z_t + o_t^z$, leading to the updating equation

$$E_t z_{T^*} - E_{t-1} z_{T^*} \quad \approx \quad \kappa_{\omega,z} \left(s_t^z - z_{t|t-1} \right) = \kappa_{\omega,z} \left(z_t - z_{t|t-1} \right) + \kappa_{\omega,z} o_t^z.$$

Again, the link between forecast errors observed by the econometrician and revisions of longer-run forecasts is weaker than in the simple model. A different interpretation of the above is that forecasters use "judgment" when forming forecasts, especially in the short-term. For example, we can re-write the forecast as

$$\tilde{E}_t z_T \quad = \quad E_t z_T + \gamma^{T-t} \tau_t$$

where for simplicity τ_t is described as a first-order autoregressive process, with $0 < \gamma < 1$, capturing any adjustment to short-term forecasts. In this case

$$E_t z_{T^*} - E_{t-1} z_{T^*} \approx \kappa_{\omega,z} \left(z_t - z_{t|t-1} \right) \quad = \quad \kappa_{\omega,y} \left(z_t - \tilde{z}_{t|t-1} \right) + \kappa_{\omega,z} \tau_t$$

where $\tilde{z}_{t|t-1}$ is the forecast observed by the econometrician inclusive of judgment. However, the long-term forecast is computed using the model and does not include the judgment component. This process also breaks the perfect correlation between observed long-term forecast revisions and short-term forecast errors.

Finally, survey-based forecasts are informative about the expectation formation process, but are likely to be measured with error, further complicating the econometrician's inference problem. This is discussed in more detail in Section 3.

2.2 Some New Stylized Facts on Individual Forecasts

2.2.1 Properties of Long-Run Forecasts

The model sketched above provides a simple framework to think about the evolution of agents' forecasts at different horizons. However, because of data limitations there is scant evidence on individuals' term structure of expectations, particularly in the context of long-run forecasts. Most surveys that ask respondents about their beliefs far in the future aggregate the individual survey

responses and provide only limited summary statistics.[1] Here we exploit a unique data set of individual long-run forecasts from the Blue Chip Economic Indicators (BCEI) survey, which provides the individual responses to the "Long-Range Consensus U.S. Economic Projections" summarized in the March and October issues. Our data cover the sample period from 1998 to 2016 and all of the variables queried by the BCEI. We focus on the following set of key macroeconomic indicators: nominal GDP, real GDP, CPI inflation, the 3-month Treasury Bill rate, and the real 3-month rate defined as the difference between the nominal rate and CPI inflation. From October 1998 and March 2006, we can link these individual forecasts to the associated short-run forecasts. However, outside this data range, we can only link forecasters across variables and horizon, not across survey dates. Nonetheless, the richness of these data allow us to introduce a number of new stylized facts about individual long-run forecasts.

To start, Figure 1 shows the consensus (i.e., cross-sectional mean) forecast for two selected horizons based on the BCEI survey, namely, three-years ahead and the average of seven-to-eleven years ahead. The sample period covers the 40 years ending in 2019. In these charts we observe two key features of these forecasts. First, the forecasts for the two horizons broadly comove, with the three-year ahead forecast generally displaying more high-frequency variability. Second, the longest-horizon forecast, predicting these key economic variables seven-to-eleven years in the future, clearly varies over time. For example, the long-horizon consensus forecast for real output growth varies between more than 3% and a bit below 2.5%, and then falls to below 2% after the Global Financial Crisis (top left chart). More dramatically, the long-run forecast for CPI inflation and the 3-month T-bill exhibit strong secular declines throughout the sample (top right and lower left chart). Despite sharing that broad-based secular decline, their difference, the real 3-month T-bill, shows variation throughout and ends the sample at about 0%. In sum, Figure 1 provides clear evidence that long-horizon forecasts show meaningful variation over time, in line with forecasters updating their estimates of the trend components of the observed data.

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[1]For example, both the Blue Chip Financial Forecasts and the BCEI present the cross-sectional average along with the average of the top-10 forecasters and bottom-10 forecasters at each forecast horizon. As another example, the Survey of Primary Dealers (SPD) provides the cross-sectional median along with the 25th and 75th percentiles.

Figure 1: **Long-Horizon Professional Forecasts.** This figure shows the time series for two long-horizon forecasts (three-years ahead and seven-eleven years ahead) from the Blue Chip Economic Indicators (BCEI) for a selection of economic indicators. The sample period is from November 1979 to December 2019.

(a) *Real GNP/GDP Growth*

(b) *CPI Inflation*

(c) *3-Month T-Bill*

(d) *Real 3-Month T-Bill*
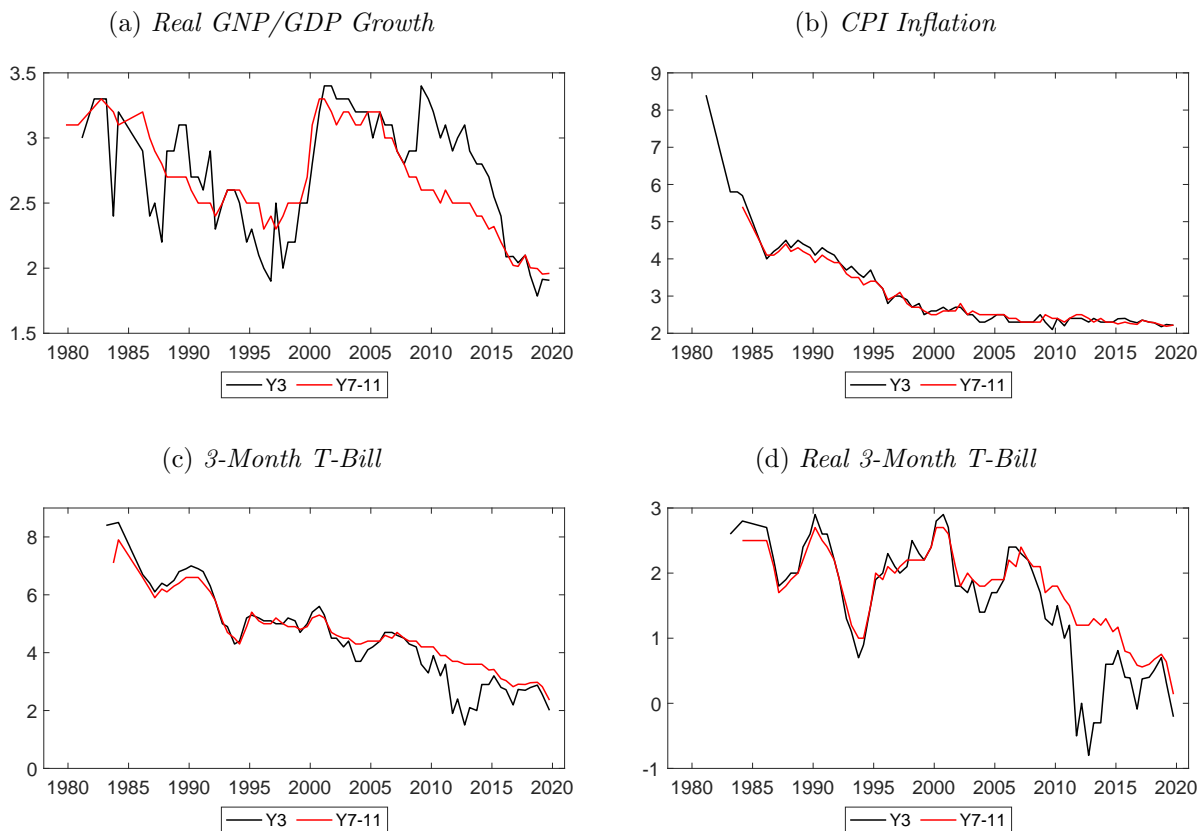
To provide further evidence in favor of a time-varying, low-frequency component in survey forecasts, Figure 2 shows that forecasters jointly update their expectations across forecasts of different horizons. Specifically, we calculate the cross-sectional rank correlation between forecasts of adjacent horizons across individual forecasters at each point in time. We then take the time series average over the sample to produce a term structure of rank correlations. Across all variables, there is a clear upward shape of the term structure with rank orderings between adjacent forecast horizons becoming more and more stable as the horizon increases. This provides evidence of the presence of a long-run component in forecasts at the individual forecasters level.

Figure 2: **Forecast Rank Correlations of Adjacent Horizons.** This figure shows the time series average of cross-sectional rank correlations of forecasts across adjacent forecast horizons. The sample period is 1998 to 2016, with the exception of the "0/1 year" and "1/2 year" pairs which are restricted to the period from 1998 to 2006.

(a) *Real GNP/GDP Growth*

(b) *CPI Inflation*

(c) *3-Month T-Bill*

(d) *Real 3-Month T-Bill*
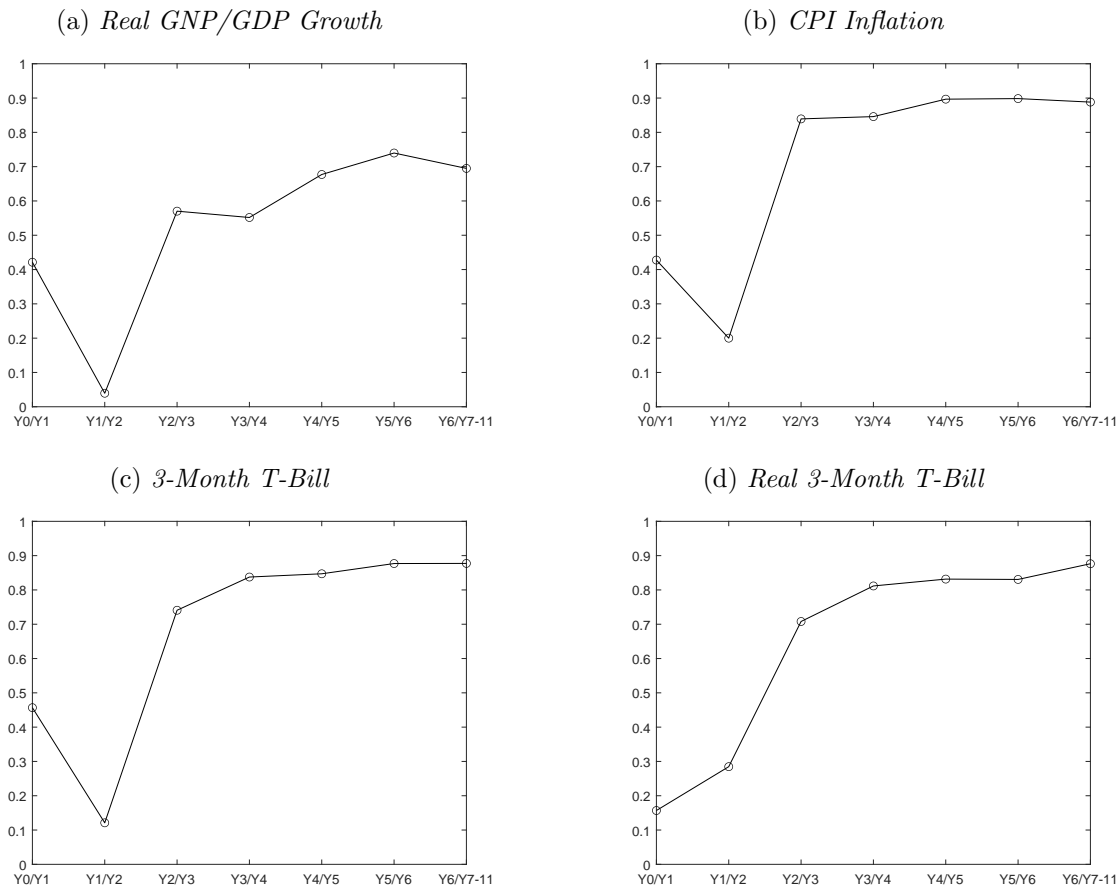
2.2.2 Long-Run Forecast Revisions and Short-Run Forecast Errors

The benchmark model introduced in Section 2.1 implies that revisions of long-horizon forecasts should be strongly related to short-run forecast errors. We can use our unique panel data set of BCEI forecasts to investigate this relation in the data. For the period between 1998 and 2006 we can construct an unbalanced panel across forecasters in the BCEI survey for forecast horizons of the current year along with forecasts one-year, two-year, three-year, four-year, five-year, six-year and seven-to-eleven years ahead. We estimate the following panel regressions,

$$\Delta \text{Exp}_{t+h|t}^i = \alpha_i + \beta \cdot \text{STFCSTERROR}_t^i + \epsilon_{it}. \tag{2.6}$$

To do so, we use data on forecasts from adjacent years to match calculated short-term forecast errors and long-term forecast revisions. Specifically, we construct short-term forecast errors by restricting ourselves to October data only and using the one-year ahead forecast from the previous

October to the current "nowcast" for the contemporaneous year.[2] To construct the change in long-horizon forecasts, we simply take the difference between the current forecast and that of the previous October. Thus, we can use the year pairs, $\{1998/1999\}, \{1999/2000\}, \cdots, \{2004/2005\}$ for a total of $T = 7$ pairs. There are 48 forecasting firms in the data, but not all firms forecast for each year and may not provide forecasts for all variables. In sum, there are a total of 164 observations at the year-firm level.

Figure 3: **Forecast Errors and Forecast Revisions.** This figure shows scatterplots of short-term forecast errors against revisions in long-run forecasts based on the BCEI data. The sample period is October 1998 to October 2005.

(a) *Real GNP/GDP Growth*

(b) *CPI Inflation*

(c) *3-Month T-Bill*

(d) *Real 3-Month T-Bill*
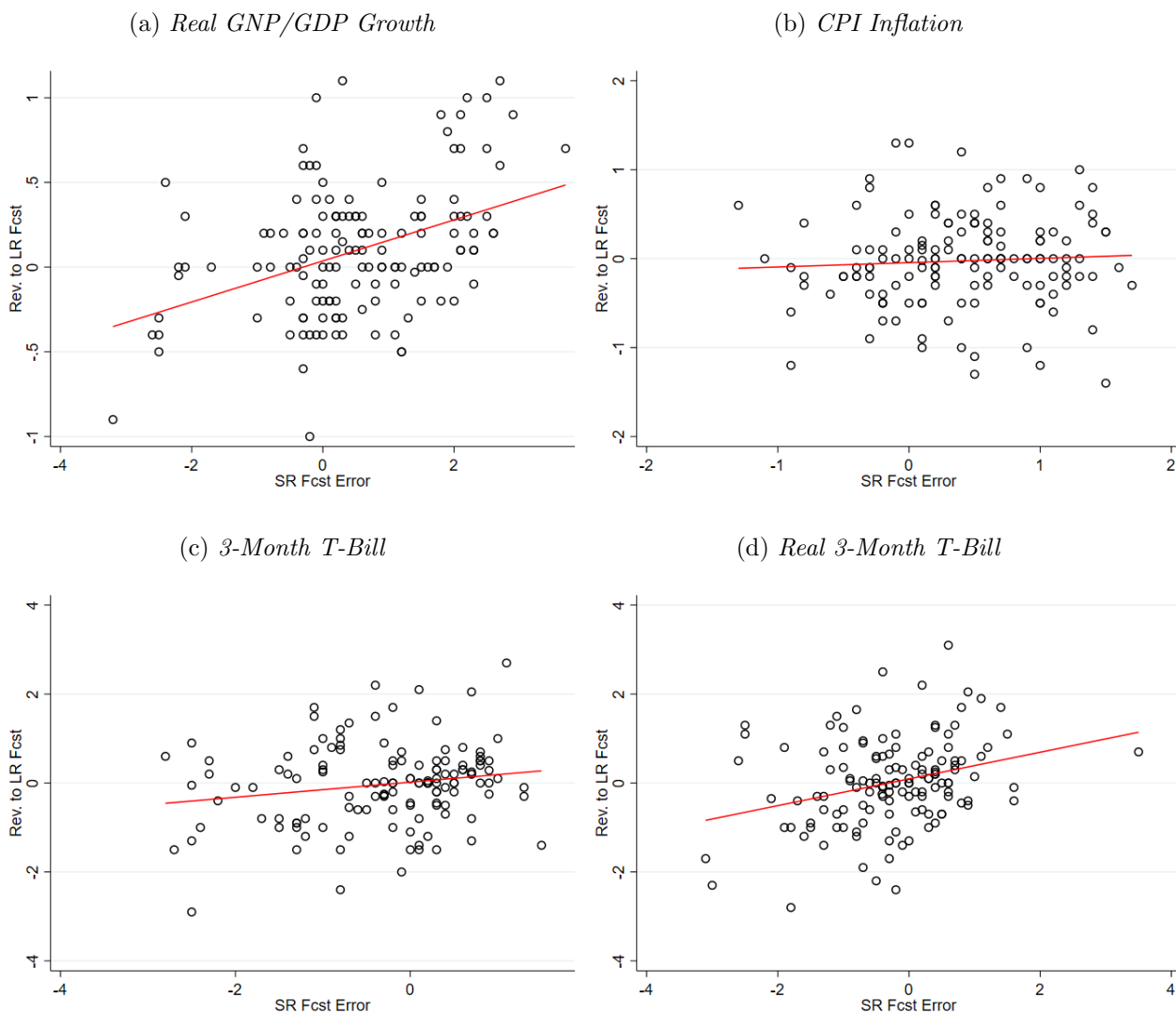
Figure 3 displays scatterplots of short-run forecast errors against forecast revisions of long-horizon

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[2]Strictly speaking, not all data have been realized by October, and so these are not exact forecast errors. However, the influence of these observations is small (see Crump et al. 2014). As a robustness check, we added time fixed effects to our regression specification and find they do not change the qualitative results.

forecasts using these individual BCEI panel data for the same four variables as in Figures 1 and 2. We superimpose the fitted OLS regression line in each of the four scatterplots. In all cases except for CPI inflation, there is a clear positive correlation between forecast errors and forecast revisions. Moreover, none of the scatters appear to have notable outliers that might unduly influence estimated relationships between the two variables.

To provide further evidence of the link between individual forecast revisions at long horizons and short-term forecast errors, we estimate 16 regression specifications, one for each of the 15 variables available in our data set, along with the implied forecast for the real 3-month T-bill which we construct as the difference between the nominal 3-month T-bill and quarterly CPI inflation forecasts. Table 1 presents the regression results for a selected set of prominent economic indicators (see Table A.1 in the Appendix for all results). We report the estimated coefficient, $\hat{\beta}$ and the associated p-value in parentheses below. For most of the 16 considered variables, there is a significant correlation between individual forecast errors and long-term forecast revisions. For example, in the case of real output growth we observe a strongly significant coefficient $\hat{\beta}$ suggesting that a rise in the short-run forecast error of 1 percentage point is associated with about a 1/8 percentage point rise in the long-run forecast for real GDP growth. More generally, we observe that all estimated coefficients are positive and almost all are statistically significant at standard significance levels. The estimated coefficients are similar regardless of whether we account for unobserved heterogeneity using the fixed-effects estimator or not. These conclusions are essentially unchanged when evaluating the larger group of indicators available in Table A.1. A notable exception is the specification for CPI forecasts which possesses positive estimated coefficients, but sufficiently large standard errors that the p-values are all far from zero. A plausible explanation for this result is that longer-horizon inflation forecasts have been relatively well anchored and thus insensitive to incoming economic information since the late 1990s which imply a weaker observed correlation between observed long-term forecast revisions and short-term forecast errors. Carvalho et al. (2021) propose a learning model which gives rise to such an insensitivity while at the same time implying a tight link between long-term forecast revisions and short-term forecast errors from the late 1970s until the 1990s. We discuss this further in section 6.

Relation to existing literature: A few papers have explored the link between forecast revisions and forecast errors using pass-through regressions of either macroeconomic news or movements in short-term expectations to long-term expectations, see for example Gürkaynak et al. (2010) and Beechey et al. (2011). In addition, Bems et al. (2021) provide time-series evidence of this link for a large set of countries. To our knowledge, we are the first to use granular panel data to explicitly demonstrate the link between short-run forecast errors and changes in long-run forecasts at the individual forecaster level.

Table 1: **Forecast Errors and Forecast Revisions** This table presents regression results from the specification of equation (2.6). Each row presents the estimate of the coefficient β for each variable along with the associated p-value below it in parentheses. The column labelled "FE" denotes the use of the fixed-effects estimator whereas "OLS" denotes a pooled-OLS estimator. Sample sizes are presented in the rightmost column. Standard errors are clustered by year. The sample period is October 1998 to October 2005.

	OLS	FE	N (OLS/FE)
Real GNP/GDP Growth	0.1212	0.1346	142 / 135
	(0.036)	(0.059)	
Nominal GNP/GDP Growth	0.1188	0.1211	138 / 131
	(0.023)	(0.013)	
CPI	0.0480	0.0937	142 / 135
	(0.483)	(0.279)	
Three-Month T-Bill	0.1698	0.1762	134 / 127
	(0.101)	(0.138)	
Real Three-Month T-Bill	0.3003	0.3375	133 / 126
	(0.016)	(0.020)	
Unemployment	0.5017	0.4810	140 / 133
	(0.000)	(0.002)	

3   Joint Behavior of Short-Term and Long-Term Forecasts

Based on the theoretical framework and empirical evidence discussed in the previous section, we now present a parsimonious reduced-form model of the term structure of expectations for three key macroeconomic variables in the U.S. economy: output growth, inflation and the short-term nominal interest rate.[3] The model serves three purposes. First, it permits evaluating whether our simple theory of expectation formation can account for the observed dynamics of expectations across a range of forecast horizons. Second, the model matches different surveys and different types of forecasts (i.e. fixed-horizon and fixed-event) in a coherent way. It also provides consistent proxies for missing survey observations. As a result, the model enables us to construct *a consensus measure of expectations at all horizons* that avoids unduly overweighing a particular survey. Third, since we observe fewer forecasts for short-term interest rates than we do for output and inflation, the multivariate nature of the model allows us to exploit the correlation structure across variables and time horizons to inform the term structure of expectations of the short-term interest rate.

3.1   A Model to Fit the Term Structure of Expectations

3.1.1   Baseline Multivariate Model

The state of the macroeconomy is defined by the vector $z_t = (\ g_t \quad \pi_t \quad i_t \ )'$ representing monthly real output growth, inflation and the short-term nominal interest rate, respectively. They evolve

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[3]The model and the analysis are based on Crump et al. (2018).

as

$$\hat{x}_t = \Phi \hat{x}_{t-1} + \nu_t \tag{3.1}$$

$$\hat{\omega}_t = \hat{\omega}_{t-1} + \eta_t \tag{3.2}$$

$$z_t = \hat{\omega}_t + \hat{x}_t \tag{3.3}$$

where the variables $\hat{x}_t \equiv x_{t|t}$ and $\hat{\omega}_t \equiv \omega_{t|t}$ are 3×1 vectors capturing agents' estimates about the underlying unobserved states. To keep the model simple, the innovations $\varepsilon_t = (\begin{array}{cc} \nu_t & \eta_t \end{array})'$ are assumed to be $i.i.d.$ across time and are normally distributed with variance covariance Σ_ε. Consistent with the model presented in Section 2.1, innovations in the drift are potentially correlated with innovations in the cyclical components of the model. The matrix Φ measures the autocorrelation properties of the stationary component x_t and consequently has eigenvalues in the unit circle. The model is defined at the monthly frequency which is the highest frequency observed across the range of surveys of professional forecasts to which we fit the model.

3.1.2 Data Overview

We seek to model the joint term structure of expectations for real output growth, inflation, and the short-term interest rate. To do this we use the universe of professional forecasts for the United States in the post-war era, obtained from nine different survey sources: (1) Blue Chip Financial Forecasts (BCFF); (2) Blue Chip Economic Indicators (BCEI); (3) Consensus Economics (CE); (4) Decision Makers' Poll (DMP); (5) Economic Forecasts: A Worldwide Survey (EF); (6) Goldsmith-Nagan (GN); (7) Livingston Survey (Liv.); (8) Survey of Primary Dealers (SPD); (9) Survey of Professional Forecasters (SPF). We focus on three sets of forecasts. For output growth we use forecasts of real GNP growth prior to 1992 and forecasts of real GDP growth thereafter. For inflation we use forecasts of growth in the consumer price index (CPI). We choose the CPI over alternative inflation measures such as the GDP deflator because CPI forecasts are available more frequently and for a longer history than alternative inflation measures. Finally, we use the 3-month Treasury bill (secondary market) rate as our measure of a short-term interest rate as it is by far the most frequently surveyed short-term interest rate available.[4]

Combined, these surveys provide a rich portrait of professional forecasters' macroeconomic expectations. Our results are based on 627 variable-horizon pairs spanning the period 1955 to 2019. While we provide more details about each individual survey in the Appendix, the survey data differ in frequency, forecast timing, target series, sample availability and forecast horizons. To ease notation we use the following conventions. Q1 represents a one-quarter ahead forecast, Q2 represents a two-quarter ahead forecast and so on. Y1 represents a one-year ahead forecast, e.g., a forecast for the year 2014 made at any time in 2013. Y2 represents a two-year ahead forecast and so on. Y0-5 represents a forecast for the average value over the years ranging from the current year to five

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[4]For example, forecasts of the Federal Funds rate, the target rate of U.S. monetary policy are only available in two of the eight surveys we consider (BCFF and SPD).

years ahead, e.g., a forecast for the average annual growth rate of GDP from 2014 through 2019 made at any time in 2014. Y1-6, Y2-7 and so on are defined similarly. Y6-10 represents a forecast for the average value over the years ranging from six years ahead to 10 years ahead, e.g., a forecast for the average annual growth rate of GDP from 2020 through 2024, made at any time in 2014. Within each of these sub-categories the exact form of the target variable may vary. For example, a forecast for the year 2014 may be queried based on annual average growth or Q4/Q4 growth. As we make clear below, throughout the paper we ensure consistency between model-implied and observed forecasts with respect to variable definition and forecast horizon.

Table 2 summarizes the survey data we use in the paper. Near-term survey forecasts (target period is up to two years ahead) are available for the longest sample with CPI forecasts from the Livingston Survey beginning in the mid-1940s. Medium- and long-term forecasts (target period includes three-years ahead and longer) are available for real output growth and inflation starting in the late 1970s. However, a more comprehensive set of long-term forecasts (a target period of five or more years ahead) for all three variables is available only starting in the mid-1980s. At all horizons there are relatively fewer forecasts for the 3-month Treasury bill than for output growth and inflation.

In the discussion of our results we focus on the period 1982–2019, covering the Great Moderation, the Great Recession following the Global Financial Crisis up to the pre-COVID period. This period includes the majority of the available survey forecasts with over 75% of the total number of series used available in this 35 year time span.

Table 2: **Summary of Surveys**

This table provides a summary of the forecast data available from each survey: Blue Chip Financial Forecasts (BCFF), Blue Chip Economic Indicators (BCEI), Consensus Economics (CE), Decision Makers' Poll (DMP), Goldsmith-Nagan Survey (GN), Economic Forecasts: A Worldwide Survey (EF), Livingston Survey (Liv.), Survey of Primary Dealers (SPD), and the Survey of Professional Forecasters (SPF). NT refers to horizons of two years or less while LT refers to horizons including more than two years in the future. For ongoing surveys, the reported frequency of questions pertaining to longer-term forecasts refer to the current scheduled frequency. Forecasts for output growth (RGDP) are based on real GNP growth prior to 1992 and real GDP growth after. M3 and M12 signify forecasts of 3-months and 12-months ahead, respectively. Entries of the form Q0-Q6 imply that horizons Q1, Q2, ..., Q6 are available; all other notation is defined in Section 3.

	BCFF	BCEI	CE	DMP	EF	GN	Liv.	SPD	SPF
Survey Sample (full)									
Frequency	*Monthly*	*Monthly*	*Monthly*	*Irregular*	*Monthly*	*Quarterly*	*Biannually*	*8 per year*	*Quarterly*
RGDP:	1984–present	1978–present	1989–present	n/a	1984–1995	n/a	1971–present	2011–present	1968–present
CPI:	1984–present	1980–present	1989–present	1978-1987	n/a	n/a	1946–present	2011–present	1981–present
TBILL:	1982–present	1982–present	1989–present	n/a	1984–1995	1969-1986	1992–present	n/a	1981–present
Survey Sample (LT)									
Frequency	*Biannually*	*Biannually*	*Quarterly*	*n/a*	*n/a*	*n/a*	*n/a*	*8 per year*	*Quarterly*
RGDP:	1984–present	1979–present	1989–present	n/a	n/a	n/a	1990–present	2012–present	1992–present
CPI:	1984–present	1984–present	1989–present	1978-1987	n/a	n/a	1990–present	2011–present	1991–present
TBILL:	1983–present	1983–present	1998–present	n/a	n/a	n/a	n/a	n/a	1992–Present
Horizons (NT)									
RGDP:	Q0-Q6, Y2 Y1-5, Y2-6	Q0-Q7, Y2, Y0-4, Y1-5, Y2-6, Y1-10	Q0-Q8, Y1, Y2	n/a	Q1-Q4	n/a	Q1-2, Q3-4, Y2, Y0-9	Q0-Q2, Y1, Y2	Q0-Q4, Y2, Y0-9
CPI:	Q0-Q6, Y2 Y1-5, Y2-6	Q1-Q7, Y2 Y1-5, Y2-6	Q2-Q8, Y1, Y2	Y1-10	n/a	n/a	Q3-4, Y2, Y0-9	n/a	Q2-Q4, Y1, Y2 Y0-4, Y0-9
TBILL:	Q0-Q6, Y1, Y2 Y1-5, Y2-6	Q1-Q7, Y1, Y2, Y1-5, Y2-6	M3, M12, Y1, Y2	n/a	Q1-Q4 Y0, Y1	M3	Q0, Q2, Q4, Y1, Y2	n/a	Q1-Q4, Y1, Y2, Y0-9
Horizons (LT)									
RGDP:	Y3, Y4, Y5, Y6 Y6-10, Y7-11	Y3, Y4, Y5, Y6, Y5-9, Y6-10, Y7-11	Y3-Y10	n/a	n/a	n/a	Y0-9	Y3, LR	Y3, Y0-9
CPI:	Y3, Y4, Y5, Y6 Y6-10, Y7-11	Y3, Y4, Y5, Y6, Y5-9, Y6-10, Y7-11	Y3-Y10	Y1-10	n/a	n/a	Y0-9	Y5-10	Y0-4, Y0-9
TBILL:	Y3, Y4, Y5, Y6 Y6-10, Y7-11	Y3, Y4, Y5, Y6, Y6-10, Y7-11	Y3-Y10	n/a	n/a	n/a	n/a	n/a	Y3, Y0-9

3.2 Mapping the Model to Survey Forecasts

The model defined by equations (3.1)-(3.3) has the state-space representation

$$Z_t = F(\Phi) Z_{t-1} + V\varepsilon_t$$

where $Z_t = (\; z_t \quad ... \quad z_{t-4} \quad \hat{x}_t \quad \hat{\omega}_t \;)'$. The presence of four lags in z_t, facilitates mapping data definitions to model concepts, as discussed further below. The heterogeneity of available forecasts makes this a non-trivial task. Start with a simple example. Suppose each month we only observe survey forecasts at monthly horizons. For example, we might measure a forecast for the n-month-ahead inflation rate at time t. Using the model, the n-step-ahead forecast of all model variables is given by

$$E_t z_{t+n} = \hat{\omega}_t + \Phi^n \hat{x}_t,$$

where the model forecast of inflation would be the second element of the vector z_t. The larger state vector satisfies

$$E_t Z_{t+n} = F(\Phi)^n Z_t$$

and provides the observation equation. The mapping between data and model is then straightforward.

In practice, however, survey participants are rarely asked to provide monthly forecasts. Rather they are queried about different types of forecasts, which involve quarterly averages, year-over-year growth rates and so on. When estimating our model we take care to match as closely as possible the observed forecasts with the correct model representation. The following examples help clarify how we do this. Consider the short-term interest rate. Forecasts for the three-month Treasury bill rate are either a simple average over a period or end of period. For the latter we assign these forecasts to the last month in the period. For real output growth and inflation, survey forecasts come in three possible forms: quarter-over-quarter annualized growth, annual average growth and Q4/Q4 growth. Let G_{2019Q1} and G_{2019Q2} be the level of real GDP in billions of chained dollars in the first and second quarter of 2019, respectively. Then, the quarterly average annualized growth rate is defined as $100 \cdot ((G_{2019Q2}/G_{2019Q1})^4 - 1)$. Our model variables define a month-over-month (annualized) real GDP growth rate series. To map the monthly series into this specific measured quarterly growth rate we follow Crump et al. (2014) and use

$$100 \cdot ((G_{2019Q2}/G_{2019Q1})^4 - 1) \approx \frac{1}{9} \left(g_{2019m2} + 2 \cdot g_{2019m3} + 3 \cdot g_{2019m4} + 2 \cdot g_{2019m5} + g_{2019m6} \right),$$

where, for example, g_{2019m2} represents the model-based month-over-month annualized real output growth in February 2019. This notation makes clear why lagged values of z_t appear in the state vector Z_t. Annual average growth rates follow a similar pattern. For example, let G_{2018} and G_{2019}

be the average level of real GDP in billions of chained dollars in the years 2018 and 2019. The annual average growth rate is $100 \cdot (G_{2019}/G_{2018} - 1)$ which we approximate via

$$100 \cdot (G_{2019}/G_{2018} - 1) \approx \frac{1}{24} \left(g_{2018m2} + 2 \cdot g_{2018m3} + 3 \cdot g_{2018m4} + \cdots + 12 \cdot g_{2019m1} \right.$$
$$\left. +11 \cdot g_{2019m2} + 10 \cdot g_{2019m3} + \cdots + 2 \cdot g_{2019m11} + g_{2019m12} \right).$$

Finally, Q4/Q4 growth rates are calculated, for example, by $100 \cdot (G_{2019Q4}/G_{2018Q4} - 1)$ and approximated via

$$100 \cdot (G_{2019Q4}/G_{2018Q4} - 1) \approx \frac{1}{12} \left(g_{2019m1} + g_{2019m2} + g_{2019m3} + \cdots + g_{2019m12} \right).$$

The above shows that certain short-term survey forecast horizons will implicitly include time periods which have already occurred. To avoid taking a stand on how forecasters treat past data (e.g., do forecasters use realized data, filtered versions or another measure?) we exclude all survey forecast horizons that include past months' values of z_t. The only exception we make is to include current quarter (Q0) and one-quarter ahead (Q1) forecasts for real output growth which extend back, at most, four months and one month, respectively. We do so to help pin down monthly real output growth since the actual series is only available at a quarterly frequency. Finally, for simplicity, forecasts which involve averages over multiple years are mapped as simple averages over the corresponding horizons.

The mapping between unobserved states and observed forecasts is then given by the observation equation

$$\mathcal{Y}_t \;\; = \;\; H_t \left(\Phi \right) \times Z_t + o_t,$$

where $\mathcal{Y}_t$ includes the survey forecasts. The observation matrix depends nonlinearly on Φ and is time-varying, reflecting missing observations in the survey forecasts series. The vector o_t denotes measurement errors. We assume individual observation errors for each survey to be mean-zero, i.i.d. and mutually independent Gaussian innovations. To ensure a parsimonious model we impose equal variances for each target variable at similar forecast horizons (but not by the specific survey). We group forecast horizons by: very short term—up to two-quarters ahead; short term—up to two-years ahead; medium term—from three-to-four-years ahead; and long term—five or more years ahead.

3.3   Estimation

We estimate the model using Bayesian methods. Despite the large number of observables, full identification of Φ is challenging given that the matrix only appears in the observation matrix $H \left(\cdot \right)$ in exponential terms. As we do not observe forecasts at the one-month horizon, the data cannot pin down Φ directly but only Φ^j, where $j > 1$ denote the forecast horizons. To aid identification we specify a prior on this set of parameters. Conditional on Σ_ε, the priors for the autoregressive

coefficient of the cyclical component, Φ, is a multivariate normal distribution

$$p\left(\Phi|\Sigma_{\varepsilon;\nu}\right) \;\; = \;\; \mathcal{N}\left(vec\left(\bar{\Phi}\right), \Sigma_{\varepsilon;\nu} \otimes \lambda^2 I_3\right),$$

where $\bar{\Phi} = 0.5 \times I_3$; $\Sigma_{\varepsilon;\nu}$ includes the first three rows and columns of Σ_{ε}; and the parameter λ is chosen to be consistent with a dispersed distribution.[5]

To assign priors on the variance-covariance matrix of innovations we decompose

$$\Sigma_{\varepsilon} \;\; = \;\; diag\left(\begin{array}{cc} \sigma_\nu & \sigma_\eta \end{array}\right) \times C_{\varepsilon} \times diag\left(\begin{array}{cc} \sigma_\nu & \sigma_\eta \end{array}\right)$$

where C_{ε} denotes the correlation matrix and where σ_ν and σ_η denote vectors of standard deviations. The prior on the correlation matrix is defined by the Lewandowski-Kurowicka-Joe (LKJ) distribution. The density function is

$$p\left(C_{\varepsilon}\right) \;\; = \;\; 2^{\sum_{j=1}^{n-1}(2(\psi-1)+n-j)(n-j)} \times$$

$$\times \prod_{h=1}^{n-1}(B(\psi + (n-h-1)/2, \psi + (n-h-1)/2))^{n-j} \times \det(\mathrm{C}_{\varepsilon})^{\psi-1}$$

where $B\left(\cdot, \cdot\right)$ is a Beta function. Here $n = 6$. Note that the marginal distribution for each off-diagonal entry in C_{ε} is a Beta distribution $\mathcal{B}\left(\psi - 1 + n/2, \psi - 1 + n/2\right)$.

In terms of moments, the mean is the identity matrix I_n and the variance of each off-diagonal entry is

$$V\left(c_{\varepsilon}^i\right) \;\; = \;\; \frac{4\left(\psi + \frac{n}{2} - 1\right)^2}{(2\psi + n - 2)^2(2\psi + n - 1)}.$$

Regarding the parameter ψ, we considered a few different choices and picked the value delivering the highest marginal likelihood. Priors on the standard deviations are set as independent inverse Gamma distributions. We choose a fairly loose prior on the standard deviation vector σ_ν.[6] In addition, we discipline the variability at the long-end of the forecast horizon. This embeds our prior that long-run forecasts shouldn't be too volatile and is necessary as there are relatively fewer survey data are available at these horizons. We therefore set relatively tighter priors on σ_η. This is particularly true for the drift in the nominal interest rate.[7] Finally, the variances of observation errors o_t have inverse gamma priors.[8]

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[5]The choice of $\lambda = 4$ guarantees priors are loose. In addition to this prior, we also enforce the restrictions that diagonal elements must be positive.

[6]Assuming an inverse gamma distribution with mean equal to 0.1 and a standard deviation of 2. Reflecting the notion that the innovations to the drift component are smaller than those to the cycle component we set tighter priors on σ_η.

[7]Specifically, we set inverse gamma priors on the individual variances: the first two elements, corresponding to the estimated drift in output growth and inflation, have a prior mean of 0.01 and a standard deviation of 0.001. The last element, corresponding to the drift in the interest rate has a smaller prior on the variance with a mean of 0.0025 and a standard deviation of 0.00025.

[8]These priors with mean equal to 0.01 and standard deviation of 4 are also fairly loose.

3.4 Discussion

The model is designed around the central mechanism driving changes in the term structure of expectations introduced in the previous section. The time-varying long-run mean captures the observed drift in survey-based forecasts. This model feature has been exploited in the previous literature with a tight focus on inflation expectations. Kozicki and Tinsley (2012), a precursor of this approach, show this class of models fits professional forecasters' inflation expectations at different horizons, including the long-run. Chan et al. (2018) conduct a similar exercise for a wide set of countries. Using a novel approach, Aruoba (2020) fits the term structure of survey-based inflation expectations by adapting the structure of the Nelson-Siegel (NS) model of the yield curve, which summarizes the yield curve with three factors (level, slope, and curvature). In contrast to the existing literature, in our model forecasters form *joint* expectations about different macroeconomic variables. As we will see below, a multivariate modeling approach that includes the common dynamics of output growth, inflation, and the short-term interest rate matches the term structure of expectations substantially better than modeling expectations individually.

However, for the sake of simplicity the model ignores some possibly important features of the expectations formation mechanism. First, the model parameters are time invariant. Shifts in the the volatility of forecast errors might have an impact on the updating of expectations by affecting the sensitivity to forecast errors via the Kalman gain. While it is has been widely documented that economic volatility has changed in the post-war U.S., this is likely less of a concern for our baseline estimation period from 1983-2019. Other sources of structural change such as regime shifts in monetary or fiscal policy can also impact the expectation formation process that the models aims at capturing. Mertens and Nason (2020) extend the framework by introducing time-varying persistence and volatility in the their underlying model of inflation expectations.[9] Carvalho et al. (2021) and Eusepi et al. (2020) allow for structural changes in the expectations formation process in general equilibrium frameworks. We revisit these ideas in Section 6.

Second, in this section we focus on a representative forecaster and disregard the forecast disagreement widely documented in surveys. However, as shown in Coibion and Gorodnichenko (2012), forecast dispersion can affect the dynamic properties of consensus measures of expectations. Researchers have introduced a rich set of informational frictions that can generate plausible degrees of forecast dispersion. Models of sticky (Mankiw and Reis (2002)) or noisy information (Woodford (2003b)) and models of rational attention (Sims (2003) and Maćkowiak and Wiederholt (2009)) assume that individual forecasters endogenously have different information sets regarding the current state of the economy. As such, they disagree in their forecasts about future economic outcomes. Mertens and Nason (2020) capture a wider set of information frictions by allowing for infrequent forecast updating of inflation expectations by individual forecasters. Andrade et al. (2016) show that one can match the term structure of disagreement of U.S. professional forecasts about infla-

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[9]Grishchenko et al. (2019) go beyond consensus inflation forecasts and use probability distributions of future inflation rates from several U.S. and euro-area surveys of professional forecasters to estimate a dynamic factor model featuring time-varying uncertainty.

tion, output growth and the federal funds rate by using a similar multivariate framework as the one described above, combined with sticky or noisy information. Similarly, Andrade and Le Bihan (2013) employ a multivariate setup and assume forecasters are subject to sticky information *and* noisy information. They fit this model to short-term survey forecasts for inflation, output growth and the unemployment rate in the euro area and find that the model cannot replicate the degree of serial correlation in consensus forecast errors and the amount of disagreement across forecasters observed in the data. We discuss the implications of additionally imposing information friction in our model setup in Section 5 which focuses on explaining the term structure of forecaster disagreement.

Third, to what degree is the model used by our representative forecaster close to the correct data generating process? Under the common assumption of rational expectations agents use the correct model. This implies the updating equations (2.4, 2.5) are based on the optimal filter.[10] Macroeconomic models embedding these assumptions have been used to study the response of the economy to changes in long-run productivity (Tambalotti (2003), Edge et al. (2007)); shifts in the long-run mean of inflation (Erceg and Levin (2003)); or movements in asset prices in response to long-run dividend growth (Timmermann (1993)). However, a growing literature assumes agents form expectations under bounded rationality. These models produce a wedge between subjective expectations and the model-consistent data generating process. Agents' inference and expectations updating is then no longer optimal. This literature includes models of adaptive learning (Marcet and Sargent (1989), Evans and Honkapohja (2021) and Eusepi and Preston (2011)), or models where expectations exhibit extrapolation bias (Fuster et al. (2010), Bordalo et al. (2020) and Angeletos et al. (2020)).[11] We discuss these additional frictions in Section 6, where we study the term structure of expectations in a structural general equilibrium model.

3.5 Results

3.5.1 Model Fit

The model is estimated over the period January 1983 to December 2019.[12] To assess the relative fit of the model, we estimate two additional specifications. The first assumes that expectations are formed independently for each variable using univariate versions of the model similar to the one discussed in Section 2.1. Formally this is achieved by restricting Φ and Σ_ε to be diagonal. The univariate specification permits us to evaluate whether professional forecasters account for the dynamic interactions between variables and to provide a direct comparison to the vast majority of work documenting the properties of survey forecasts making this assumption. The second model specification makes the common assumption that innovations in the unobserved trend and cyclic

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[10]Under this assumption the surprises measured by $z_t - z_{t|t-1}$ are uncorrelated with information available at $t-1$, as we assume here for convenience. In particular, the Kalman filter produces innovations to trend (η_t) and cycle (ν_t) that are *i.i.d.* across time.

[11]See Angeletos et al. (2020) which offers a comprehensive discussion on the literature and introduces a model featuring both disperse information and extrapolation bias. This model reproduces the observed response of survey-based forecast to an identified business cycle shock.

[12]The parameters' posterior distribution is obtained using a standard Metropolis Hastings algorithm; the unobserved states are drawn with the standard Carter and Kohn smoother.

components are independent. By comparing the model fit of the baseline and the restricted specification, we can then evaluate whether the predicted link between short-term developments and long-term forecast revisions is consistent with observed survey forecasts.

In terms of marginal likelihood, our baseline specification outperforms the alternative featuring independent innovations by over 30 log-points, strongly supporting our proposed expectation formation mechanism. The baseline model also dominates (by over 1000 log points!) the alternative specification where forecasts are modeled independently for each macroeconomic variable.[13]

Figure 4 sheds further light on the absolute and relative fit of the model by showing a scatter plot of the fitted values (x-axis) and observed survey data (y-axis) for both the baseline specification (in red) and the univariate specification (in black). If the model perfectly explained the data with no observation error, each data point would lie on the forty-five degree line. The baseline model does a very good job at fitting the 627 times series, especially for the short-term nominal rate. The slightly worse performance for real GDP and inflation reflects the high volatility in those forecasts at very short horizons which the model does not fully capture. The fact that this short-term volatility does not translate to longer-term forecasts suggests it is driven by shocks perceived to be temporary. The estimation procedure then attributes a large fraction of variation to measurement error because the model's innovations affect both short and longer horizons. This short-term volatility problem does not affect significantly the fit of interest rate forecasts, perhaps because the central bank is believed to respond to the underlying trend in the economy, rather than to temporary shocks.

Figure 4: **Model fit: Baseline vs. univariate model.** The figure shows the scatterplot of the prediction error for the baseline model (red) and the univariate specification (black).

| (a) *Real GDP* | (b) *CPI Inflation* | (c) *3-Month T-Bill* |
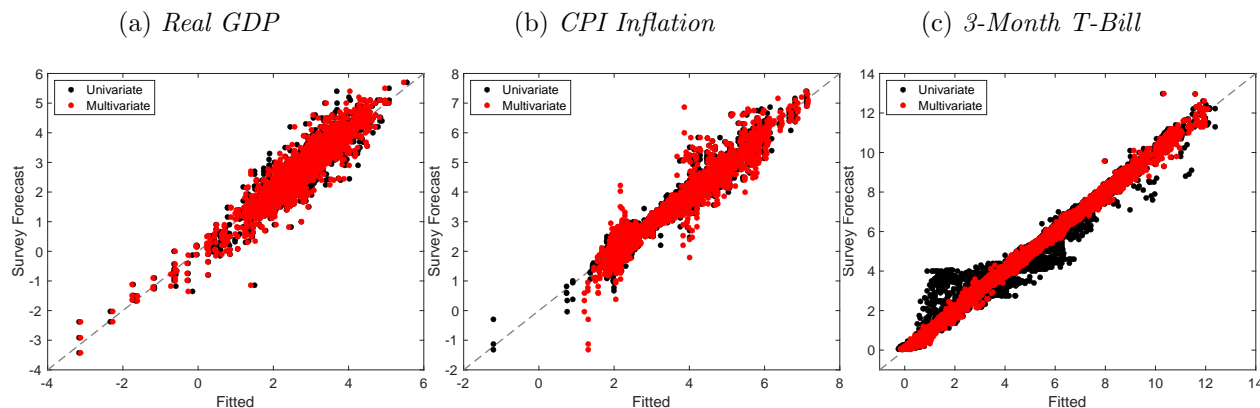
When comparing our baseline model (red) with the independent forecasts model (black) we see that the single-equation model performs particularly poorly for forecasts of the short rate. The much better fit of the multivariate model suggests that market participants form expectations about the short rate jointly with those of output and inflation. While the difference in fit appears less striking for inflation and real GDP forecasts, simple measures of forecast performance show that

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[13]In detail, the marginal likelihood for the baseline is 850.13, compared to 818.26 for the model with independent innovations; and to -1771 for the model with independent univariate forecasts.

our baseline model outperforms the alternative of independent forecasts in terms of relative mean-square error (MSE) by about 20% for almost all horizons. One exception is inflation forecasts at very short horizons, i.e. up to the one-quarter-ahead forecast. Here, the univariate model, which is just the sum of a random walk and an i.i.d. shock, delivers a better performance because measurement errors and short-term disturbances are not separately identified. The univariate model then uses short-term shocks rather than observation shocks to fit the data, resulting in smaller measurement errors.[14] Summing up, the model's relative fit confirms that a multivariate model is needed to characterize the expectation formation mechanism of professional forecasters.

Given the large number of series involved in the estimation, it is not straightforward to illustrate the fit of our model comprehensively in the time series domain. We therefore relegate the 627 figures detailing the model's fit for each individual survey forecast series to the online appendix. Figure 5 offers a subset of this information, detailing three forecast horizons for each variable: the short term (two-quarters ahead); the medium term (two-years ahead); and the long term (five-years ahead and beyond). In each panel, we show a collection of survey forecasts from different sources that match the appropriate forecast horizon (we use about sixty time series in total). The model does a remarkable job. Perhaps not surprisingly given the vast number of survey forecasts available for this time period, the grey areas capturing the 95 percent coverage interval are very tight. Moreover, the model-implied forecast values closely track the data, with a few exceptions for real GDP long-term forecasts during the late 1980s and the 2009 recession.

In addition to fitting the observed survey forecasts over the estimation period 1983-2019, we back-cast the individual model-implied forecast series and report smoothed estimates of expectations going back to 1970. Over this earlier period the availability of survey forecasts is scarce and, for longer-range forecasts, nonexistent. Therefore, there is considerable uncertainty about the term structure of forecasts. One additional caveat with this exercise is that the expectations formation process has most likely undergone structural change across the full sample. As discussed in Section 6, the evolution of the perceived drifts has changed and has become less responsive to short-term developments over time. Also, economic volatility and, possibly, the perceived policy regime could have shifted over time.[15]

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[14]To give a sense of the relative fit, once we exclude nowcasts and one-quarter-ahead forecasts, the mean squared error for real GDP and CPI forecasts for the univariate model is 22% and 18% higher, respectively. For interest rate forecasts, the univariate model produces a mean square error more than 600% higher than our baseline model.

[15]A potential extension to our framework that we do not pursue here involves incorporating explicitly time variation in both the systematic and stochastic components of our model. For example, Garnier et al. (2015) estimate a model for trend inflation on different countries and allow time variation in the volatility of the trend. Primiceri (2005) and Bianchi and Ilut (2017) estimate VARs with time-varying coefficients on U.S. data in order to account for structural change.

Output Growth **Inflation** **Interest Rate**

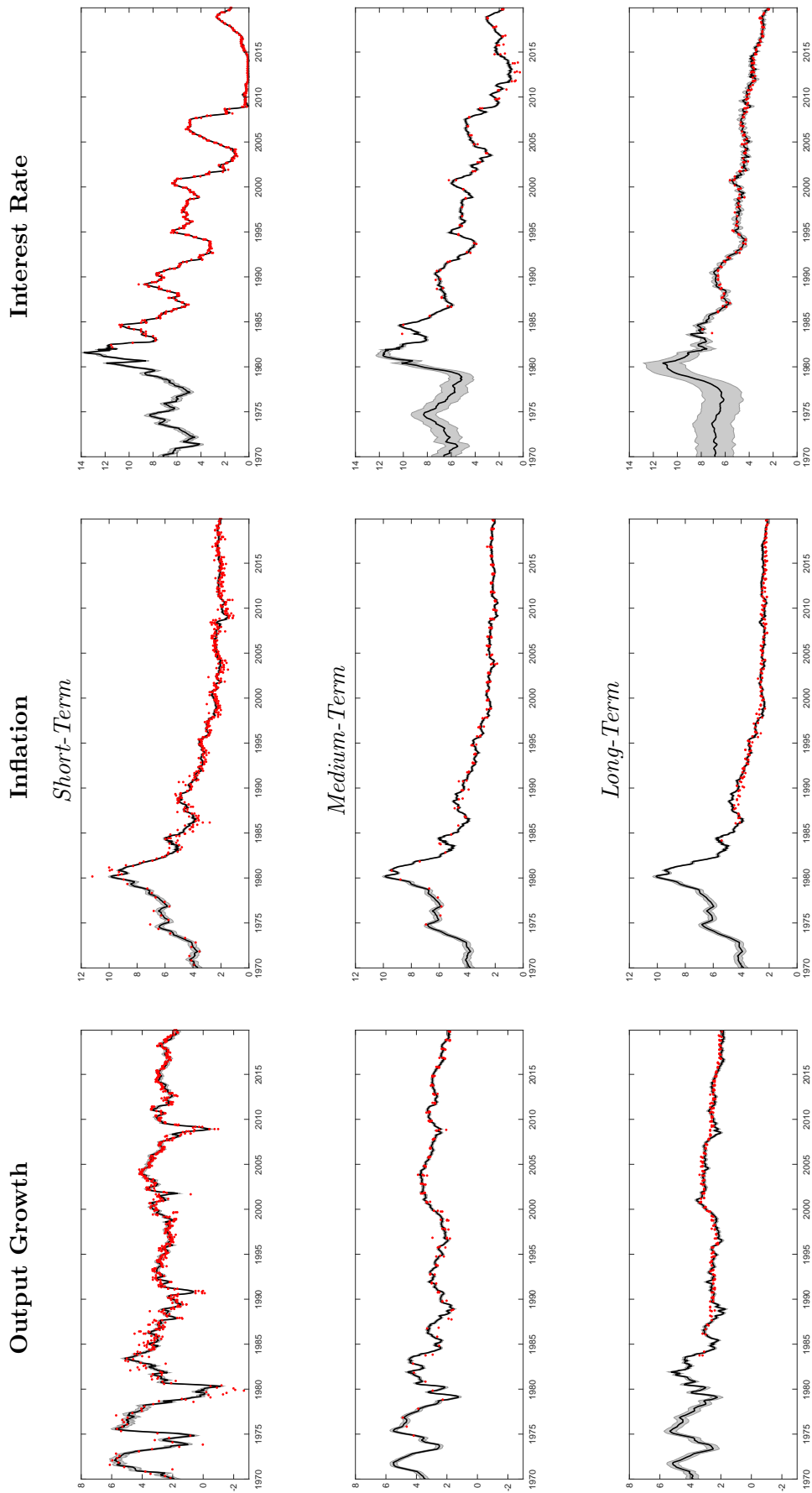*Short-Term*

Medium-Term

Long-Term

Figure 5: **Fitting the term structure of expectations**

The panels contrast model predictions with survey data at different forecast horizons. The solid line shows median predictions and the grey shade shows the 95 percent coverage interval. The squares represent survey-based forecasts at short-term (two quarters ahead), medium-term (two years ahead) and long term (five years ahead and beyond) from different surveys.

These caveats notwithstanding, the model also fits the few observed survey forecasts for output growth and inflation that are available before 1983 reasonably well. Not surprisingly, the uncertainty around the estimates increases as we move backwards in time. However, model predictions accord with conventional wisdom, with an increase in inflation and interest rate expectations over the mid-1970s, peaking in the early 1980s. While predicted long-term forecasts for real GDP are possibly too volatile, they capture the higher growth rate during that period. Overall, our simple and highly parsimonious model fits the term structure of survey-based expectations, especially after the mid-1980s, exceptionally well.

3.5.2   Evolution of the Term Structure of Expectations

The estimated model allows us to study the implied expected paths of the fitted variables at any specific point in time. Figure 6 shows a number of "hair charts" which are a convenient way to summarize the evolution of these forecast paths. The top panel displays real GDP growth; the middle panels show the nominal interest rate and the underlying rate of CPI inflation;[16] and in the bottom panel we provide the real short-term interest rate computed as the difference between the nominal short rate and and expected inflation.[17] The black solid lines show the actual realized data while the grey lines show the expected paths of the respective variable over the next ten years once every twelve months.

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[16]In order to smooth the high volatility in series we plot the model-based measure of CPI which does not transitory shocks captured by observation errors.

[17]In particular, since our model is at the monthly frequency we define the real rate as the current interest rate less the one-month ahead forecast of inflation.

Figure 6: **Estimated forecast paths.** The figure shows model estimates of survey-based expectations up to ten years horizons.

(a) *Real GDP Growth*

(b) *Short-term Nominal Interest Rate*

(c) *Inflation*

(d) *Short-term Real Interest Rate*
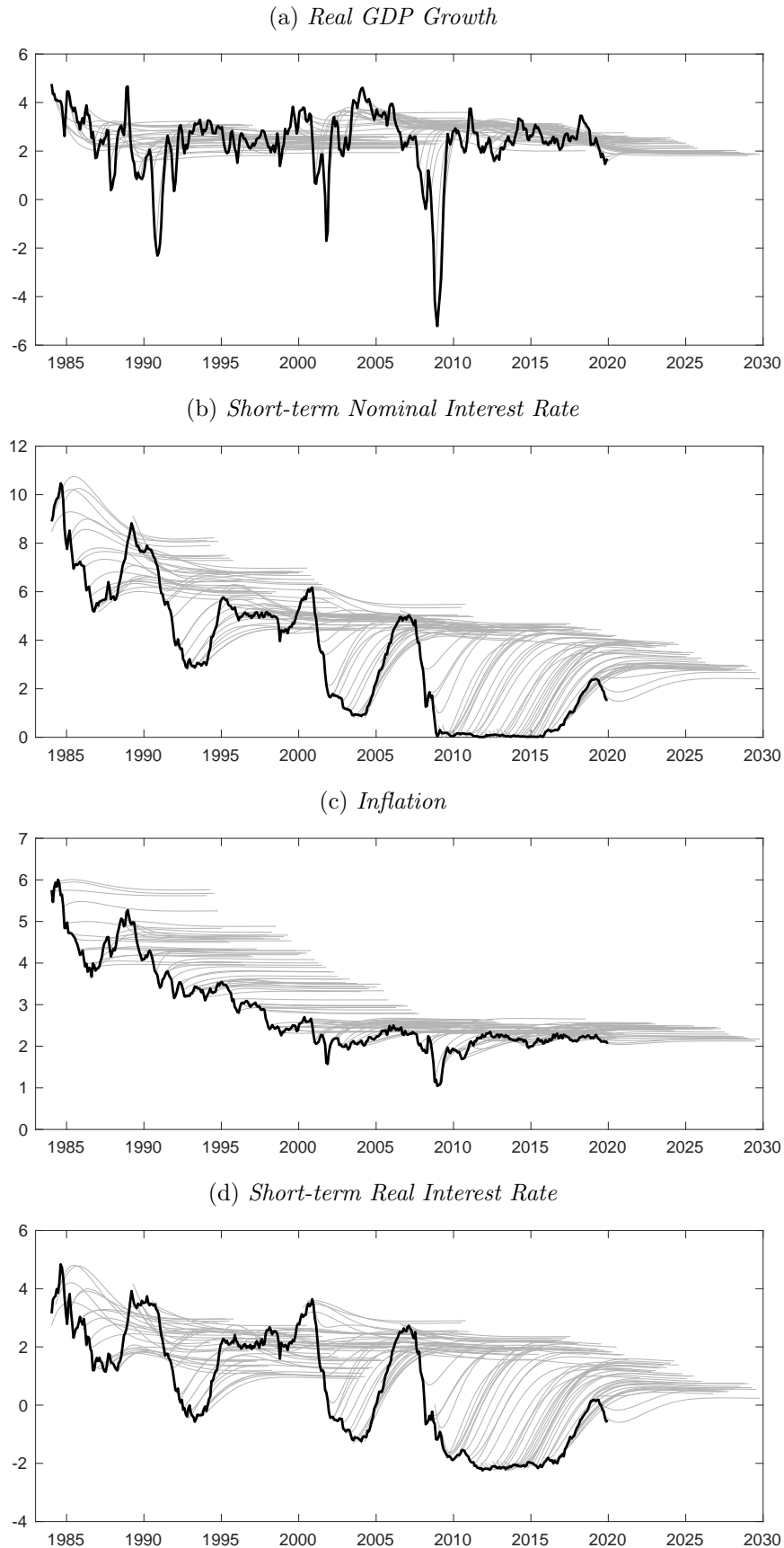
The forecast paths display substantial volatility over time, typically flattening (and often inverting) at the end of economic expansions and steepening in the aftermath of recessions. This pattern is starker for nominal and real interest rates, as professional forecasters respond to the predictable component of monetary tightening and easing cycles. For example, the term structure of short rate expectations inverts in early 1989 when short rates reached their local peak leading into the 1990-91 recession. A flattening and slight inversion is also observed at the end of the 2004–2006 tightening cycle. Importantly, these estimated measures of short rate expectations based on survey forecasts, in contrast to many model-based expected short rate paths, are consistent with a perceived zero lower bound (ZLB) on nominal interest rates. After the short rate reached the ZLB in 2008, the term structure first flattened and then steepened again as forecasters continued to expect an eventual lift-off. This "over-optimism" about lift-off that is apparent in the short rate expectations is mirrored by over-optimistic real GDP forecasts during the same period.

While expected nominal short rates display a significant degree of volatility, the shape of the expected path of inflation (third panel) exhibits far less variation, remaining mostly flat around the prevailing level of inflation. Professional forecasters perceive the persistent component of inflation to approximately follow a random walk. An important implication is that movements in expected nominal short rates translate almost one-to-one to expected real short rates (bottom panel), consistent with nominal rigidities preventing prices from adjusting in the short term.

The expected ten-year paths of short rates and inflation converge to each variable's time-varying long-run mean extracted from all available surveys of professional forecasters. These long-run projections reflect forecasters' perceptions of macroeconomic fundamentals rather than cyclical variation. Long-run forecasts have all varied substantially over the past thirty years. The long-run expected nominal short rate has gradually fallen from about 8 percent in the mid 1980s to about 2.5 percent in 2019. Much of this decline is accounted for by a secular decline in the expected long-run level of inflation, which dropped from about 6 percent in the early 1980s to a level of around 2.5 percent in the late 1990s. Since then, the perceived inflation target has remained extremely stable, only showing a small dip around the Great Recession and over the last two years in our sample.

Using the survey-implied term structures of expectations for the nominal short rate and inflation, the final panel shows the evolution of the expected real short rate. The long-term expected real short rate has remained fairly stable around 2 percent over the thirty year period starting in 1983, but has begun to decline after 2010, falling below 1 percent by the end of 2014. This is consistent with long-run real GDP growth forecasts which have fallen modestly over the past ten or so years, reaching slightly below 2 percent by the end of the sample.

3.5.3   Perceived Natural Rate of Interest

The reduction of expected long-run real rates is consistent with recent evidence on the decline of the natural real rate of interest. Summers (2014), Johannsen and Mertens (2016), Holston et al. (2017), Del Negro et al. (2018), among others, have argued that long-run equilibrium real rates in the U.S. have seen a secular decline over the past decades, and dropped suddenly around the

financial crisis. A persistently depressed long-run or equilibrium level of the real short rate would have important implications for macroeconomic analysis. For example, the level of the equilibrium real short rate is important for assessing the stance of monetary policy. The size of monetary stimulus is determined by the level of current real interest rates relative to the long-run level. Given inflation expectations, falling long-run real rates thus imply less nominal space for monetary policy to respond to an economic contraction.

We compare professional forecasters' real-time predictions for the long-run value of the real short rate with two prominent measures of the natural real rate.[18] Figure 7 plots our survey-based measure of the long-run equilibrium real rate (red line) with these alternative estimates. In the top panel, the solid and dashed black lines show the evolution of the five- and thirty-year forecasts of the natural rate of interest from the dynamic stochastic general equilibrium model (DSGE) in Del Negro et al. (2018), updated until the end of our sample. For the period up to 2018, Del Negro et al. (2018) show that estimates from a Bayesian VAR deliver similar estimates.[19]

All estimates share a similar broad pattern: the long-run equilibrium real rate fluctuated around 2% until the early 2000s and then started to decline, reaching about 1% by 2012. However, two differences emerge between our estimate and those implied by the DSGE model. First, the survey-based estimate fluctuates considerably over the sample reaching below -1% after 1992 and rebounding to nearly 3% in early 2000. In contrast, the DSGE estimates are fairly stable around 2% over the period 1982-2000. This suggests professional forecasters' long-run predictions were fairly responsive to current economic conditions during that period, an issue that we revisit in Section 6. Second, while the DSGE estimates suggest the estimated long-run natural rate to bottom in 2012 and to begin reverting afterwards, professional forecasters continued to revise their forecasts downwards, reaching below -0.5% by the end of 2019. Lastly, the term structure of the DSGE-implied real short rate forecasts exhibit starkly different behavior across horizons. At the five-year horizon the natural rate of interest is predicted to be negative until the end of 2017, while the 30-year forecast predicts above 1.5%. In contrast, survey-based forecasts display almost identical dynamics for forecast horizons above five years, as movements at such long horizons mimic shifts in the drift component.

The middle panel in Figure 7 displays the updated estimates of the natural real interest rate from two versions of the Laubach and Williams (2003) model as the solid and dashed black lines along with the survey-based expected long-run real short rate as the red solid line. The different measures broadly co-move but also feature some differences. Most importantly, the natural rate estimates drop sharply around the financial crisis and have hovered around zero since then, while professional forecasters long-run expectations have fallen to such low levels only very recently.

Explanations of the secular decline in the real rate include demographic changes, a widespread productivity slow-down, an increased demand for liquid Treasury securities and other secular shifts in global savings and investment decisions. Our reduced-form model does not allow us to disentangle

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[18]While we show the forecast for the ten-year forward horizon, we emphasize that the estimated drift looks nearly identical. In other words, the survey-based forecast reaches its long-run estimate at horizons around five to ten years.

[19]Data available at https://github.com/FRBNY-DSGE/rstarBrookings2017.

these different explanations. However, use of some basic macroeconomic relationships provides
further insight. Consistent with the Fisher equation, define the perceived long-run real rate as

$$\bar{r}_t \;\; = \;\; \hat{\omega}_t^i - \hat{\omega}_t^\pi \tag{3.4}$$

where $\bar{r}_t$ is the long-run estimate of the real short rate and $\hat{\omega}_t^i$ and $\hat{\omega}_t^\pi$ are the estimated drifts from
our model, defined in Equation (3.2). Models of long-run economic growth also predict real interest
rates are determined by the rate of time preference and the expected growth rate of output. This
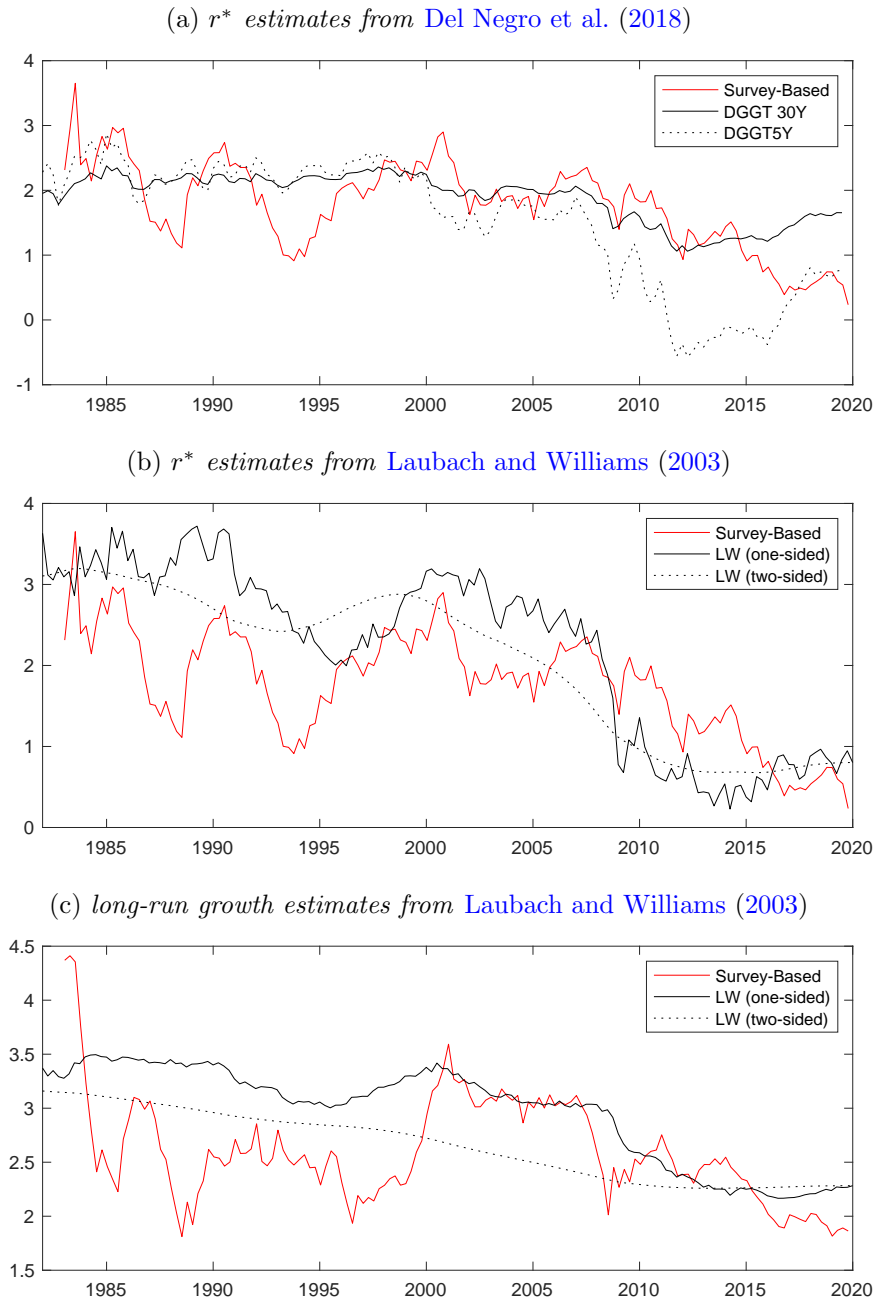provides the relationship

$$\bar{r}_t \;\; = \;\; \sigma^{-1}\hat{\omega}_t^g + \hat{\beta}_t \tag{3.5}$$

where $\hat{\omega}_t^g$, the estimated drift for output growth, proxies expected growth; $\sigma > 0$ measures the
intertemporal elasticity of substitution, the sensitivity of expected consumption to changes in the
real interest rate; and $\hat{\beta}_t$ a residual that captures expected movements in the discount rate, affecting
the equilibrium supply of savings. Combining these two expressions gives a decomposition of real
interest rates

$$\hat{\omega}_t^i - \hat{\omega}_t^\pi \;\; = \;\; \sigma^{-1}\hat{\omega}_t^g + \hat{\beta}_t. \tag{3.6}$$

The bottom panel in Figure 7 plots the perceived growth rate of real GDP in the long-run (red
line). It is significantly more volatile than the Laubach and Williams (2003) estimates. Further-
more, consistent with Del Negro et al. (2018), the evolution of long-run expectations of both the
real rate and output growth share a downward trend over the middle part of the sample, suggesting
that expected growth has partially driven the decline in the perceived real rate. This factor plays
less of a role in the estimates of Laubach and Williams (2003). However, the sharp decline of the
expected real rate over the last ten years is unmatched by a decline in expected output growth of a
similar magnitude, suggesting that other factors, captured by the residual $\hat{\beta}_t$, explain this decline.
Through a similar decomposition Del Negro et al. (2018) show that rising premiums for the liq-
uidity and safety of Treasury bonds (referred to as the "convenience yield" by Krishnamurthy and
Vissing-Jorgensen (2012)) have been the main drivers of the decline in the estimated natural rate of
interest. We come back to this decomposition in Section 5, when we discuss forecaster disagreement
about long-run interest rates.

Figure 7: **Alternative measure of r^*.** The figure shows the evolution of Laubach and Williams (2003). and Del Negro et al. (2018) and compares them with the survey-based expectations.

(a) *r^* estimates from* Del Negro et al. (2018)

(b) *r^* estimates from* Laubach and Williams (2003)

(c) *long-run growth estimates from* Laubach and Williams (2003)

4 Expectations and the Term Structure of Interest Rates

Monetary policy affects the aggregate economy via the term structure of interest rates. While central banks have tight control over short-term rates, the efficacy of monetary policy depends on the ability to affect longer-maturity interest rates which drive the saving and investment decisions of households and firms. Standard macroeconomic models assume that the transmission mechanism of monetary policy is given by the expectations hypothesis: yields on longer-term government bonds reflect the average short rate that investors expect to prevail over the life of the bond.

We now use the fitted term structure of expectations to evaluate how well movements in future short rate expectations explain long-term yields. We do this by decomposing observed government bond yields into an expectations hypothesis component and a residual component which we interpret as a measure of the term premium perceived by professional forecasters. The term premium captures the compensation an investor in longer-term bonds would require for bearing the risk that the short rate evolves differently than expected. We document that the expectations component fails to explain the term structure of interest rates for medium to longer-run maturities. Despite sizable time-series variation, expected short rate paths explain little variation in medium to long-term Treasury yields. Rather, the term premium explains the bulk of this variation and has a strong common component across maturities. At the same time, term premiums are only weakly correlated with the term structure of short rate expectations. These findings suggest that monetary policy – through its effects on expected future short rates – has imperfect control over the term structure of interest rates.

4.1 Decomposing The Term Structure of Interest Rates

We obtain zero coupon U.S. Treasury yields from the Gurkaynak et al. (2007) dataset available on the Board of Governors of the Federal Reserve's research data page. The sample period is March 1983–December 2019. Following Fama and Bliss (1987) and Campbell and Shiller (1991), we decompose Treasury yields into two components: investor expectations about average future short rates as measured from our model and a residual component which we label the "term premium." Because our survey-based term premiums represent the residual between yields and expected short rates, we can remain agnostic about what specifically they represent. For example, they might reflect shifts in investor risk attitudes, differences between the expectations of the marginal investor and consensus expectations, or frictions in the bond market which prevent the elimination of arbitrage opportunities.

Let $y_t(n)$ be the continuously compounded yield on an n-month discount bond and i_t the risk-free nominal short rate at time t.[20] To separate longer-term from short-term expectations, we conduct our analyses in terms of *forward rates*, defined as the current yield of an n-year bond maturing in $n + m$ years:

$$f_t(n, m) = \frac{1}{n}[(n + m)y_t(n + m) - my_t(m)]. \tag{4.1}$$

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[20]The Appendix provides further details on the relevant notation along with specific examples.

Because our empirical model of expectations is estimated at a monthly frequency, we construct annual forward rates as the annual average of monthly forward rates. For example, a 4Y1Y forward would set $n = 12$ and $m = 48$. We then define forward term premiums as the difference between $f_t(n, m)$ and the expected nominal short-term rate over the n months from m months hence, which we can further decompose into the expected real short rate and expected inflation:

$$
\begin{aligned}
tp_t^{fwd}(n, m) &= f_t(n, m) - \frac{1}{n} \sum_{h=m+1}^{n+m} \mathbb{E}_t \left[i_{t+h} \right] \\
&= f_t(n, m) - \frac{1}{n} \sum_{h=m+1}^{n+m} \mathbb{E}_t \left[r_{t+h} + \pi_{t+h+1} \right].
\end{aligned}
\tag{4.2}
$$

In other words, the forward term premium is the difference between observed forwards and what would be the yield predicted by the expectations hypothesis, i.e. the average expected future short rate over the n months beginning in m months. Note that this is an identity: there are no implicit assumptions about the rationality or bias of expectations or the data generating process for yields, expectations, or term premiums.

Figure 8: **Forward rates vs. Expected interest rates.** The figure shows the evolution of the terms structure of forward rates up to then years out (red lines); and the term structure of expectations (blue line).
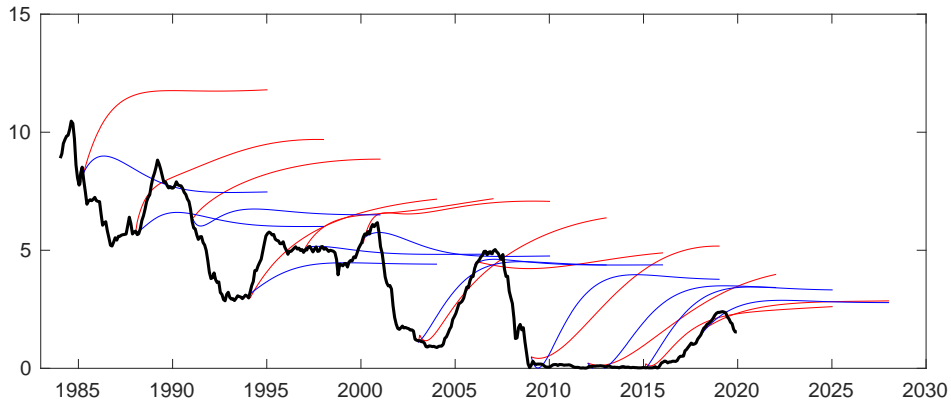

Figure 8 shows the now familiar term structure of nominal short rate expectations (blue line) discussed in the previous section and superimposes the interest rate path implied by the forward curve as defined in Equation 4.1 (red line). The latter represents the expected path of short rates under the pricing (risk-adjusted) measure. The short rate paths implied by the two measures are quite different. Forward rates imply both a substantially steeper and higher path compared to our measure of expectations, at least until the early 2000s. Conversely, the forward path becomes flatter afterwards, while the gap between the end-points of the two measures shrink substantially.

To shed further light on the differences, Figure 9 uses equation 4.2 to decompose nominal Treasury forward rates into expected future real short rates, expected future inflation, as well as the forward

term premium. The figure displays the 1Y1Y, 4Y1Y and 9Y1Y forward horizons in the top, middle, and bottom panel, respectively. All three components of bond yields contribute to the secular decline in Treasury yields observed over the past several decades, albeit with different timing. At the 1Y1Y horizon, the term premium declined from about three percent in the early 1980s and stabilized around zero in the early 2000s, mimicking the path of expected inflation. At longer maturities, forward term premiums display a similar pattern, falling over the 1980s and 1990s and stabilizing in the 2000s. Since about 2010, however, longer-maturity forward term premiums again declined together with the expected real short rate. Term premiums have remained at negative levels since 2010, except for a brief uptick around the "taper tantrum" episode of 2013.

Overall, forward term premiums account for more than half of the secular decline in longer-maturity forwards. This finding of a secular decline in term premiums is consistent with the evidence in Wright (2011) who uses an affine term structure model to show that term premiums in the U.S. and in other developed economies have experienced sizable and persistent declines between 1990 and mid-2009. He attributes this decline to a broad-based reduction of inflation uncertainty. Our survey-based decomposition shows that even when one takes full account of the expected short rate path of well-informed economic agents (professional forecasters), there is a secular decline of term premiums. We offer one potential explanation based on heterogeneous beliefs about short rates in Section 5.3 below.

Term premiums and survey forecasts: existing literature. Our measure of the term premium is model-free, in the sense that we simply obtain it as a residual from observed yields and observed or tightly fitted expected short rates. A few other studies have estimated term premiums using information from surveys of professional forecasters. However, they all obtain term premium estimates from no-arbitrage term structure models, fitted using observed yields and some survey forecasts of interest rates. For example, Kim and Wright (2005), Kim and Orphanides (2012) and Bauer and Rudebusch (2020) employ survey forecasts of the nominal short rate at select horizons to discipline their estimates. Similarly, Piazzesi et al. (2015) combine survey forecasts of the short rate, inflation, and of longer-term Treasuries to distinguish subjective beliefs (i.e. surveyed forecasters), objective beliefs (i.e. those of a statistician endowed with full-sample information) and subjective risk premiums. All these models assume a small-scale stationary VAR governing the dynamics of short rates and term premiums and, therefore, do not explicitly allow for low-frequency variation in expected short-rate paths which we have shown is a key element of actual short rate expectations.

Figure 9: **The Components of Treasury Forward Rates.** These figures show the decomposition of Treasury forwards into the expected short-term real interest rates, expected inflation and the nominal forward term premium. Treasury forwards are (based on) the zero coupon bond yields from the Gurkaynak et al. (2007) dataset available on the Board of Governors of the Federal Reserve's research data page. The sample period is March 1983–September 2016.
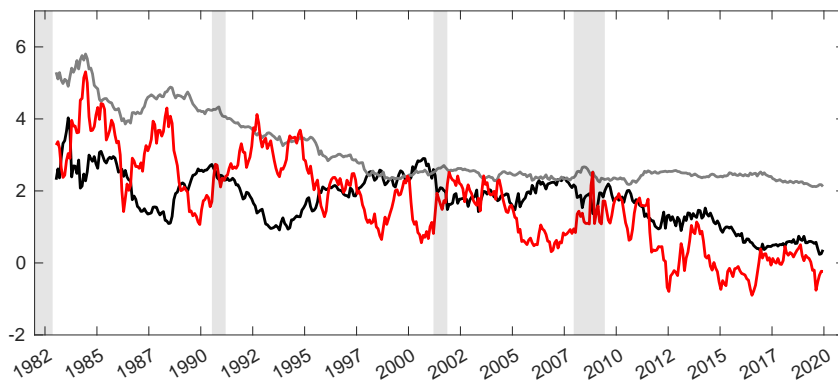
(a) *1Y1Y Forward Rate*
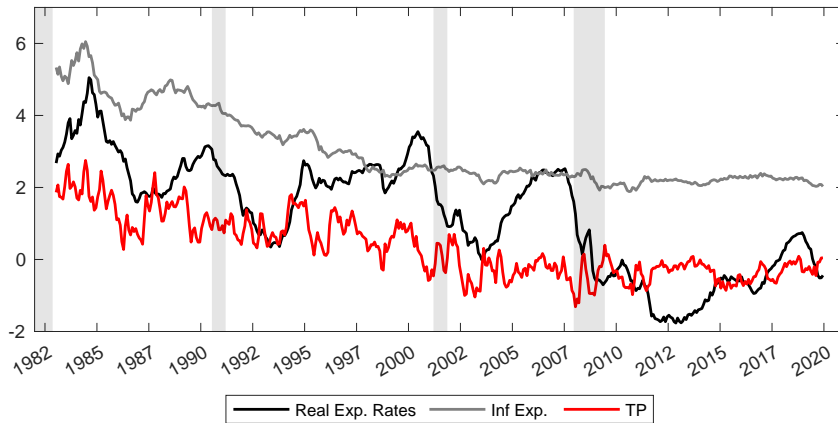
(b) *4Y1Y Forward Rate*
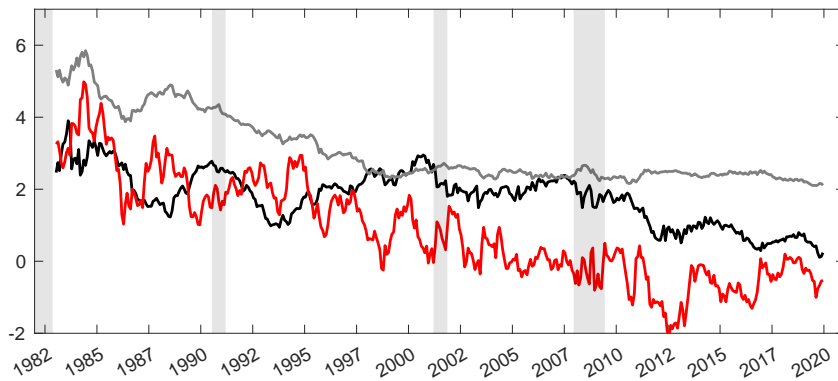
(c) *9Y1Y Forward Rate*

4.2 Expectations and Term Premiums

Figure 9 shows that at higher frequencies, forward term premiums and expected real rates feature significant variability across all maturities. In contrast, expected inflation shows little variability beyond its underlying trend. We can make these informal observations more concrete using a variance decomposition of forward rates based on the following identity

$$\mathcal{S}\left(n^{-1}\sum\nolimits_{h=m+1}^{n+m}\mathbb{E}_t\left[r_{t+h}\right]\right) + \mathcal{S}\left(n^{-1}\sum\nolimits_{h=m+1}^{n+m}\mathbb{E}_t\left[\pi_{t+h+1}\right]\right) + \mathcal{S}\left(tp_t^{fwd}(n,m)\right) = 1$$

where

$$\mathcal{S}(y_t) = \frac{\mathbb{C}\left(f_t(n,m),y_t\right)}{\mathbb{V}(f_t(n,m))}$$

is the ratio between the corresponding covariance ($\mathbb{C}$) and variance ($\mathbb{V}$). Table 3 provides variance decompositions for both the level (upper panel), as well as monthly, quarterly (middle panels) and annual changes (lower panel) of the one-year yield and one-year forward rates from one through nine years out.

These decompositions highlight the pivotal role of term premiums in accounting for yield variation. Expected real rates explain about 60 percent of the variance of the one-year yield while expected inflation and the term premium account for about 30 and 10 percent, respectively. Expected real rates also explain just shy of 50 and 30 percent of forward rates at the one- and two-year ahead horizon. However, their importance then declines sharply at longer maturities, accounting for less than 20 percent at forward horizons beyond four years. In contrast, term premiums only explain a small amount of variation at the very short end, but account for about 50 percent of the variation in forward rates at intermediate and longer maturities. The share of variance explained by expected inflation is relatively stable at around 30 percent across the maturity spectrum.

Since yields and forward rates are quite persistent, it is instructive to look also at the decomposition of their monthly, quarterly and annual *changes*. The contribution of term premiums to the variation of monthly changes in forward rates is substantial at all horizons and increases from 75 percent at the one-year forward horizon to over 90 percent at longer forward horizons. In contrast, expected real short rates only account for 20 percent of the month-to-month variation at the one-year forward horizon, and this contribution quickly drops to zero at longer maturities. Expected inflation also accounts for a negligible share of the variance of forward rate changes across maturities. The last two panels show that the finding holds also for quarterly and annual changes. The share of the variance explained by the term premium component declines somewhat at shorter maturities but remains above 75 percent for horizons beyond four years. Table A.2 in the Appendix shows that these conclusions are not altered by restricting the sample to the pre-crisis period.
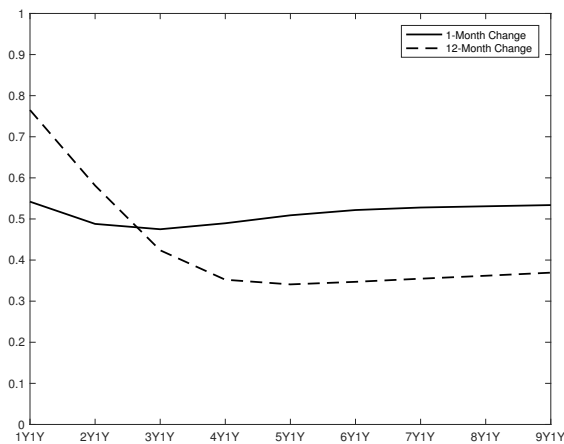
Table 3: **Variance Decompositions for Yield Components**

This table presents variance decompositions for the one-year yield and one-year forward rates ranging from one though ten-years out. For each maturity, the numbers shown represent the ratio of the covariance of the respective forward with its individual components (average expected real short rate, average expected inflation, and term premium) divided by the variance of the forward. The top panel provides variance decompositions for forward rates in levels, and the following panels the 1-month, 3-month and 12-month change in forward rates. The sample period is March 1983–December 2019.

	Y1	1Y1Y	2Y1Y	3Y1Y	4Y1Y	5Y1Y	6Y1Y	7Y1Y	8Y1Y	9Y1Y
Levels										
Avg Exp Nominal SR	0.90	0.76	0.64	0.55	0.50	0.48	0.47	0.48	0.49	0.50
Avg Exp Real SR	0.61	0.47	0.34	0.25	0.19	0.17	0.16	0.16	0.17	0.18
Avg Exp Inflation	0.30	0.29	0.29	0.30	0.30	0.31	0.31	0.32	0.32	0.32
Fwd Term Premium	0.10	0.24	0.36	0.45	0.50	0.52	0.53	0.52	0.51	0.50
1-Month Changes										
Avg Exp Nominal SR	0.61	0.28	0.14	0.09	0.07	0.06	0.05	0.04	0.04	0.04
Avg Exp Real SR	0.49	0.22	0.10	0.06	0.03	0.02	0.01	-0.00	-0.01	-0.01
Avg Exp Inflation	0.11	0.06	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05
Fwd Term Premium	0.39	0.72	0.86	0.91	0.93	0.94	0.95	0.96	0.96	0.96
3-Month Changes										
Avg Exp Nominal SR	0.79	0.43	0.26	0.18	0.14	0.11	0.09	0.08	0.07	0.07
Avg Exp Real SR	0.64	0.34	0.19	0.11	0.07	0.04	0.02	0.00	-0.01	-0.01
Avg Exp Inflation	0.15	0.09	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08
Fwd Term Premium	0.21	0.57	0.74	0.82	0.86	0.89	0.91	0.92	0.93	0.93
12-Month Changes										
Avg Exp Nominal SR	0.92	0.71	0.50	0.34	0.24	0.19	0.15	0.13	0.12	0.12
Avg Exp Real SR	0.75	0.55	0.36	0.22	0.12	0.07	0.03	0.01	-0.00	-0.01
Avg Exp Inflation	0.17	0.15	0.14	0.13	0.12	0.12	0.12	0.12	0.12	0.13
Fwd Term Premium	0.08	0.29	0.50	0.66	0.76	0.81	0.85	0.87	0.88	0.88

Figure 10: **Term Structures of Expectations and Forwards.** These figures show different aspects of the term structure of various second moments of forward expectations and forward rates. The top left panel displays the relative standard deviation of changes in expectations compared to changes in forward rates by forward maturity. The top right panel shows the correlation coefficient between changes in expectations compared to changes in forward rates by forward maturity. The black solid line denotes 1-month changes whereas the dotted line denotes 12-month changes.

(a) *Relative Vol. of Expectations wrt Forwards*   (b) *Corr. b/t Expectations & Forwards*
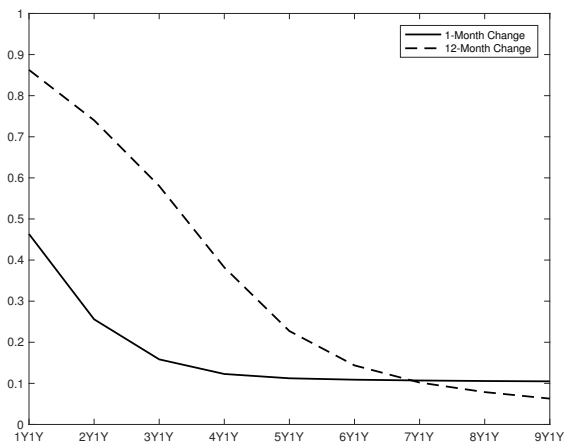
Co-movement across time. Given the considerable volatility of expected short rates, how can we explain the prominent role of term premiums in accounting for the variability of longer-maturity bond yields? Figure 10 sheds light on this question. The left-hand chart reiterates that nominal

rate expectations are fairly volatile at all forecast horizons when compared to actual forward rates: at horizons beyond three years, their volatility ranges from over 40% for 12-month changes to 50% for monthly changes. However, the right-hand chart shows that changes in expectations *co-move very little* with changes in yields, except at short forecast horizons. Since the variance share of the yield components ($\mathcal{S}$) can be re-expressed in terms of variances and correlations

$$\mathcal{S}\left(y_t\right) = \text{Corr}\left(f_t(n,m), y_t\right) \cdot \left(\frac{\mathbb{V}(y_t)}{\mathbb{V}(f_t(n,m))}\right)^{1/2},$$

the low shares of variance explained by real rate and inflation expectations must be due to the fact that expectations are only weakly correlated with forward yields. This is consistent with aggregate shocks affecting the components of the yield curve in different ways.[21]

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[21]Note that the importance of term premiums for variations in Treasury yields is not driven by the recent financial crisis and the large-scale asset purchases undertaken by the Federal Reserve. When repeating the variance decompositions ending the sample in 2007, we find that term premiums played an even larger role before the financial crisis.

Figure 11: **Short Rate Forecast errors vs. Changes in LR Forecasts.** This figure compares forecast errors with changes in long-run objects. Nominal and real short-rate forecast errors are calculated as the backward-looking 12-month cumulative sum of monthly forecast errors. Revisions to long-run forecasts and changes in term premiums are defined as changes relative to 12 months ago. The sample period is January 1984–December 2019.
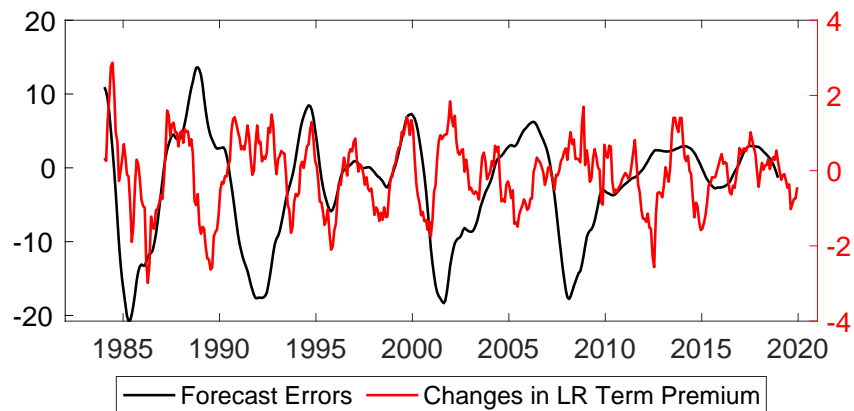
(a) *Nominal short rate forecast Errors vs. Revisions in Long-run (9Y1Y) Forecasts*
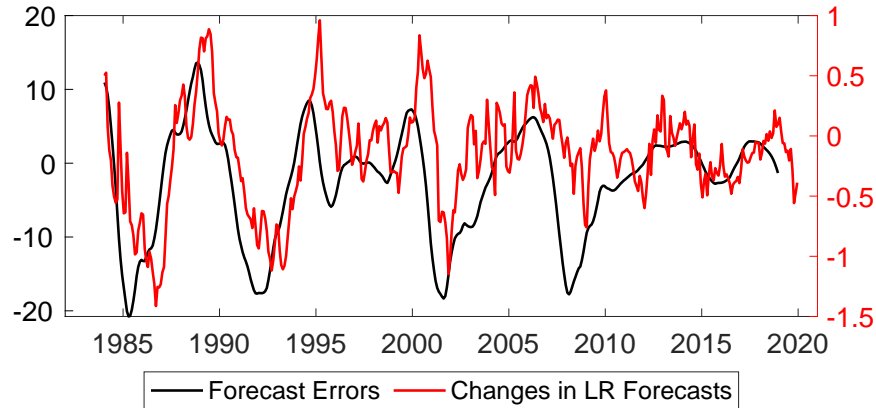
(b) *Real short rate forecast errors vs. Revisions in Long-run (9Y1Y) Forecasts*
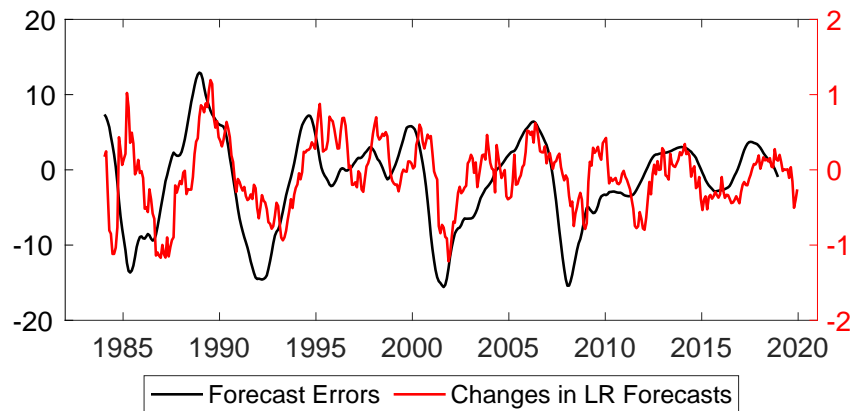
(c) *Nominal short rate forecast errors vs. Changes in the 9Y1Y Term premium*

To gain further intuition about the determinants of the longer-term interest rates we return to the simple model of Section 2.1. Revisions in long-term expectations should be positively related to short-term forecast errors. The two top panels in Figure 11 show this is indeed the case for both nominal and real interest rates. The in-sample correlation is 58% and 43% for nominal and real short rates. This implies that market participants update their views about the long-term mean of the nominal and real short rate in response to new information captured by forecast surprises. The bottom panel that shows the correlation between short-term forecast revisions and our measure of the term premium tells a different story. A correlation of only -9% suggests that other forces determine interest rate term premiums.

Co-movement in the cross-section. We have discussed the weak co-movement of expected short rates and forward rates in the time dimension. Next, we uncover another important difference by looking across bond maturities. A long literature in finance has documented that government bond yields feature substantial co-movement across maturities (e.g., Garbade 1996, Scheinkman and Litterman 1991). This is also true in our sample: the first two principal components extracted from the ten maturities shown in Table 3 explain 97 and 3 percent of their joint variation. The loadings of these principal components confirm the common interpretation as level and slope of the yield curve.

Based on our decomposition of forwards into expected short rate and term premium components, we can parse out the sources of the strong cross-sectional correlation. In line with the results in Table 3, almost half of the variance of the level factor is explained by term premiums, one third by expected inflation and the remaining 20 percent by expected real short rates. Also consistent with the variance decompositions for individual forwards, almost 90 percent of the month-to-month variation in the level factor and more than three quarters of the year-over-year variation are explained by term premiums. The expectations components are somewhat more important for the slope factor: 85 percent of its variation is accounted for by expected real short rates, about 10 percent by expected inflation and the remainder by term premiums. However, more than two thirds of the month-to-month variation of the slope factor is explained by term premiums, in line with the above finding that only a small share of the yield curve variation at higher frequencies is driven by expectations.

Figure 12: **Co-Movement of Expected Rates and Term Premiums.** These figures show 12-month changes in forward rates (top chart), expected forward nominal short-term rates (middle chart) and the forward term premium (bottom chart). The sample period is January 1984–December 2019.

(a) *Forward Rates*

(b) *Expected Forward Rates*

(c) *Forward Term Premiums*
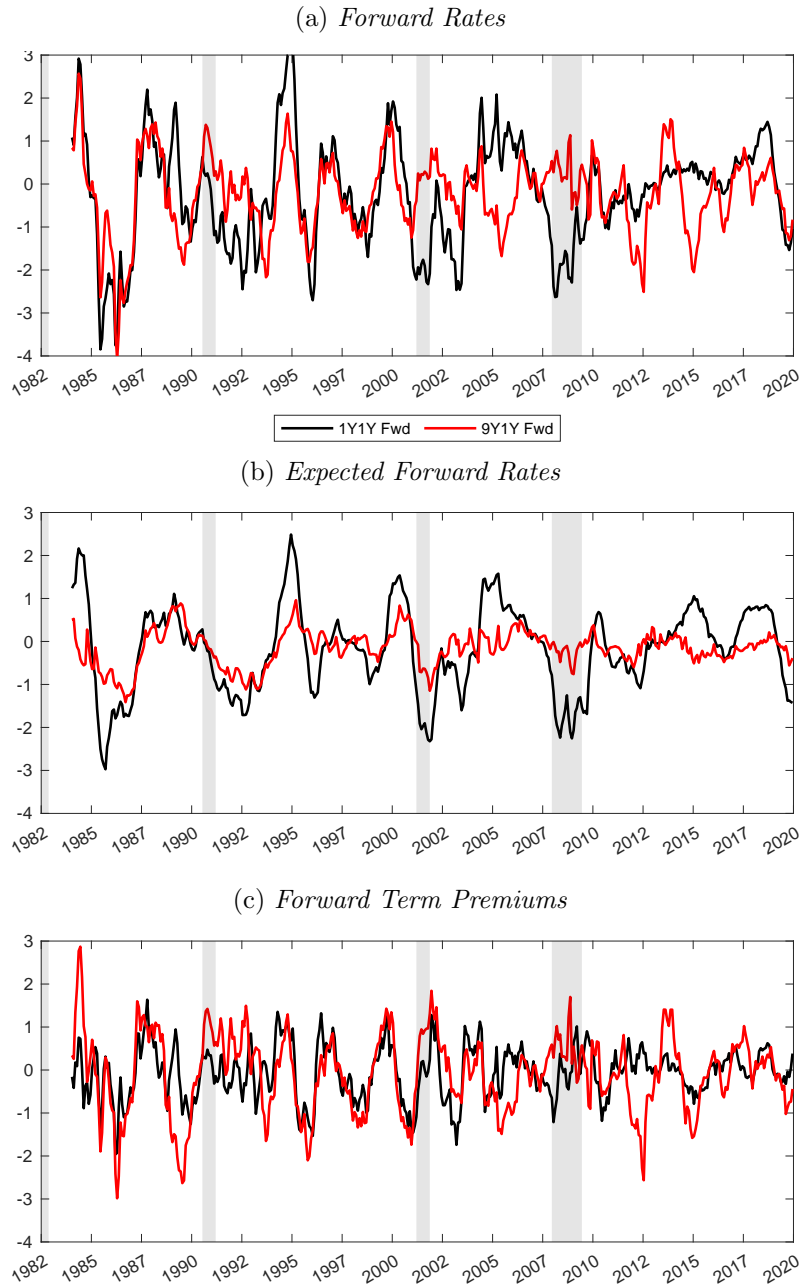
Figure 12 visualizes the importance of term premiums for the strong co-movement across maturities. It shows twelve-month changes in short and long-maturity forward rates (top panel), expected rates (middle panel), and forward term premiums (bottom panel) for the 1Y1Y and 9Y1Y forward maturities. The figure documents that across maturities survey-based term premiums co-move much more strongly than survey-based expected future short rates, or forwards themselves. Twelve

month changes in expected rates at short and long horizons are only weakly correlated, whereas changes in forward term premiums co-move in lockstep, at least until the mid to late 2000s.

Note that the strong co-movement of term premiums is a feature of the data and is not imposed in any way in our analysis as term premiums are obtained as residuals between observed forwards and expected average short rates. Term premiums equal average expected short-term excess holding period returns over the life of a bond, see Equation (5) in Cochrane and Piazzesi (2008). Hence, our finding of a strong comovement of term premiums across maturities is consistent with a strong factor structure in expected excess returns as also documented by Cochrane and Piazzesi (2005). Interestingly, we observe a break in this co-movement around the financial crisis. This might be capturing the unconventional monetary policy actions undertaken during that period, with particularly strong effects on term premiums of longer-term bonds.

Forecast performance. The previous results based on the decomposition of longer-term interest rates into expected short rate paths and term premiums rely on the quality of survey-based short rate forecasts. While a formal forecast evaluation is beyond the scope of this chapter, we illustrate the precision of survey-based short rate expectations by visually comparing them with the expected short rates implied by the forward curve.[22] As discussed above, these expected short rate paths are consistent with the expectations hypothesis in the absence of any term premiums.
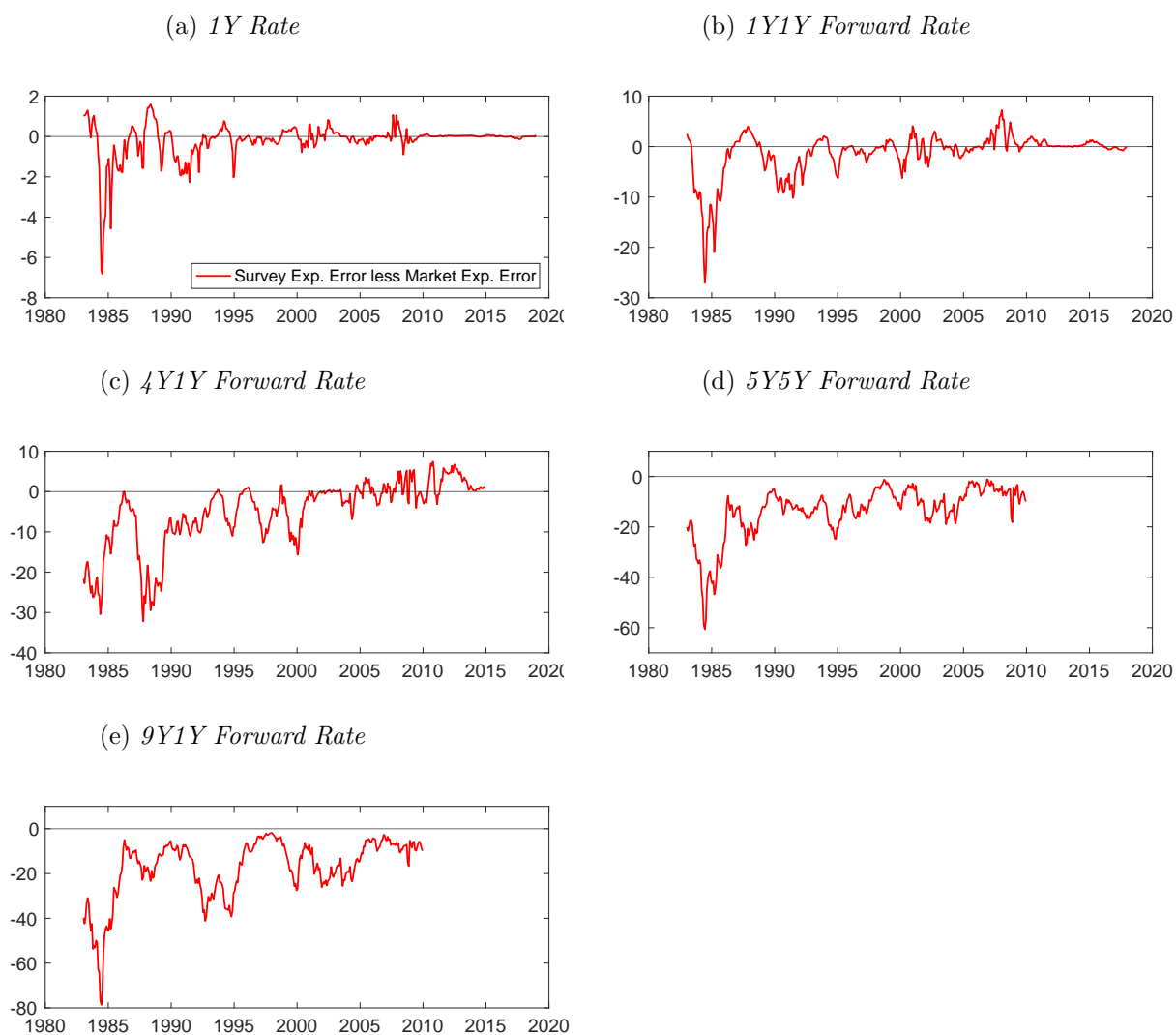
In fact, forward rates are often interpreted as market-based (or risk-neutral) expectations and used as an alternative to professional surveys. Cochrane (2017) argues that "risk-neutral probabilities are a good sufficient statistic to make decisions." We have seen above that forward-based short rate paths display very different dynamics compared to our survey-based short rate expectations. They are typically steeper and lie above those of surveys, in line with the notion of a time-varying term premium that is positive on average. Figure 13 compares the forecast performance, as measured by the difference of squared errors over time for different forward horizons. The main takeaway is that our survey-based measure of short rate expectations has on average performed substantially better in predicting short rates than the forward curve, and the performance gap widens with the forecast horizon. This finding is consistent with the notion that forecasters do not simply report risk-neutral expectations extracted from the forward curve when being surveyed about short rates, a conclusion also shared by Adam et al. (2021). At the same time both measures have come closer in the past 20 or so years, reflecting the overall decline of term premiums. In addition, forward rates appear to have performed somewhat better at intermediate horizons since around 2010: this is consistent with the notion that professional forecasters were more optimistic about the normalization process at that time, while terms premiums where compressed.

Summing up, we offer two conclusions. First, we document that the term structure of short rate expectations is fairly volatile and its behavior is consistent with market participants frequently

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[22]Crump et al. (2011) study the forecast accuracy of professional forecasts for the federal funds rate based on BCFF data. They show that forecast accuracy is negatively correlated with the variation in the federal funds rate (i.e., forecast accuracy is generally worse during easing and tightening cycles). Moreover, they show that forecast accuracy has been consistently improving across tightening cycles over their sample period (1982 through mid-2011). In contrast, there is no such improvement when the forecast evaluation is restricted to easing cycles. Cieslak (2018) discusses the relationship between short-rate forecast errors and the dynamics of bond returns.

updating their beliefs about the medium- to long-term evolution of the policy rate. Second, we show that the term structure of interest rates is only partly driven by short rate expectations. The residual component, the term premium, plays a key role in determining the equilibrium evolution of interest rates at longer maturities. This component is often left unaccounted for in standard macroeconomic models used for monetary policy analysis. In the next section, we argue that disagreement among forecasters about the long-run mean of the short rate can partly explain the secular decline of term premiums documented here and in previous work.

Figure 13: **Forecast errors.** This figure shows the difference between the mean-square forecast error (MSFE) of survey expectations less the MSFE of market expectations. Values below zero indicate that survey expectations have a lower MSFE. The sample period is January 1983–December 2019.

(a) *1Y Rate*

(b) *1Y1Y Forward Rate*

(c) *4Y1Y Forward Rate*

(d) *5Y5Y Forward Rate*

(e) *9Y1Y Forward Rate*

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5   The Term Structure of Disagreement

Throughout the paper we have focused on a consensus measure of expectations. While this term structure of expectations can be enormously useful in many economic applications, it neglects the dispersion in individual forecasts that has been widely documented for professional forecasters, households, firms and policymakers. A growing literature studies the implications of this empirical observation for macroeconomics and finance.[23] In this section, we document substantial disagreement about the expected nominal short rate path, particularly in the long-run. We show that the term structure of disagreement of short rates starkly differs from that observed for output and inflation and discuss a modeling framework that can explain these differences. We then use this framework to study the determinants of short rate disagreement. Finally, we link short rate disagreement to the evolution of term premiums in a heterogeneous agent term structure model. Along the way, we selectively review the literature on forecast disagreement.

5.1   Disagreement About the Short Rate, Real Output Growth, and Inflation

We now document some basic facts about the term structure of disagreement using the Blue Chip Financial Forecasts (BCFF) survey. A strength of this data-set is that it contains forecasts at very different horizons, from one-quarter ahead to ten-years ahead. This provides a clear picture of the term structure of expectations over a long sample. While it does not provide the full distribution of survey responses at medium to long horizons, the BCFF reports the average of the top and bottom deciles of all respondents at these longer horizons.

Andrade et al. (2016) propose using the difference between the average response of the top and bottom deciles of forecasters as a measure of disagreement. They show that at shorter forecast horizons for which the full distribution of individual forecasts is available, this measure is strongly correlated with other commonly used measures of forecast dispersion such as the cross-sectional standard deviation or the interquartile range. Figure 14 reproduces some stylized facts from Andrade et al. (2016). The left panel shows the term structure of disagreement. That is the average disagreement at each forecasting horizon. The nature of disagreement is fundamentally different across macroeconomic variables: the term structure of disagreement is upward-sloping only for short rate forecasts while it is flat across horizons for inflation and downward-sloping for output growth. The right panel shows the time series of disagreement at long horizons for output, inflation and the short-term interest rate. Evident are both cycle and trend components, which differ across variables.

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[23]For a review of the literature see Angeletos et al. (2020), Andrade et al. (2016) and references therein.

Figure 14: **The Term Structure of Disagreement.** This figure shows selected statistics for forecaster disagreement from the Blue Chip Financial Forecasts survey. Disagreement is defined as the average forecast of the highest ten responses minus that of the lowest ten responses of survey participants (in percent). The left panel shows the term structure of disagreement averaged across time for real output growth, CPI inflation, and the federal funds rate for various forecast horizons. Q1Q4 denote the one- through four-quarter ahead forecasts, Y2Y5 denote the two- through five-year forecasts, and Y6-11 captures the average forecast for horizons from 6-to-11 years ahead. The right panel displays the time series of the 6-to-11 years ahead forecast disagreement for the three variables.
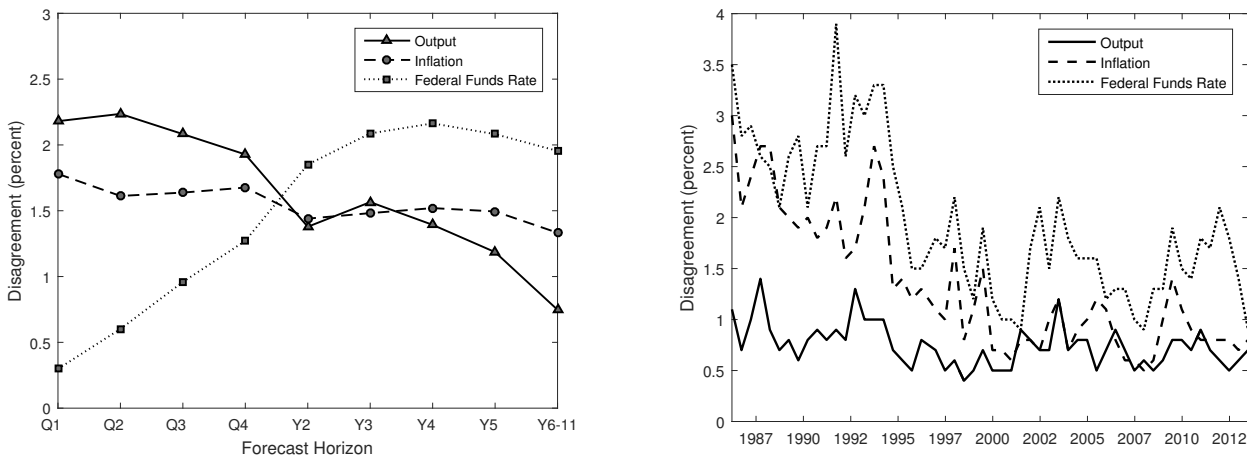
Disagreement in professional forecasts: selected further evidence. Other data sources have been used to discuss features of forecast dispersion. Among the earliest contributions is Mankiw et al. (2003) who document that the disagreement about U.S. inflation expectations from various surveys of consumers and professional forecasters (not including the Blue Chip survey) is time varying. They also study the correlation of inflation disagreement with changes in macroeconomic variables such as inflation and GDP growth and find weak evidence of a link. Lahiri and Sheng (2008) and Patton and Timmermann (2010) use the Consensus Economics survey to discuss the evolution of output growth and inflation forecast disagreement for the U.S. up to two years in the future.[24] A few other papers have studied various aspects of the disagreement among forecasters. Dovern et al. (2012) study the behavior of one-year ahead forecasts of real GDP growth, inflation, and nominal short rates for G7 countries and find that short-term disagreement differs across the three variables and across countries. Wright (2011) shows that disagreement of one-year ahead inflation forecasts from the Consensus Economics survey is correlated with nominal term premia in a number of countries. He measures disagreement as the cross-sectional standard deviation of individual inflation forecasts and argues that this variable captures inflation uncertainty.

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[24]As discussed in Andrade et al. (2016), their term structure of disagreement does not have the same features as that in the BCFF because of key differences between the two sources of survey data, in particular in terms of the definition of forecasts as fixed-horizon (e.g. one year ahead) or fixed-event forecasts (e.g. for a particular calendar year).

Using data on individual point as well as density forecasts from the U.S. Survey of Professional Forecasters, Zarnowitz and Lambros (1987) study the relationship between consensus forecasts and measures of uncertainty while Rich and Tracy (2010) show that disagreement about U.S. inflation is not systematically related to measures of inflation uncertainty. Lastly, Boero et al. (2008) study the relation between forecast uncertainty and disagreement up to two years into the future for a UK survey of professional forecasts and find a sustained reduction of inflation uncertainty after the introduction of a formal inflation targeting regime by the Bank of England.

5.2 What Drives Disagreement About the Short Rate?

Section 3.5.3 showed the secular trend in long-run forecasts of the nominal and real short rate and discussed some potential factors behind this decline. Here we provide a similar investigation of the drivers of long-run disagreement about the short rate. Consistent with the treatment of consensus expectations, forecasters must disentangle slow-moving changes from short-term fluctuations. Following Andrade et al. (2016), each individual forecaster has imperfect information about the current state of the economy because they only infrequently update their information set; this is an economy with sticky information similar to Mankiw and Reis (2002).[25]

In terms of the model outlined in Section 3.1, at any point in time individual forecasters see only a subset of new observations about the three variables in the vector $z_t = (\ g_t \quad \pi_t \quad i_t\ )'$ with a constant probability.[26] This implies the Kalman filter involves a time-varying observation equation

$$z_t^{(j)} \ = \ h_t^{(j)} \left(\begin{array}{c} x_t \\ \omega_t \end{array} \right)$$

where $z_t^{(j)}$ denotes the subset of variables observed by forecaster j in period t, and x_t and ω_t are the unobserved cycle and trend components. The dimension of the observation matrix $h_t^{(j)}$ changes across agents and across time depending on which elements of z_t are observed. At any point in time only a fraction of agents will observe a particular element of z_t. When agents do not observe the full vector z_t they use the Kalman filter with missing observations. Updating of the unobserved trend component in this model is then captured by the following recursion

$$\hat{\omega}_t^{(j)} \ = \ \hat{\omega}_{t-1}^{(j)} + \kappa_{\omega,t}^{(j)} \times \left(z_t^{(j)} - z_{t|t-1}^{(j)} \right)$$

where $\kappa_{\omega,t}^{(j)}$ denotes the time-varying Kalman gain that changes size with $z_t^{(j)}$. These assumptions deliver dispersed forecasts at all forecast horizons because agents observe a different slice of available information at any point in time.

Calibrated to survey forecasts of realized output growth, inflation and the nominal short rate, the model replicates closely the term structure of disagreement observed in the data. Importantly, Andrade et al. (2016) show that the combination of sticky information and the trend-cycle infer-

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[25] Andrade et al. (2016) show that an information friction where agents only observe noisy signals of the true variables they intend to predict (Sims (2003) or Woodford (2003a)) yields similar results.

[26] This probability is allowed to be different across variables.

ence problem is the key ingredient to matching both the large movements in long-run forecasts of the short-term interest rate and the substantial disagreement across forecasters particularly at longer horizons. Moreover, a multivariate model trend-cycle inference problem is required to explain the upward-sloping term structure of short rate disagreement. To see this, note from Figure 14 that there is essentially no disagreement about the policy rate at the shortest forecast horizon of one-quarter ahead. This is consistent with the notion that the Federal Reserve has been communicating its policy intentions carefully over the past several decades. As shown by Andrade et al. (2016), a univariate sticky information model with a trend-cycle decomposition would not be able to match the strongly upward-sloping term structure of short rate disagreement under reasonable calibrations of information stickiness. However, in a multivariate forecasting model the larger disagreement about inflation and real output growth at short horizons combined with the trend-cycle decomposition generates the strongly upward-sloping term structure of short rate disagreement.

To quantify the relative importance of disagreement about the long-run fundamentals of inflation and real growth for short rate disagreement, we take a more structural approach and postulate that each forecaster j has in mind a monetary policy rule of the form

$$
\begin{aligned}
i_t &= \rho \cdot i_{t-1} + (1-\rho) \cdot i_t^{\star,(j)} + \varepsilon_t \\
i_t^{\star,(j)} &= \hat{\omega}_t^{i,(j)} + \varphi_\pi \cdot \left(\pi_t - \hat{\omega}_t^{\pi,(j)} \right) + \varphi_g \cdot \left(g_t - \hat{\omega}_t^{g,(j)} \right).
\end{aligned}
$$

where ρ measures the degree of interest rate smoothing, φ_π defines the response to inflation in deviation from its long-run perceived mean and φ_g captures the response to output growth in deviation from its mean. For simplicity, we assume agents know and agree on the policy rule coefficients but are uncertain about the long-term drifts, summarized in the term $i_t^\star$. As before, we impose the Fisher equation holds in the long run and identify the following components driving the nominal short rate in the long run

$$
i_{\infty|t}^{\star,(j)} \equiv \omega_t^{i,(j)} = \omega_t^{\pi,(j)} + \omega_t^{g,(j)} + \beta_t^{(j)} \tag{5.1}
$$

where the long-run mean of the nominal short rate is driven by the perceived long-run mean of inflation, output growth and the residual factor $\beta_t^{(j)}$ capturing changes in the aggregate discount factor. Andrade et al. (2016) show that the reduced-form model used to estimate the term structure of disagreement is consistent with the policy rule above with coefficients that are similar to those commonly used in the literature.

Figure 15 shows the decomposition of the term structure of disagreement in interest rate forecasts into its three long-run components. The red line displays the term structure of disagreement in the data ranging from the average dispersion at the one quarter horizon to the average disagreement $6 - 10$ years ahead. The measure of disagreement used here is again the difference between the top-10 and bottom-10 average forecasts from the BCFF survey. The black line measures the model fit, which captures tightly the size and upward slope of disagreement. Our decomposition suggests that all of the long-run components of the policy rule (black lines) contribute to disagreement at

long horizons. First, a model with a constant long-run component ($i_\infty^{\star,(j)} = \bar{i}$, dotted line) fails to deliver a meaningful slope of the term structure of disagreement. Second, disagreement about the long-run rate of inflation ($i_{\infty|t}^{\star,(j)} = \omega_t^{\pi,(j)} + \bar{r}$, dashed line) goes about one-third of the way toward explaining policy rate disagreement. While this is consistent with inflation expectations playing an important role, it also suggests that other factors are important, especially starting in the late 1990s when longer-run inflation expectations stabilized.

Similarly, disagreement about the real growth rate of output ($i_{\infty|t}^{\star,(j)} = \omega_t^{\pi,(j)} + \omega_t^{g,(j)} + \bar{\beta}$, solid line) plays an important role in boosting long-term disagreement about policy rates. In a structural model, its contribution depends on the intertemporal elasticity of substitution which here is assumed to be unity. A lower intertemporal elasticity of substitution would increase the role of disagreement about long-run growth in explaining observed short rate disagreement. However, the residual factor β_t, affecting the demand and supply of saving, plays an important role in driving long-run interest rate expectations and forecast dispersion. This is consistent with the suggestive evidence discussed in Section 3.5.3 that movements in risk premia or convenience yields are needed to explain the secular decline in real rate expectations.

Summing up the evidence presented in this section, the framework presented in this chapter proves to be useful not only in capturing the behavior of consensus expectations, but it is also able to capture key features of the observed term structure of forecaster disagreement.

Figure 15: **Decomposing policy rate disagreement** This figure displays the model-implied disagreement for different values of $(\rho; \rho_\phi; \rho_g)$ along with the Blue Chip Financial Forecasts survey (red) and with different assumptions about $i_\infty^{\star,(j)}$. The "standard rule" is given by $(\rho; \rho_\phi; \rho_g) = (0.90; 2.0; 0.5)$.
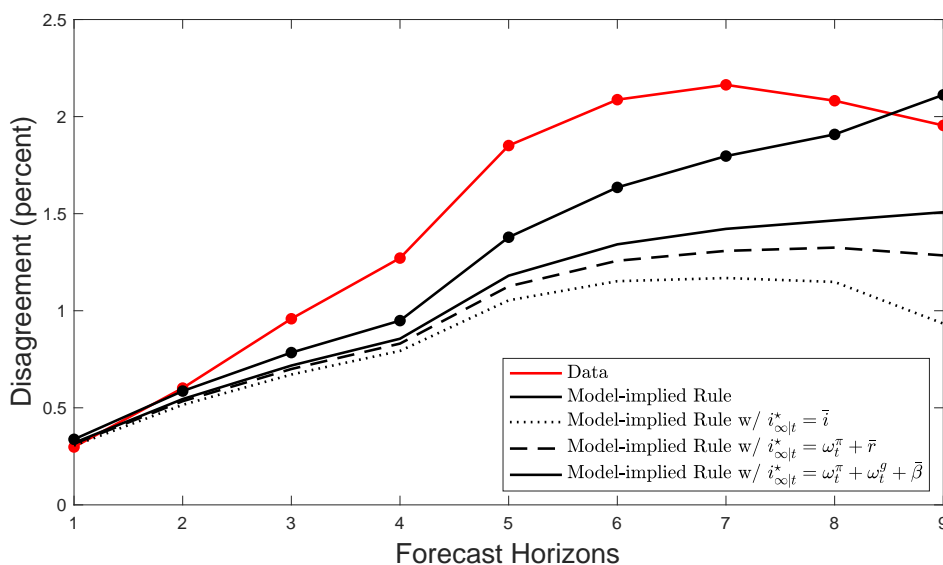
5.3 Interest Rate Disagreement and the Term Premium

In Section 4 we showed that while revisions of long-term policy rate forecasts are strongly correlated with short-term forecast errors of the short rate, changes in the term premium, measured as the difference between observed yields and survey-based consensus forecasts of the short rate, are essentially uncorrelated with these short-term forecast errors. Here we explore the link between the term premium and forecast dispersion about policy rates. Figure 16 plots the ten-year term premium along with the difference between the top-10 and the bottom-10 average forecasts of the federal funds rate from the BCFF survey for two horizons: five years ahead and 7-11 years ahead. The chart shows a strong correlation between the consensus term premium and short rate disagreement. Both have been declining over the past three decades, but have also been comoving at higher frequencies. This evidence is suggestive of a link between market-wide measures of risk premiums and short rate disagreement.

Figure 16: **Term premiums and short rate disagreement.** This chart shows the 5-year/5-year forward Treasury term premium together with a measure of policy rate disagreement at similar horizons. Disagreement is measured as the difference between the top-10 and bottom-10 average forecasts of the 3-month T-bill at the five-year or six-year ahead ("Y6") and 6-10 or 7-11 years ahead ("Y7-11") horizons from the Blue Chip Financial Forecasts survey.
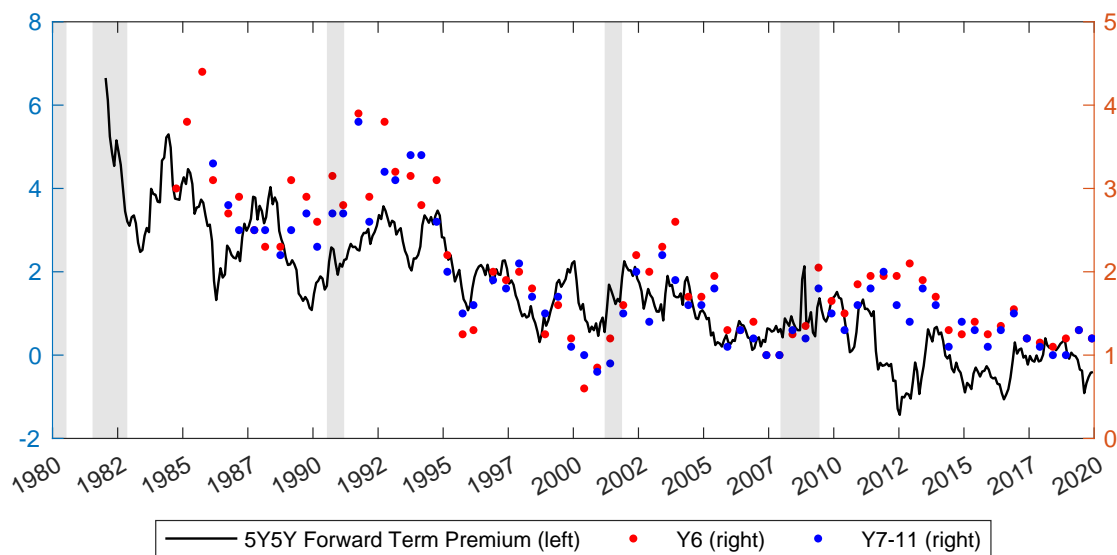
While the traditional approach to modeling the term structure of interest rates, both in structural and reduced-form models, relies on a representative agent formulation, a recent literature has made progress incorporating beliefs heterogeneity in asset pricing models. In general, in this literature heterogeneity drives a wedge between the marginal and average investor, with implications for asset prices. As shown by Jouini and Napp (2007) and Bhamra and Uppal (2014), among others, in models with heterogeneous beliefs investors with a higher share of the total wealth have a greater impact on the marginal valuation of risky assets. Xiong and Yan (2010) apply this idea to a model of bond pricing. Consider an econometrician measuring an aggregate term premium in a

heterogeneous belief economy under the incorrect assumption of a representative investor. They show that the estimated term premium can display time-varying fluctuations simply because of changes in relative wealth and disagreement, even if individual term premiums are constant.

We build on this insight but focus on the role of disagreement about the long-run mean of the policy rate for variations in measured term premiums. Cao et al. (2021) estimate a heterogeneous expectations affine term structure model in which bond yields are driven by three observable factors: level, slope, and curvature. In contrast to the extant literature, and consistent with the modeling framework described in Section 3, they assume that the level factor has a time-varying long-run mean which follows a random walk. Investors have different beliefs about this long-run mean, implying different views about the long-run level of the short rate and therefore differences in the perceived risk-return trade-off of longer-term bonds. Their estimated model fits observed yields and short rate forecasts of the top-10 and the bottom-10 average of respondents from the BCFF survey closely.

Figure 17 displays the evolution of expected short rate paths for the average top-10 and bottom-10 investors as implied by the model of Cao et al. (2021). Three insights are immediate. First, the term structure of expectations across these two types of investors are markedly different. Indeed, at various times the term structure of expectations are rising for one group, while falling for the other. Second, disagreement about the long-term evolution of the short rate has been declining over time: from more than four percentage points in the mid 1980s to around one percentage point at the end of the sample. Third, the term structure of disagreement, defined as the average disagreement over the sample period over different forecast horizons is upward-sloping: forecasters have more dispersed interest rate expectations in the long run. For example, while the average wedge between the near-term short rate forecasts of the top-10 and bottom-10 average BCFF survey respondents is less than 0.5%, it exceeds 2% for forecast horizons beyond three years.

Figure 17: **Forecast Paths Disagreement across BCFF forecasters.** This figure shows the fitted short rate rate forecast paths for the top-10 and bottom-10 average of BCFF respondents from Cao et al. (2021). The sample period is March 1983–August 2015.
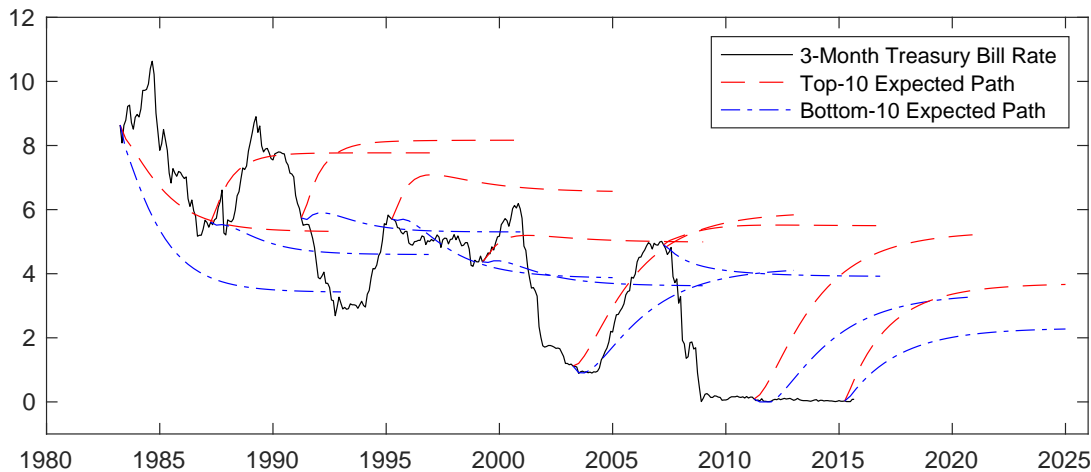

A feature of the Cao et al. (2021) model is that given equilibrium yields, investors expecting higher future short rates expect lower excess returns on longer-term bonds, and vice versa. Accordingly, the expected short rate path of the top-10 average BCFF forecaster translates into slightly negative subjective term premiums at all maturities. In contrast, the bottom-10 average expected short rate path implies a subjective term premium that ranges from about 50 basis points at the one-year maturity to about two percent at the ten-year maturity. In accordance with these perceived risk-return tradeoffs, the top-10 investor would tilt their portfolio towards short-term bonds while the bottom-10 investor would invest more in longer-term bonds. Against the background of a secular decline in yields over the sample period from 1983-2015, Cao et al. (2021) show that these portfolio allocations result in a strongly increasing wealth share of the bottom-10 investor. Compounded with a decline of this investors' perceived term premium, the model generates a wealth-weighted market-wide term premium that is fairly stable over time. This is in contrast to the literature based on representative agent term structure models which commonly estimate term premiums that feature a strong secular decline (e.g. Wright (2011), Kim and Orphanides (2012), Adrian et al. (2013). The heterogeneous beliefs bond pricing model in Cao et al. (2021) suggests that this secular decline captures expected returns of those investors that better anticipated the observed persistent decline of short rates over the sample.

To illustrate this, Figure 18 displays the ten-year term premium implied by the consensus BCFF short rate path from Section 4 along with an aggregate term premium derived from weighting the two investors' subjective term premiums with their respective wealth share. In contrast to the consensus term premium, this aggregate heterogeneous beliefs term premium displays essentially no trend.

In sum, the observed heterogeneity of short rate expectations implies a large wedge between the

beliefs of the marginal and the average investor. The findings thus show that the term structure of disagreement about policy rate expectations is highly informative about the pricing of risk in the bond market.

Figure 18: **Aggregate term premiums**. The figure compares the consensus term premium (red line) from Section 4 to the aggregate term premium obtained as the wealth-weighted average of the term premiums perceived by the top-10 and bottom-10 investors (black line) from Cao et al. (2021).
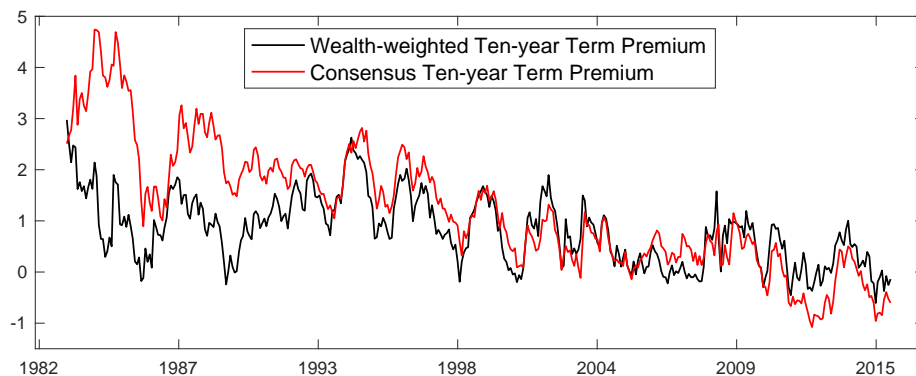
Disagreement and term premiums: other work. A growing recent literature has linked forecast disagreement and the bond market. As already mentioned above, Wright (2011) shows that the secular decline of term premiums implied by a standard affine model and model-free term premiums implied by consensus survey expectations across ten developed economies coincide with a decline of inflation forecast dispersion since the 1990s. Ehling et al. (2018) document empirically that the dispersion of inflation expectations has a strong effect on real and nominal bond yields over and above the impact of expected inflation. Buraschi and Whelan (2016) study the interactions between risk aversion and disagreement and present a model where heterogeneous beliefs arise because agents have different views about the (constant) long-run growth rate of consumption and because their perceptions of the correlation of shocks differs. More recently, Buraschi et al. (2021) aggregate individual expected excess bond returns based on forecasters past accuracy in predicting interest rates. In line with the findings presented here, they document that disagreement about bond risk premiums is time-varying and persistent. Barillas and Nimark (2019) build a structural model of the term structure in which investors with heterogeneous information sets form higher-order expectations about the beliefs of all other investors. Equilibrium bond prices then reflect a speculative component which depends on investors beliefs about the error that the average investor makes when predicting future short rates. Their model suggests that the speculative component explains a sizable fraction of the variation in U.S. Treasury yields. Barillas and Nimark (2017) generalize this model to allow for richer price of risk specifications as used in the empirical term structure literature. In their model, investors observe heterogeneous signals of the state variables driving bond yields. Finally, Giacoletti et al. (2021) build a dynamic term structure model in which a representative investor updates her beliefs about future bond yields. They find that when this

updating is conditioned on the dispersion in bond yield forecasts, the model produces substantially smaller forecast errors.

6 The Term Structure of Expectations in Structural Models

The analysis of the term structure expectations has yet to take a stand on the rationality of expectations. We now develop a dynamic structural general equilibrium model in which the subjective beliefs of households and firms are consistent with our reduced-form forecasting model. The structural model has the equilibrium property that subjective beliefs are more persistent than the true data generating process. Household and firm expectations exhibit extrapolation bias, consistent with empirical and laboratory evidence. That subjective and objective forecasting models differ represents a departure from full-information rational expectations.

An advantage of a structural model is that we can estimate how different economic shocks determine forecast errors and the term structure of expectations. In principle, this permits addressing some earlier questions—such as the determinants of real neutral rates and interest rate terms premia—left unanswered by the reduced-form model. Having a structural theory of long-term expectations permits analysis of important practical policy questions, that rational expectations can not. For example, we argue that our framework provides a coherent definition of expectations anchoring, and clear predictions of the economic conditions in which expectations will be anchored or un-anchored. We show this has important implications for monetary and fiscal policy.

6.1 A General Structural Model

Dynamic stochastic general equilibrium (DSGE) models describe the behavior of economic agents solving infinite-horizon intertemporal decision problems in a market economy. The equilibrium behavior of the economy therefore depends primarily on the expected path of aggregate variables such as prices, aggregate quantities and policy variables. The log-linear solution of a typical DSGE can then be generally expressed as

$$A_0 z_t = \sum_{s=1}^{n} A_n \left(E_t \sum_{T=t}^{\infty} \lambda_s^{T-t} z_{T+1} \right) + A_{n+1} z_{t-1} + A_{n+2} \varepsilon_t$$

where the vector z_t collects all models variables in deviation from their steady-state values; the vector ε_t collects exogenous innovations; $\lambda_s \in 1, ..., n$ are discount factors resulting from the agents's decisions rules; A_i for $i \in 1, ..., n+2$ coefficient matrices; and

$$E_t = \int_0^1 E_t^i di$$

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average beliefs. The representation holds for arbitrary beliefs, including rational expectations.[27] Under this latter assumption the model has the equilibrium solution

$$z_t = \Phi z_{t-1} + \Phi_\varepsilon \varepsilon_t.$$

Agents are fully informed about the economy steady-state; they face no information frictions leading to fluctuating long-run beliefs. The matrix Φ measures the transitional dynamics around the steady state, and Φ_ε captures the economy's impact response to innovations.

Now introduce an information friction to this full information benchmark. Consistent with our simple model in sections 2.1 and 3.1 agents are uncertain about the long-run. Expectations are then formed using the forecasting model

$$
\begin{align}
z_t &= \omega_t + \Phi z_{t-1} + \nu_t \tag{6.1} \\
\omega_t &= \omega_{t-1} + \eta_t \tag{6.2}
\end{align}
$$

where both the drift ω_t and the innovations ν_t are unobserved to agents.[28] Reflecting the perceived slow-moving drift we further assume agents' priors imply $E\left[\eta_t \eta_t'\right] = \kappa_\omega^2 \times E\left[\nu_t \nu_t'\right]$ where $1 >> \kappa_\omega > 0$. This signal extraction problem delivers the now familiar Kalman filter updating

$$
\begin{align}
\hat{\omega}_{t+1} &= \hat{\omega}_t + \kappa_\omega \left(z_t - \hat{\omega}_t - \Phi z_{t-1}\right) \tag{6.3} \\
&= \hat{\omega}_t + \kappa_\omega \left(z_t - z_{t|t-1}\right) \tag{6.4}
\end{align}
$$

where $z_t - z_{t|t-1}$ denotes the short-term forecast error. Given the estimate $\hat{\omega}_t$, forecasts at any horizon $T > t$ are determined as

$$E_t z_T = \Phi^{T-t} z_t + \sum_{j=0}^{T-t} \Phi^j \hat{\omega}_t \tag{6.5}$$

while their estimate of the time-varying mean is

$$\lim_{T \to \infty} E_t z_T = (I - \Phi)^{-1} \hat{\omega}_t. \tag{6.6}$$

Evaluating expectations in the structural equations and combining with the belief updating equation gives the true data generating process

$$
\begin{align}
z_t &= \Gamma \times \hat{\omega}_t + \Phi z_{t-1} + \Phi_\varepsilon \varepsilon_t \\
\hat{\omega}_{t+1} &= \left[I + \kappa_\omega \left(\Gamma - I\right)\right] \hat{\omega}_t + \kappa_\omega \Phi_\varepsilon \varepsilon_t \\
z_t - z_{t|t-1} &= (\Gamma - I)\hat{\omega}_t + \Phi_\varepsilon \varepsilon_t
\end{align}
$$

---

[27] All variables are expressed in log-deviations from their non-stochastic steady-state values. See Eusepi et al. (2020) for a detailed example and derivations. For an analysis of real business cycle theory see Eusepi and Preston (2011).

[28] For simplicity we assume the matrix Φ which governs short-run dynamics is known to agents.

where Γ is composite of structural parameters. Provided $\hat{\omega}_t$ is stationary, the rational expectations equilibrium represents a limiting case of this model with $\kappa_\omega \to 0$.[29]

Comparison with the reduced-form model of earlier sections reveals four new properties of structural models. First, forecast errors are determined by long-run drifts and model innovations. This permits giving a structural interpretation to the economic determinants of the term structure of expectations. Second, because economic decisions depend on the estimated drifts, dynamics exhibit self-referentiality. Beliefs affect realized data, and the data in turn affect beliefs. The matrix Γ determines the extent to which equilibrium outcomes depend on beliefs.[30] Third, and related, the model displays *extrapolation bias* (Fuster et al. (2010), Bordalo et al. (2020)) as an equilibrium property.[31] Subjective beliefs have a unit root, while the true data-generating process implies beliefs evolve as an auto-regressive process of order one, with eigenvalues determined by

$$I + \kappa_\omega \left(\Gamma - I \right).$$

The wedge between subjective and objective beliefs provides a metric of the importance of the information friction from belief formation. And again, the use of a structural model permits understanding how structural shocks and economic policy affect this wedge. Fourth, short-term forecast errors as are not i.i.d., a further manifestation of bounded rationality.

We now consider two applications of this general framework. In both applications these general properties are vital to being able to answer policy-relevant questions as well as providing a coherent account of the data.

6.2 Application 1: Anchored Inflation Expectation

An important practical concern for policymakers is whether long-term inflation expectations are well anchored or not. Greater stability of long-term inflation expectations improves the short-run trade-off between inflation and output, improving welfare. Carvalho et al. (2021) propose a structural model of inflation and inflation expectations which determines the economic conditions under which long-term expectations will be well anchored. The model permits answering questions like: what determines trend inflation? Will chronic undershooting of inflation targets lead to downward drift and unanchoring of long-term inflation expectations? How can we reconcile large negative output gaps and stable inflation during the Great Recession with positive output gaps and high inflation during the Great Inflation?

The model comprises a new Keynesian aggregate supply curve

$$\pi_t = \mu_t + E_t \sum_{T=t}^{\infty} (\xi\beta)^{T-t} \left[\kappa x_T + (1-\xi) \beta \pi_{T+1} \right] \tag{6.7}$$

---

[29]That is the eigenvalues of $I + \kappa_\omega \left(\Gamma - I \right)$ are inside the unit circle.

[30]In the case that $\Gamma = I$ the model would have a self-confirming equilibrium (see Sargent 1999). In general $\|\Gamma\| < 1$ so that beliefs are only partially self-fulfilling. This implies beliefs in equilibrium are stationary variables

[31]Recall the discussion in section 3.4

where π_t is inflation, x_t the output gap, μ_t an exogenous i.i.d. cost-push shock. The parameters $0 < \beta, \xi < 1$ are the household discount factor and the probability that the firm cannot re-optimize their price in any given period. These parameters determine the slope of the aggregate supply curve as $\kappa \equiv (1 - \xi\beta)(1 - \xi)/\xi > 0$. The central bank is assumed to control aggregate demand directly, implementing monetary policy using the targeting rule

$$\pi_t - \pi^* + \lambda x_t = \varphi r_t^n$$

where π^* denotes the inflation target, the long-run mean of inflation, normalized to zero; λ_x the stabilizing weight given to output gap stabilization; and $\varphi > 0$ potential deviations from optimal policy.[32] With the exception of the inflation target π^*, the policy rule is known and understood by agents so that when combined with the aggregate supply curve, inflation is the only endogenous variable to be forecast to make pricing decisions. The model is closed with a forecasting model that has the same structure as above, though permits the Kalman gain to vary over time, as discussed further below.

Evaluating expectations and some simple algebra gives the state-space representation

$$\pi_t = \Gamma_\pi \hat{\omega}_t^\pi + \Phi r_{t-1}^n + \Phi_\varepsilon \varepsilon_t$$
$$\hat{\omega}_{t+1}^\pi = [1 + \kappa_{\omega,t}(\Gamma_\pi - 1)]\hat{\omega}_t^\pi + \kappa_{\omega,t}$$

where $\kappa_{\omega,t}$ is a time-varying gain and the coefficient

$$0 < \Gamma_\pi = \frac{1}{1 + \kappa\lambda_x^{-1}}\frac{(1 - \xi)\beta}{1 - \xi\beta} < 1$$

measures the degree of self-referentiality of expectations. Firms raise their optimal price when they believe long-run inflation has risen, raising aggregate inflation. In turn, higher inflation leads firms to markup their long-term inflation expectations. This interaction engenders an exogenous inflation trend, determined by monetary policy and shocks to the natural rate of interest and markups. Monetary policy regulates the strength of this connection. When $\lambda_x \to 0$ the target criterion is equivalent to strict inflation targeting, eliminating self-referential inflation dynamics. When $\lambda_x \to \infty$ the target criterion is equivalent to output gap targeting, and, for standard parameter values implies values of Γ_π near unity. Beliefs are near self-confirming.

How do firm's form an estimate of the inflation target? Following Marcet and Nicolin (2003), firms are unsure about the correct model to forecast inflation. They choose between two estimators of the long-run mean of the inflation rate: either a decreasing gain algorithm, $\kappa_{\omega,t} = \kappa_{\omega,t-1} + 1$; or a constant gain algorithm, $\kappa_{\omega,t} = \kappa_\omega$. The first estimator is ordinary least squares. The gain is the inverse of the sample size, so that accumulating evidence of a stationary mean leads to declining

---

[32]With $\varphi = 0$ and in absence of markup shocks the central bank implements optimal discretion where both output gap and inflation are at their zero objective in every period while the nominal interest rate moves one-to-one with the natural rate of interest. See Woodford (2003b).

sensitivity of long-run expectations to new information. The second estimator implies a constant and relatively high sensitivity to new information. In the spirit of CUSUM tests, firms test the hypothesis of a constant mean every period using a criterion based on cumulative forecast errors.[33]

Together these assumptions imply long-term inflation expectations display time-varying sensitivity to short-run forecast errors. This permits a formal definition of anchored expectations which is directly tied to beliefs about inflation. Expectations are said to be anchored when beliefs are consistent with the policy regime in place: a fixed inflation target. This occurs when firms do not reject the hypothesis of a time-invariant inflation mean. Long-term inflation expectations display decreasing sensitivity to forecast errors and, if they remain anchored, converge to the inflation target. However, large and persistent forecast errors can lead firms to abandon the belief of a constant mean. Long-run beliefs are then unanchored and tightly connected to short-term forecast errors.

Figure 19 describes the model's predictions for long-term inflation expectations for the U.S. and three other countries. Parameters are estimated using nonlinear Bayesian methods on inflation U.S. data, including survey-based professional short-term forecasts. However, measures of long-term forecasts *were not used* in the estimation. The top panels shows how the model does a remarkable job at predicting 5-10 year ahead expectations for all countries.[34] For most of the sample, the survey-based forecasts are inside the 95% credible set. The bottom panels display the evolution of the learning gain. For all countries the sensitivity of long-term inflation expectations to short-term forecast errors has changed over time and, in particular, signaled episodes of poor anchoring. In the U.S. the 1970s marked over twenty years of volatile expectations, followed by firm anchoring starting at the end of the 1990s throughout the financial crisis. Conversely, Japan has consistently displayed poorly anchored expectations since the mid-1990s and Spain experienced a brief episode of un-anchoring during the financial crises. Finally, Swedish long-term inflation expectations become anchored following its adoption of inflation targeting in 1992.

Summing up, we have provided further evidence of a link between short-term forecast errors and revisions in long-term expectations. Perhaps even more important, the structural model highlights the structural breaks in the expectations formation process and how they capture the evolution of measured long-term expectations in different countries.

6.3 Application 2: The Term Structure of Expectations

In practice, central banks use short-term interest rates to influence aggregate demand. To model the transmission of monetary policy we supplement the aggregate supply equation with the aggregate demand equation

$$x_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - (i_T - \pi_{T+1} - r_T^n) \right] \tag{6.8}$$

---

[33]Carvalho et al. (2021) provides details. This approach to model selections builds on Marcet and Nicolini (2003), Milani (2014) and Cho and Kasa (2015).

[34]Model's predictions are made using the same parameters. The mean of inflation is, however, estimated separately for each country.

where i_t is the one-period nominal interest rate, the instrument of monetary policy. The entire future sequence of anticipated short rates matters for aggregate demand. Drfiting long-run interest rate expectations represents an information friction that helps explain macroeconomic data and is a challenge for stabilization policy.

For simplicity assume the central bank adopts the simple rule

$$i_t = \phi \pi_t \tag{6.9}$$

where $\phi > 1$ is a policy parameter. Combined with the aggregate supply equation (6.7) and the aggregate demand equation (6.8) the model has a rational expectations equilibrium of the form

$$z_t = \Phi r_{t-1}^n + \Phi_\epsilon \epsilon_t^n$$

where $z_t = (\pi_t, x_t, i_t)'$ and ϵ_t^n the innovation to the natural rate of interest. Assume that under imperfect information beliefs are given by (6.1) and (6.2) with $\omega_t = (\omega^\pi, \omega^x, \omega^i)$, S an indentity matrix, and lagged state vector replaced by the natural rate of interest. Long-term interest rate forecasts are revised in response to short-term forecast errors according to

$$\hat{\omega}_{t+1}^i = \hat{\omega}_t^i + \kappa_{\omega_i} \left(i_t - \hat{\omega}_t^i - \Phi_i r_{t-1}^n \right) \tag{6.10}$$

$$= \hat{\omega}_t^i + \kappa_{\omega_i} \left(i_t - i_{t|t-1} \right). \tag{6.11}$$

where Φ_i is the element of Φ corresponding to the rational expectations solution for the interest rate. Shocks to the natural rate of interest induce interest rate surprises leading to shifts in long-term beliefs—indeed, the whole term structure of expectations.

Extrapolation bias is a equilibrium property of the model driving a wedge between subjective and objective beliefs. To illustrate in the case of interest rates, consider an economy where prices are nearly flexible, that is $\xi \to 0$, and monetary policy is understood to be implement using the simple rule. Taking the difference between agent interest-rate forecasts, E_t, and model-consistent projections under true data-generating process, $\hat{E}_t$, gives the wedge in any future period $T > t$

$$
\begin{aligned}
E_t i_T - \hat{E}_t i_T &= \left(1 - \frac{\phi^{-1} - \beta}{1 - \beta} \right) \hat{\omega}_t^i \\
&= \kappa_{\omega_i} \times \left(1 - \frac{\phi^{-1} - \beta}{1 - \beta} \right) \sum_{j=0}^{\infty} \left(1 - \kappa_{\omega_i} \frac{1 - \phi^{-1}}{1 - \beta} \right)^j \epsilon_{t-1-j}^n
\end{aligned}
\tag{6.12}
$$

where the second equality follows from writing current interest rate beliefs as a function of the entire past history of natural rate innovations. Both the size of the learning gain and monetary policy regulate the degree of extrapolation bias. Depending on the expectations formation mechanism and monetary policy, transitory natural-rate shocks may have long-lived effects. For example, a sequence of positive shocks to the natural rate leads to an increase in the wedge, leading to a steeper path of the interest rate compared to that path which would be expected under the true

data-generating process. This wedge has empirical and policy implications.

Eusepi et al. (2020) estimate a medium-sized DSGE model using U.S. macroeconomic data, including the term structure of interest rate and inflation expectations from professional forecasters. The wedge between subjective and objective beliefs is vital to understanding and fitting the evolution of macroeconomic data as well as the Federal Reserves' ability to pursue active stabilization policy.[35] For example, Figure 20 shows the fit of five-to-ten (black line) and one-to-ten (blue line) expectations for both inflation and nominal short-term interest rate from a blend of Blue Chip surveys (red and blue dot respectively). The DSGE model fits well the behavior of expectations since the mid-1980s, while at the same time fitting shorter-term survey forecasts.[36]

The bottom panel plots agents' *subjective* one-to-ten-year-ahead expectation for the short-term nominal rate (black line) together with the *model consistent* expectation (blue line) held by an outside observer knowing the true data-generating process. Because of extrapolation bias subjective expectations display weaker mean reversion and higher volatility compared to objective expectations. This wedge has economic content. Model consistent expectations would correctly predict short-term rates to fall more quickly from the peak of the Great Inflation over the subsequent Great Moderation period: a lower expected path of the short-term rate in turn would deliver lower equilibrium long-term interest rates. Conversely, at the beginning of the sample, loose monetary policy flattens the term structure of interest rate and forecasters systematically under-predicted the significantly higher interest rates observed over the 1980s. It is also clear the the wedge between subjective and objective expectations varies over time: over the last decade of the sample this wedge shrinks significantly as long-term expectations stabilize. Finally, the figure also illustrates that ten-year Treasury yields (red line) move together with subjective expectations, but still leave a significant residual, as discussed in Section 4.2.

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[35]See Giannoni and Woodford (2004).

[36]See Eusepi et al. (2020) for further details. The estimation allows a structural break in the learning gain at the end of the 1990s.

Figure 19: **Expectations Anchoring**

These panels show model predictions for long-term inflation forecasts (top) and the learning gain (bottom). Black solid lines denote median; gray areas measure 70th and 95th credible intervals; red dots denote a blend of five-to-ten and other long-term forecast from our U.S. dataset; and they denote five-to-ten years inflation forecasts from Consensus Economics for other countries.

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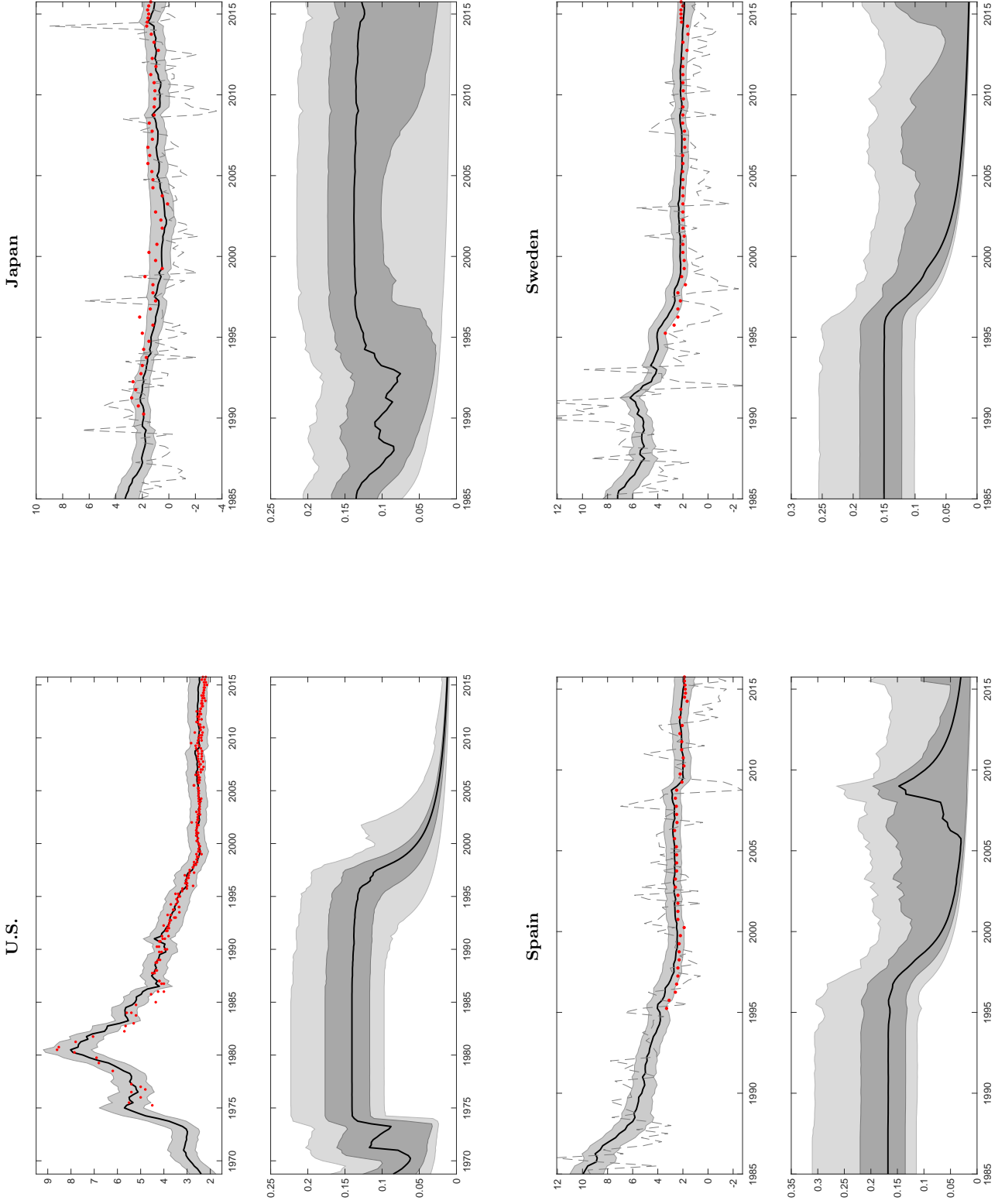
6.4 Implications for Monetary and Fiscal policy

The previous examples adduce further evidence that long-term movements in expectations inflation and interest rate expectations are tied to short-term forecast errors and that this relationship is time-varying. Eusepi et al. (2020) show that these properties of long-term expectations have implications for monetary policy. In contrast to a full-information rational expectations model, a central bank cannot fully stabilize the macro-economy, even in the case of demand shocks. The degree to which stabilization policy is compromised depends on how well anchored are long-term expectations. As long-term expectations become less stable, aggressive aggregate demand management becomes infeasible: large movements in overnight policy rates translate into volatility in long rates and therefore aggregate demand. However, when long-term expectations are well anchored, model predictions are much closer to those of a rational expectations analysis. The constraint posed by the term structure of interest rate expectations is quantitatively important.

That activist stabilization policy is undesirable, contrasts with earlier papers by papers by Ferrero (2007), Orphanides and Williams (2007) and Molnar and Santoro (2013). These papers emphasize how non-rational expectations can alter the short-run trade-off in the new Keynesian Phillips curve and conclude that optimal policy should be more aggressive than a rational expectations analysis. However, this literature makes the assumption that the central bank can directly control aggregate demand. Accounting for the transmission mechanism of monetary policy turns this result on its head, because aggressive adjustment of overnight rates creates volatility in the term structure of interest rate expectations.

Building on Eusepi et al. (2020), Eusepi et al. (2021) explore the implications for optimal monetary policy at the zero lower bound. Their model addresses the common practical concern that long-term inflation expectations might become un-anchored and drift downwards in response to a large negative demand shock. They show unanchored expectations complicate monetary policy requiring an extended period of zero interest policy because expectations themselves, if sufficiently pessimistic, can cause the zero lower bound to be a constraint on policy actions. The optimal forward guidance policy is front-loaded and displays an insurance principle: aggressive responses to a negative demand shock are required to support inflation expectations in the case of a persistent deterioration in economic conditions. However, policy is too stimulatory in the case of transitory disturbances. These policy implications are strikingly different to Eggertsson and Woodford's (2003) rational expectations analysis.

Introducing fiscal and debt management policy further complicates inflation control in this class of model. Eusepi and Preston (2012) and Eusepi and Preston (2018) show that when households are uncertain about their long-run tax obligations, modeled in the same way as uncertainty about long-run inflation or interest rates, then Ricardian equivalence fails to hold, even when the fiscal authority has access to lump-sum taxation. Movements in expectations about taxes, inflation and interest rates generate shifting valuations of the public debt and the expected tax burden attached to that debt. The resulting wealth effects on aggregate demand complicates stabilization policy, fundamentally changing the economy's response to different kinds of disturbances. These effects

are larger for higher levels of average debt and for moderate debt maturities, and quantitatively important for debt levels observed for the United States and other countries since the Great Recession. These results raise obvious concerns about inflation policy over the coming decade, given the substantial debt-financed stimulus packages in response to the global pandemic.

7 Conclusions and Further Directions

This chapter has provided a simple expectations formation mechanism to study and measure of the term structure of expectations. Observed survey-based measures of expectations are consistent with forecasters frequently revising their long-term outlook. In particular, these revisions are partly associated to their short-term forecast errors, as predicted by standard statistical models of trend and cycle decomposition. This holds for individual forecasts as well as for our measure of the consensus term structure of expectations. In addition to fitting consensus expectations, the proposed model provides valuable insights in the sources of forecast dispersion from short to long horizons.

In the final section, we consider the implications of this expectations formation mechanism in DSGE models. Here additional information frictions are imposed. In particular, economic agents are boundedly rational and do not know the correct data generating process: consistent with much empirical evidence, they tend to over-extrapolate from recent developments. Having estimated the models using the term structure of expectations, we show that the proposed expectations formation mechanism has profound implications for monetary and fiscal policy design when compared with the benchmark of rational expectations.

The work explored also suggests interesting future avenues of investigation. First, both reduced-form and structural model should allow time-varying components to the expectations formation process. As shown in the last section, agents' model validation process can lead to state-dependent sensitivity of revisions of long-term expectations to short-term forecast errors. But time- and state-dependence of information frictions can be more general. For example, the agents' ability or willingness to process information likely depends on the associated costs and benefits which may change with the state of the economy.

Second, while models of information frictions with dispersed information are now ubiquitous in macroeconomics, these models mostly focus on forecast dispersion over the short-term. In contrast, our analysis suggests that disagreement about the long term is a key feature of survey data. Incorporating learning about the long run in structural models of dispersed information could deliver important implications for both business cycle analysis and monetary policy design.

Third, our measure of consensus expectations suggests that the term structure of interest rates is driven only partly by the term structure of policy rate expectations. An overwhelming majority of monetary models used for policy analysis instead assumes that the only transmission channel of monetary policy is via short rate expectations. The results in this chapter help better quantify the importance of this channel but also highlight the importance of additional channels, including

variation in term premiums, forecast dispersion or failure of equilibrium asset pricing restrictions grounded on the assumption of perfect information and rational expectations.

Finally, while the reduced-form framework we have introduced offers a fairly flexible specification that is able to provide a tight fit to the observed survey data, when incorporating the expectations formation mechanism in a DSGE setup we made specific assumptions about information frictions. While the literature has made remarkable progress in using survey data to select among competing theories of expectations formation, the jury is still out. The measure of the term structure of expectations that this chapter provides can be used to make further gains in selecting the models that best describe the data, for example by studying the dynamic response of the term structure of expectations to specific macroeconomic shocks.
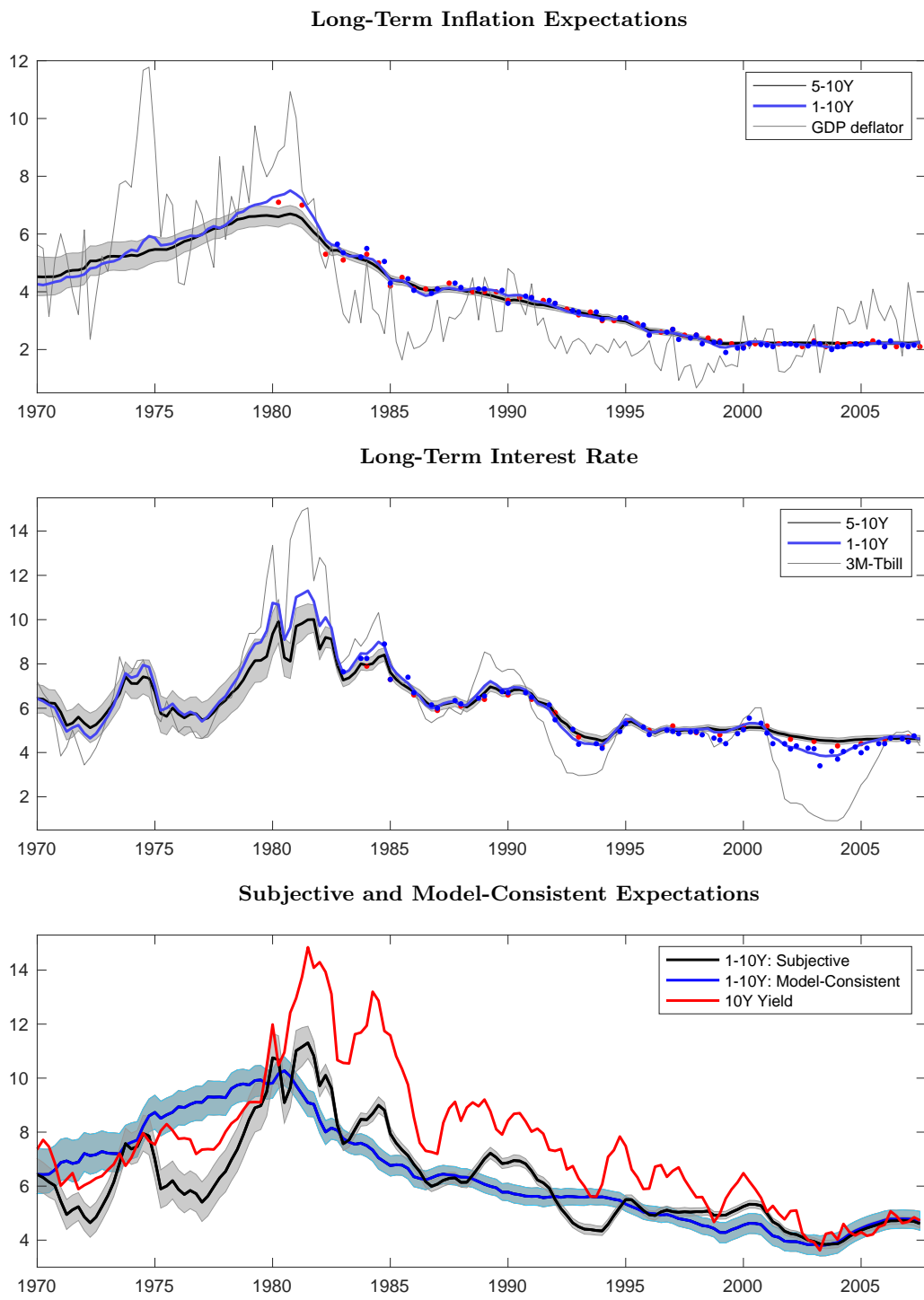
Figure 20: **Long Term Expectations from a DSGE model.**

The top and middle panels show the evolution of long-term survey expectations data for inflation and the short-term nominal rate of interest. Actual variable (grey), the two survey expectations measures (red and blue dots), the model implied 1-10 year average expectations (the blue line); and the model implied 5-10 year average expectations with 95% posterior probability bands (black line). The bottom panel shows the model subjective (black) and model consistent expectations (blue) with 95% coverage interval. Then red line defines the 10-year yield on treasury bonds.

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Appendix A Defining Term Premiums

The term premium for an n period bond can be obtained from observed yields and expectations via the following identity:

$$y_t(n) = \frac{1}{n}\mathbb{E}_t\left[i_t + i_{t+1} + \cdots + i_{t+n-1}\right] + tp_t(n), \tag{A.1}$$

where $y_t(n)$ is the continuously compounded yield on an n-month discount bond, i_t is the risk-free nominal short rate at time t, and $tp_t(n)$ is the nominal term premium. The term premium is thus simply given by the difference between observed yields and what would be the yield predicted by the (pure) expectations hypothesis, i.e. the average expected future short rate over the life of the bond. It is important to emphasize that this is simply an identity; there are no implicit assumptions about the rationality or bias of expectations or the data generating process for yields, expectations, or term premiums.

In order to separate longer-term from short-term expectations, we conduct our analyses in terms of forward rates, defined as the current yield of an n-month bond maturing in $n + m$ months:

$$f_t(n,m) = \frac{1}{n}[(n+m)y_t(n+m) - my_t(m)]$$

Since the model is estimated at a monthly frequency, we construct annual forward rates as the annual average of monthly forward rates. We then define forward term premiums as the difference between $f_t(n,m)$ and the consensus expected short-term rate over the n months m months hence (i.e., a forward version of equation (A.1)):

$$tp_t^{fwd}(n,m) = f_t(n,m) - \frac{1}{n}\sum_{i=m+1}^{n+m}\mathbb{E}_t\left[i_{t+i}\right]$$

For example, at our monthly sampling frequency the 9Y1Y forward term premium, i.e., the term premium embedded in a one-year bond, nine years in the future, would be defined as:

$$tp_t^{fwd}(12,108) = f_t(12,108) - \frac{1}{12}\sum_{i=109}^{120}\mathbb{E}_t\left[i_{t+i}\right]$$

A convenient way to gain intuition about forward rates versus yields is to consider the case where term premiums are zero at all maturities. Then 1-period forward rates, $\{f_t(1,i) : i = 1, \ldots\}$ would be given by $\mathbb{E}_t\left[i_t\right], \mathbb{E}_t\left[i_{t+1}\right], \mathbb{E}_t\left[i_{t+1}\right], \ldots$, whereas yields, $\{y_t(n) : n = 1, \ldots\}$ would be

$$\mathbb{E}_t\left[i_t\right], \frac{1}{2}\left(\mathbb{E}_t\left[i_t\right] + \mathbb{E}_t\left[i_{t+1}\right]\right), \frac{1}{3}\left(\mathbb{E}_t\left[i_t\right] + \mathbb{E}_t\left[i_{t+1}\right] + \mathbb{E}_t\left[i_{t+3}\right]\right), \ldots$$

In other words, once adjusted for term premiums, forwards reflect the expectation of the short rate at a specific horizon in the future whereas yields reflect the average expected short rate up to that horizon. Accordingly, the term premium on a bond with n months to maturity simply reflects the average one-month forward term premium from 1 through n:

$$tp_t(n) = \frac{1}{n}\sum_{i=1}^{n} tp_t^{fwd}(1,i).$$

Since we collect data on inflation expectations we can further decompose expected nominal future short rates into expected real short rates and expected inflation,

$$tp_t^{fwd}(n, m) = f_t(n, m) - \frac{1}{n} \sum_{i=m+1}^{n+m} \mathbb{E}_t \left[r_{t+i} + \pi_{t+i+1} \right],$$

where r_t is the *ex-ante* real short rate, i.e., $i_t = r_t + \mathbb{E}_t \left[\pi_{t+1} \right]$.

Appendix B   Additional Results

Table A.1: **Forecast Errors and Forecast Revisions: All Variables** This table presents regression results from the specification of equation (2.6). Each row presents the estimate of the coefficient β for each variable along with the associated p-value below it in parentheses. The column labelled "FE" denotes the use of the fixed-effects estimator whereas "OLS" denotes a pooled-OLS estimator. Sample sizes are presented in the rightmost column. Standard errors are clustered by year. The sample period is October 1998 to October 2005.

	OLS	FE	N (OLS/FE)
RGDP	0.1212	0.1346	142 / 135
	(0.036)	(0.059)	
NGDP	0.1188	0.1211	138 / 131
	(0.023)	(0.013)	
CPI	0.0480	0.0937	142 / 135
	(0.483)	(0.279)	
3M TBill	0.1698	0.1762	134 / 127
	(0.101)	(0.138)	
Real Short Rate	0.3003	0.3375	133 / 126
	(0.016)	(0.020)	
Unemployment	0.5017	0.4810	140 / 133
	(0.000)	(0.002)	
GDP Price Index	0.0707	0.0943	140 / 133
	(0.209)	(0.067)	
Ind. Production	0.0964	0.1109	132 / 126
	(0.033)	(0.058)	
Disposable Pers. Income	0.0338	0.0135	130 / 122
	(0.585)	(0.784)	
Pers. Consumption Exp.	0.1071	0.1130	137 / 132
	(0.030)	(0.066)	
Non-Res. Fixed Investment	0.1241	0.1305	138 / 128
	(0.012)	(0.026)	
10Y T-Note	0.0447	-0.0776	136 / 129
	(0.681)	(0.540)	
Corporate Profits	0.0217	0.0207	113 / 106
	(0.139)	(0.347)	
Housing Starts (mil)	0.1857	0.2184	131 / 124
	(0.247)	(0.335)	
Auto Sales (mil)	0.2289	0.3952	110 / 102
	(0.387)	(0.439)	
Net Exports	0.4350	0.4708	132 / 126
	(0.195)	(0.333)	

Table A.2: **Variance Decompositions for Yield Components: Pre-Crisis Sample**

This table presents variance decompositions for the one-year yield and one-year forward rates ranging from one though ten-years out. For each maturity, the numbers shown represent the ratio of the covariance of the respective forward with its individual components (average expected real short rate, average expected inflation, and term premium) divided by the variance of the forward. The top panel provides variance decompositions for forward rates in levels, and the following panels the 1-month, 3-month and 12-month change in forward rates. The sample period is March 1983–December 2007.

	Y1	1Y1Y	2Y1Y	3Y1Y	4Y1Y	5Y1Y	6Y1Y	7Y1Y	8Y1Y	9Y1Y
Levels										
Avg Exp Nominal SR	0.86	0.70	0.59	0.52	0.49	0.48	0.48	0.49	0.50	0.50
Avg Exp Real SR	0.50	0.33	0.20	0.12	0.07	0.05	0.04	0.04	0.04	0.04
Avg Exp Inflation	0.37	0.38	0.39	0.40	0.41	0.42	0.44	0.45	0.46	0.47
Fwd Term Premium	0.14	0.30	0.41	0.48	0.51	0.52	0.52	0.51	0.50	0.50
1-Month Changes										
Avg Exp Nominal SR	0.59	0.27	0.14	0.09	0.07	0.07	0.06	0.06	0.06	0.05
Avg Exp Real SR	0.47	0.20	0.09	0.04	0.02	0.01	-0.00	-0.01	-0.01	-0.01
Avg Exp Inflation	0.12	0.06	0.05	0.04	0.05	0.06	0.06	0.07	0.07	0.07
Fwd Term Premium	0.41	0.73	0.86	0.91	0.93	0.93	0.94	0.94	0.94	0.95
3-Month Changes										
Avg Exp Nominal SR	0.76	0.41	0.25	0.17	0.14	0.12	0.11	0.09	0.08	0.08
Avg Exp Real SR	0.61	0.31	0.17	0.09	0.05	0.02	-0.00	-0.02	-0.03	-0.03
Avg Exp Inflation	0.16	0.10	0.08	0.08	0.09	0.10	0.11	0.11	0.11	0.11
Fwd Term Premium	0.24	0.59	0.75	0.83	0.86	0.88	0.89	0.91	0.92	0.92
12-Month Changes										
Avg Exp Nominal SR	0.90	0.67	0.49	0.35	0.26	0.20	0.17	0.14	0.13	0.12
Avg Exp Real SR	0.71	0.50	0.32	0.18	0.09	0.03	-0.01	-0.03	-0.05	-0.05
Avg Exp Inflation	0.18	0.18	0.17	0.17	0.17	0.17	0.18	0.18	0.18	0.18
Fwd Term Premium	0.10	0.33	0.51	0.65	0.74	0.80	0.83	0.86	0.87	0.88