Tracking the New Economy: Using Growth Theory to Detect Changes in Trend Productivity
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Abstract

The acceleration of productivity since 1995 has prompted a debate over whether the economy's underlying growth rate will remain high. In this paper, we propose a methodology for estimating trend growth that draws on growth theory to identify variables other than productivity—namely consumption and labor compensation—to help estimate trend productivity growth. We treat that trend as a common factor with two "regimes," high-growth and low-growth. Our analysis picks up striking evidence of a switch in the mid-1990s to a higher long-term growth regime, as well as a switch in the early 1970s in the other direction. In addition, we find that productivity data alone provide insufficient evidence of regime changes; corroborating evidence from other data is crucial in identifying changes in trend growth. We also argue that our methodology would be effective in detecting changes in trend in real time: In the case of the 1990s, the methodology would have detected the regime switch within two years of its actual occurrence according to subsequent data.

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Discerning underlying trends in productivity growth has long been a goal of both policymakers and economists. At least since Solow's (1956) pioneering work on long-term growth, economists have understood that sustained productivity growth is the only source of long-term growth in living standards. It is also important for short-term policy analysis, as any assessment of "output gaps" or growth "speed limits" ultimately derives from some understanding of what the trend is. On a quarterly basis, however, measured productivity growth is extremely volatile. Over the postwar period the average quarterly growth rate of nonfarm productivity has been 2.2 percent (annualized), but the standard deviation has been 3.9 percent. Moreover, the volatility is not confined to high frequency fluctuations. Productivity growth is also cyclical, typically declining at the onset of a recession and rising during a recovery. Thus it is often only years after the fact that any change in its long-term trend will be apparent.

It is widely believed that the difficulty of detecting a change in trend growth contributed significantly to the economic instability of the 1970's, as policymakers were unaware of the slowdown in productivity growth for many years, and only much later were able to date the slowdown at approximately 1973.[1] This resulted in overestimating potential GDP (at least so the conventional wisdom goes) and setting interest rates too low, and double-digit inflation followed not long after.

In recent years, attention has turned once again to productivity because of speculation that its trend growth rate may be picking up again. The growth rate of nonfarm output per hour increased by approximately 1 percent beginning in 1996 relative to the period 1991-1995, and by about 1.3 percent relative to 1973-1995. The acceleration of productivity puts its growth rate during this 5-year period close to where it was during the most recent period of strong growth,

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[1] See, for example, Sims (2001), who writes that during the 1970's, "unemployment rose and inflation rose because of real disturbances that lowered growth. . . . Since such 'stagflation' had not occurred before on such a scale, they faced a difficult inference problem, which it took them some years to unravel."

from roughly 1948 to 1973. This has provoked a debate over whether we can expect an extended period of more rapid productivity growth. Robert Gordon (2000), for example, attributes about half of the acceleration to a "cyclical" effect, and much of the remainder to measured productivity growth in the technology sector. Others (e.g. Stiroh, 2002) find evidence that productivity growth has spilled over into other sectors through capital deepening.

Much of the difficulty in evaluating the arguments in this debate relates to the issue of separating trend from cycle in the data. For example, if Gordon had assumed an acceleration in trend GDP, then he would have found a smaller or non-existent output gap, and consequently would be less likely to attribute the productivity acceleration to cyclical effects. Thus without more information, either story (new economy: accelerating productivity and output; or old economy: increased productivity growth confined to IT sector, all else is cyclical) seems consistent with the productivity data. This is a problem that plagues any effort to distinguish trend from cycle in a single time series over a relatively short period of time. Moreover, even apart from the difficulty of distinguishing trend from cycle, it is reasonable to question whether much of anything can be learned from five or six years of data on a series as volatile as productivity growth.

In this paper we attack this problem by drawing on standard neoclassical growth theory to help us identify variables other than productivity itself—namely consumption and labor compensation—that should help to estimate the trend in long-term growth. We treat that trend as a stochastic process whose mean growth rate has two "regimes," high and low, with some probability of switching between the two at any point in time. We model the business cycle as a second process common to all of the variables in the analysis, also with two regimes of its own, based on the so-called "plucking" model of Friedman (1969,1993).

There are several advantages to our approach. First, we show that aggregate productivity data alone do not provide as clear or as timely a signal of structural change as does the joint signal from the series we examine. Second, we do not have to choose break dates *a priori*, as we let the data speak for themselves. Third, the model not only provides information about when regime switches occurred, it also provides estimates of how long the regimes are likely to last. This last property contrasts with even the most sophisticated structural break tests, such as those described by Hansen (2001).

Also worth emphasizing is that our use of theory enables us to restrict our analysis to a low dimensional system of variables and to impose parameter restrictions in the estimation procedure. Our approach contrasts with atheoretical applications of factor models that involve a large number of variables or that do not place theory-based restrictions on estimated coefficients.[2] Here there are both advantages and disadvantages: Our model may not provide as tight a fit to the data as would a more eclectic approach, but it is likely to be more robust to structural changes in the economy.

Our analysis picks up striking evidence of a switch in the mid-1990's to a higher long-term growth regime, as well as a switch in the early 1970's in the other direction. While these conclusions may come as no big surprise, our analysis has further implications. First, one could not conclude that there was a switch to a higher regime on the basis of productivity data alone, or even with the addition of a variable to control for the business cycle. Only the corroborating evidence from consumption and labor compensation can swing the balance in favor of a regime switch. Second, the ability to discern the switch to a higher-growth regime based on our analysis has appeared relatively recently. Only with data through the end of 1999 could one conclude that with fairly high probability a switch had occurred.

[2] See for example, Stock and Watson (1989, forthcoming).

I. Background: The Neoclassical Growth Model

Over forty years ago, Nicholas Kaldor (1961) established a set of stylized facts about economic growth that have guided empirical researchers ever since. His facts are: (1) labor and capital's income shares are relatively constant; (2) growth rates and real interest rates are relatively constant; (3) the ratio of capital to labor grows over time, and at roughly the same rate as output per hour, so that the capital-output ratio is roughly constant. To these facts, more recent research has added another: that measures of work effort show no clear tendency to grow or shrink over time on a per capita basis. The important implication of this additional fact is that wealth and substitution effects roughly offset each other. This means, for example, that a permanent change in either the level or growth rate of labor productivity has no permanent impact on employment.

Of course, closer inspection suggests that none of the above "stylized facts" is literally true. Indeed the premise of much work on U.S. productivity is that productivity growth was systematically higher from 1948-1973 than it was over the subsequent 20-plus years. But they still provide a starting point for modeling economic growth. That starting point is generally referred to as the neoclassical growth model. The linchpins of this approach are typically a constant returns-to-scale Cobb-Douglas production technology in capital and labor, constant elasticity-of-substitution preferences for consumption, and exogenous labor-augmenting technological progress.

In our analysis we allow for exogenous changes in preferences between consumption and leisure to account for any long-term movements in work effort (as measured by hours) that show up in the data. Specifically, let C denote aggregate consumption, Y aggregate output, N population (measured in person-hours and growing at rate n), K capital, E effective labor per unit of labor input, and L aggregate labor input (in hours). We also assume that there is a production function

$$Y = K^{\alpha}(LE)^{1-\alpha}$$

and preferences defined in terms of a present discounted value of utility

$$U = \ln(C/N) + \Lambda v(1-\ell),$$

where $\ell \equiv L/N$ represents the proportion of available hours devoted to work. The term $\Lambda v(1-\ell)$ therefore represents the utility of leisure, where v is a concave function, and Λ is a taste parameter that might shift over time.

We assume that the economy evolves as if a planner solves the following problem:

$$\max \int_0^{\infty} e^{-\rho t} U(C/N, L/N)\,dt,$$

subject to $$C + \dot{K} \leq F(K, LE),$$

where ρ is a discount rate. Let $c \equiv C/(NE)$, $y \equiv Y/(NE)$, and $k \equiv K/(NE)$. Also, let W denote labor compensation, and $w \equiv W/(NE)$. If we assume for the moment that E grows at a constant rate g and that Λ is constant, then first-order conditions for the above maximization problem are as follows:

$$\begin{aligned} c + \dot{k} + (n+g)k &= F(k,\ell) \\ 1/c &= \lambda \\ \Lambda v'(1-\ell) &= \lambda F_2(k,\ell) \\ \dot{\lambda}/\lambda &= n + g - F_1(k,\ell) \end{aligned}$$

where λ is the shadow value of the resource constraint embodied in the first equation, and the other variables are as defined in the paper. In a steady state both λ and k will be constant, so the system

$$c + (n + g)k = F(k, \ell) \qquad (1)$$
$$1 / c = \lambda \qquad (2)$$
$$\Lambda v'(1 - \ell) = \lambda F_2(k, \ell) \qquad (3)$$
$$0 = n + g - F_1(k, \ell) \qquad (4)$$

can be solved for the steady state values of c, k, λ and ℓ for fixed values of g and Λ. This implies that the capital labor ratio $K/L = Ek/\ell$ and the ratio of consumption to hours of work $C/L = Ec/\ell$ will both grow at rate g, as will output per hour $Y/L = Ey/\ell$. Also, if we assume that labor is compensated according to its marginal product, then labor compensation per hour W/L will simply equal $(1-\alpha)Y/L$ and will also grow at the same rate.

To summarize:

Proposition 1: The per capita quantities C/L, K/L, W/L, and Y/L all grow at rate g in the steady state.

Proof: This follows from the constancy of c, k, ℓ and y, and their definitions.

The preceding result neatly characterizes the relationships between variables of interest, under the assumption that g and Λ are constant over time (i.e. within a given regime). Of course this paper is predicated on the possibility that both (and g in particular) may have experienced important changes in the postwar period. To dispense with the easy case first, level changes in Λ have no impact on the conclusions of the previous paragraph. The permanent components of the capital-labor ratio, output per hour, consumption relative to hours, and labor compensation per hour should all continue to move together except for transitory variation in response to changes in labor supply. For example, if Λ falls, the result will be an increase in L/N, but K and C will adjust proportionally. So we have

Proposition 2: The steady-state values of c/ℓ, k/ℓ, and y/ℓ do not depend on Λ.

Proof: The homogeneity of *F* implies that (1) and (4) can be expressed in terms of c/ℓ, k/ℓ, and y/ℓ. Since Λ only enters (3), and $w/\ell = F_2(k,\ell)$ is only a function of k/ℓ, ℓ and λ can vary with Λ to keep $\Lambda v'(1-\ell)/\lambda$ constant, and *c* can vary with λ to keep (2) satisfied. ■

Thus even permanent movements in Λ are neutral with respect to the trend in the ratios described in Proposition 1.

Changes in *g*, however, are not "neutral," except with regard to work effort. We have

Lemma: The steady-state value of ℓ does not depend on *g*.
Proof: Cobb-Douglas technology implies $y = k^{\alpha}\ell^{1-\alpha}$. Then $F_1 = \alpha(k/\ell)^{\alpha-1}, F_2 = (1-\alpha)(k/\ell)^{\alpha}$, and $y/\ell = F(k,\ell)/\ell = (k/\ell)^{\alpha}$. It follows from (1) and (4) that $c/y = (1-\alpha)$ for any value of *g*. Substituting this result and (2) into equation (3) proves that ℓ is constant. ■

Together with the Proposition 2, the Lemma is particularly helpful for understanding the relationship between employment and long-term growth: Any impact in either direction is transitory. Trends in employment are primarily a function of labor supply, i.e. demographic and "taste" factors.[3] The practical implication of this is that we can safely detrend employment without fear of discarding important information about underlying trends in per capita growth.

The Lemma also leads immediately to the following non-neutrality result:

Proposition 3: The steady-state values of c/ℓ, k/ℓ, y/ℓ, and w/ℓ vary with *g*.
Proof: Equation (4) implies immediately that k/ℓ depends (negatively) on *g*. Since y/ℓ is a function of k/ℓ, it changes as well. Equation (3) then implies that λ must change with *g*, since the Lemma shows that ℓ is constant. Equation (2) then implies that c/ℓ must vary with *g*. Finally w/ℓ is proportional to y/ℓ. ■

The intuition behind Proposition 3 is straightforward: higher growth makes individuals feel wealthier, and therefore increases desired consumption. The interest rate (i.e. the marginal product of capital) must then rise to keep the level of consumption in line with the current level of technology. This changes the steady state capital-labor ratio, and consequently the steady state values of all variables that are linked to it. Faster growth, for example, raises future consumption relative to current levels, and therefore discourages saving and capital accumulation. The result is that the capital-labor ratio, while ultimately growing faster as well, also experiences a downward shift in its level. A corollary of this effect is that the output-capital ratio shifts upward. This effect can be seen rather strikingly in Figure 1A, which depicts the output-capital ratio over the period 1948-95. This ratio should shift in the same direction as the underlying growth rate, which it clearly does. Moreover, the size of the shift is roughly what one would expect from the model using plausible parameter values.

Proposition 3 thus breaks the exact link between the growth rate of E and the growth rates of C/L, Y/L, K/L, and W/L (if the growth rate of E experiences permanent[4] changes). It does not, however, imply that these four ratios do not themselves have common long-term components, only that the common component is not the same as the common component of E. In fact, the level shifts in output per hour, labor compensation per hour, and consumption/hours should all be the same, so that those three variables should have the same permanent component even in the face of regime shifts in growth. This result is summarized in the following:

Proposition 4: While the steady-state value of k/y depends on g, the steady-state values of c/y and w/y do not.

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[3] Of course, it is possible that growth affects demographics, and vice-versa. For example, sustained higher growth seems to be related to lower fertility rates. But such relationships are unlikely to be apparent over a few decades.

Proof: The first-order conditions (1)-(4) imply that $c/y = 1 - \alpha$, and $w/y = \alpha$. On the other hand, $k/y = (k/\ell)^{1-\alpha}$, which is obviously a function of the capital-labor ratio, which we have seen depends on g. ■

The upshot of this review of the neoclassical growth model is that other data, notably labor compensation per hour and consumption relative to hours, may provide auxiliary information about the trend in output per hour. This is not to say that the information is completely independent, and indeed the source of errors in one series may be present in the other series as well. For example, an inaccurate price deflator could result in common mismeasurement across multiple series. Nonetheless, the theory suggests that considering these series together may provide better information about underlying trends than consideration of any of them in isolation.

II. Econometric Specification

Our estimation strategy draws upon the regime-switching dynamic factor model recently proposed by Kim and Murray (2002). The essence of our approach is to examine a number of related economic time series and to use their comovement to identify two shared factors: a common permanent component and a common transitory component. The common permanent component relates to the movements in the series associated with their trend growth rate. On the other hand, the common transitory component relates to movements in the series associated with the business cycle. These components provide a basis to distinguish between long-run and medium-run changes in the series as well as to gauge their relative importance over particular historical episodes.

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[4] We use the term "permanent" loosely to mean any change that is sufficiently long-lasting that the economy moves arbitrarily close to a new steady state.

While there are other time series models that also incorporate permanent and transitory components, the Kim and Murray (2002) model is unique in that it allows for the possibility that the components may be subject to changes in regime. That is, there may be periodic changes in the underlying process generating the common permanent and common transitory components.

For the purpose of analyzing the trend and cyclical movements in productivity, the regime-switching aspect of the model has several attractive features. First, the regime-switching permanent component allows us to account for sustained changes in trend growth without making the growth process itself nonstationary. Consequently, we can not only address the issue of a post-1973 productivity growth slowdown, but also a post-1995 productivity growth resurgence. Second, the regime-switching common transitory component allows us to account for possible asymmetries across booms and recessions. In particular, we can capture the idea proposed in Friedman's (1964, 1993) "plucking model" model that economic fluctuations are largely permanent during expansions and largely transitory during recessions. Last, while the model allows for periodic changes in the processes generating the common permanent and common transitory components, the timing of the switches does not need to be specified prior to estimation. Rather, the timing of the switches is determined as one of the outcomes from the estimation procedure.

Following Kim and Murray (2002), we can describe the regime-switching dynamic factor model as follows.[5] Suppose we consider a number of time series indexed by *i*. Let ΔQ_{it} denote the growth rate of the *i*th individual time series. It is assumed that the movements in each series are governed by the following process:

$$\Delta Q_{it} = D_i + \gamma_i \Delta \Pi_t + \lambda_i \Delta x_t + z_{it} , \tag{5}$$

where D_i is the average growth rate of the series, $\Delta \Pi_t$ denotes the growth rate of a permanent component that is common to all series, Δx_t denotes the growth rate of a common transitory component, and z_{it} denotes an idiosyncratic component. The parameter γ_i (the permanent "factor loading") indicates the extent to which the series moves with the common permanent component. Similarly, the parameter λ_i indicates the extent to which the series is affected by the transitory component.[6]

The common permanent component is assumed to be subject to the type of regime-switching proposed by Hamilton (1989) in which there are periodic shifts in its growth rate:

$$\Delta\Pi_t = \mu(S_{1t}) + \phi_1 \Delta\Pi_{t-1} + \ldots + \phi_p \Delta\Pi_{t-p} + \upsilon_t, \upsilon_t \sim iidN(0,1) \qquad (6)$$

$$\mu(S_{1t}) = \begin{cases} \mu_0 & if \quad S_{1t} = 0 \\ \mu_1 & if \quad S_{1t} = 1 \end{cases}, \qquad (7)$$

$$\Pr[S_{1t} = 0 \mid S_{1,t-1} = 0] = q_{1,} \qquad \Pr[S_{1t} = 1 \mid S_{1,t-1} = 1] = p_1 \qquad (8)$$

where S_{1t} is an index of the regime for the common permanent component. The transition probabilities p_1 and q_1 indicate the likelihood of remaining in the same regime. Under these assumptions, the common permanent component Π_t, grows at the rate $\mu_0/(1-\phi_1-\ldots-\phi_p)$ when $S_{1t}=0$, and at the rate $\mu_1/(1-\phi_1-\ldots-\phi_p)$ when $S_{1t}=1$.

The common transitory component x_t is also subject to regime-switching:

$$x_t = \tau(S_{2t}) + \phi_1^* x_{t-1} + \phi_2^* x_{t-2} + \ldots + \phi_p^* x_{t-p} + \varepsilon_t, \varepsilon_t \sim iidN(0,1) \qquad (9)$$

$$\tau(S_{2t}) = \begin{cases} 0 & if \quad S_{2t} = 0 \\ \tau & if \quad S_{2t} = 1 \end{cases}, \qquad (10)$$

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[5] Additional details are provided in the Appendix.
[6] As we demonstrate shortly, the formulations for the common permanent component and common transitory component have different implications for their effects on the individual time series. Specifically, the level of the common permanent component can increase over time, while the level of the common transitory component is stationary.

$$\Pr[S_{2t}=0 \mid S_{2,t-1}=0]=q_2 \qquad \Pr[S_{2t}=1 \mid S_{2,t-1}=1]=p_2 \tag{11}$$

where S_{2t} is an index of the regime for the common transitory component, with transition probabilities p_2 and q_2. The permanent and transitory regimes are assumed to be independent of each other. The restriction of unit variance for the error terms of the two processes is an identifying restriction, since Π and x are of indeterminate scale. The parameter τ represents the size of the "pluck," with $\tau<0$ implying that the common transitory component is plucked down during a recession. Finally, the idiosyncratic components are assumed to have the following structure:

$$z_{it}=\psi_{i1}z_{i,t-1}+\psi_{i2}z_{i,t-2}+\ldots+\psi_{ip}z_{i,t-p}+\eta_{it}, \qquad \eta_{it}\sim iidN(0,\sigma_i^2), \tag{12}$$

where all innovations in the model are assumed to be mutually and serially uncorrelated at all leads and lags.

While the two regimes are not directly observable, it is nevertheless possible to estimate the parameters of the model and to extract estimates of the common components.[7] An important byproduct from the estimation procedure is that we can draw inferences about the likelihood that each common component is in a specific regime at a particular date. The inferences can be based on a "real time" assessment, which provides an estimate of the current regime as of date *t* based only on information through date *t*, or a "retrospective" assessment that incorporates information for the entire sample.[8] Of course the retrospective assessment is a more reliable estimate of past regimes because it incorporates subsequent data, but the real-time assessment is useful as a gauge of what analysts could realistically have known at the time.

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[7] Kim and Murray (2002) discuss how the regime-switching dynamic factor model can be cast in a state-space representation and estimated using the Kalman filter. More details of how we applied their methods to the present study are in the Appendix.

[8] The designation "real time" is potentially misleading on two counts. First, the parameter estimates are based on the entire sample, though results based on rolling parameter estimates going back to 1997 gave very similar results. Second, the data may have undergone subsequent revisions that would have been unavailable as of the time of the assessment.

Our data consist of four quarterly series on labor productivity, real compensation per hour, consumption deflated by hours of work, and hours of work itself. All of the data except aggregate consumption refer to the private nonfarm sector. The selection of the series and their subsequent transformation for the estimation are based on the implications of the growth model in the previous section, which provides guidance on their usefulness for identification of the common permanent and transitory components. With regard to the hours series, as we have seen, the growth model implies that its permanent component is unrelated to the permanent component of the other three series. Rather than estimate a second permanent component, we simply detrend this series using the Hodrick-Prescott (1980) filter and include it solely to help refine the estimate of the common transitory component.

With regard to the dynamic specification of the common and idiosyncratic components, our diagnostic checking procedure suggested that the common permanent component should include one lagged value of $\Delta\Pi_t$, the common transitory component should include two lagged values of x_t, and that the idiosyncratic component should include one lagged values of z_{it} for each series. The results from Section I (Proposition 4 in particular) imply restrictions on the γ_i, which represent the permanent factor loadings in the model. Specifically, we restricted the estimated permanent factor loadings for productivity, real compensation per hour, and consumption per hour to be equal. In addition, we set the value of the permanent factor loading for detrended hours to be equal to zero.

III. Results

As described above, our benchmark model uses the three variables that growth theory suggests should have approximately the same permanent components: Real output per hour (variable 1), real labor compensation per hour (variable 2), and consumption relative to hours (variable 3). The inclusion of detrended hours (variable 4) is designed to help capture the transitory or business cycle component of the data. As we have seen, while the capital-labor ratio should also exhibit the same growth as output per hour within a growth regime, both theory and the data suggest that it can jump substantially between regimes, so we exclude it from our system.

One final specification issue relates to the synchronization of the data. The three trending variables were selected primarily with a view to helping identify the common permanent component. They are not necessarily "coincident indicators" with respect to the transitory component. In theory it would be possible to allow for a more general lead/lag structure in our system, but this would greatly increase the number of parameters to estimate. As an alternative, we first examined the cross-correlations of the four series. We found that the first two variables (productivity and labor compensation per hour) both tended to lead the other two series by about three quarters. To capture this asynchronization in the estimation we lagged variables 1 and 2 by three quarters in the system described above.[9]

A. Parameter Estimates

The first set of results in Table 1 provides the parameter estimates for our benchmark model with these four variables.[10] As already mentioned, we restrict γ (the loading factor on the common permanent component) to be the same for the three trending variables, a restriction that we test and fail to reject for this specification. The data cover 1947:Q1-2002:Q2, though because the first two variables are lagged three quarters, their growth rates cover the period

[9] A minor drawback to this procedure is that the last three observations of variables 1 and 2 are not used in estimating the parameters of the model. We do incorporate them in the period-by-period assessments of the state variables, as described below.

[10] Estimates of the idionsyncratic variances are omitted from Table 1 for the sake of brevity.

1947:Q2-2001:Q3, while the growth rates of consumption and detrended hours variables cover 1948:Q1-2002:Q2 .

The results indicate that the model yields precise estimates of most of the parameters of interest: The loading factors on both the permanent and transitory components, the transition probabilities, and the shift parameters connected with the regimes (μ_0, μ_1, τ) all come in significant. The difference between the low and high permanent regimes works out to be $\gamma(\mu_0 - \mu_1)/(1-\phi) = 0.367$. This corresponds to roughly 1.5 percent on an annualized basis, very close to the difference between the 1948-73 and 1973-95 growth rates. The transition probabilities for the permanent regimes imply that the unconditional probability of being in the high growth regime is 0.593, which as we shall see reflects the inference that the economy was in the high growth regime 59.3 percent of the time between 1947 and the present.

The transitory process is estimated to be a "hump-shaped" autoregressive process typical of the business cycle, but with a significant negative pluck. The four λ_i coefficients, which represent the loading factors on the transitory process, are all estimated very precisely, and have the expected signs. Note that λ_3, the loading factor for consumption/hours, is negative, reflecting the fact that hours are more cyclical than consumption. The positive estimates for the loading factors on the real compensation and productivity variables indicate that those two variables associate positively with the transitory component, albeit leading by three quarters relative to the other two variables (since they enter the system lagged by three quarters).

B. Growth Regime Assessments

Before further describing the permanent and transitory components of productivity growth, however, it is instructive to examine the inferred probability of being in the high- or low-growth state over time. There are two ways to examine this. The first is in "real time": At each point in time, using only data through that point in time, what probability would one have assigned to being in the high-growth state? The second way is retrospectively: Given what we now know has happened through 2002:Q2, what can we say looking back over time about the likelihood of being in the high growth state? (Obviously the two assessments coincide at the end, i.e. as of 2002:Q2.)[11]

Even within the "real time" approach, two practical issues arise. First, there is the matter of data revisions. The historical series we have today have been revised numerous times over the years, so the data truncated at some date does not correspond to what anyone would actually have known. We ignore this problem in the present study, but plan to address it in subsequent work. Second, it is not practical to have rolling estimates of parameters, especially if regime shifts occur infrequently. Over a shorter sample few if any regime switches will be observed, making it impossible to estimate transition probabilities. And other parameters will be difficult to estimate precisely as well. Consequently in looking at real time assessments of the state vector we will rely on parameter estimates from the full sample.[12]

There is also a technical issue related to the timing of the data series. Since Q_1 and Q_2 are lagged by three periods, if we were simply to assess the state variable given data through a given observation number, we would be ignoring the most recent three data points for those two series. For example, a standard application of the Kalman filter at date t would yield an assessment of the state given data through period t,

$$\hat{\xi}_{t|t} = E(\xi_t \mid Q_{1t}, Q_{2t}, Q_{3t}, Q_{4t}; Q_{1t-1}, Q_{2t-1}, Q_{3t-1}, Q_{4t-1}; \ldots) .$$

[11] Hamilton (1994) refers to these two approaches as using a "zero-lag" versus "full-sample smoother."
[12] We experimented with rolling parameter estimates going back five years and found that the results were not significantly different.

In terms of the actual underlying data, however, it would really be providing

$$E(\xi_t \mid \widetilde{Q}_{1t-3}, \widetilde{Q}_{2t-3}, \widetilde{Q}_{3t}, \widetilde{Q}_{4t}; \widetilde{Q}_{1t-4}, \widetilde{Q}_{2t-4}, \widetilde{Q}_{3t-1}, \widetilde{Q}_{4t-1}; \ldots),$$

where the "~" denotes the variable indexed by its true time period. Fortunately it is relatively straightforward to "partially update" the state and regime assessments. This involves conditioning on three subsequent observations of Q_1 and Q_2 at each point in time, to get

$$\begin{aligned}\hat{\xi}_{t|t+3'} &= E(\xi_t \mid Q_{1t+3}, Q_{2t+3}, Q_{3t}, Q_{4t}; Q_{1t+2}, Q_{2t+2}, Q_{3t-1}, Q_{4t-1.}; \ldots) \\ &= E(\xi_t \mid \widetilde{Q}_{1t}, \widetilde{Q}_{2t}, \widetilde{Q}_{3t}, \widetilde{Q}_{4t}; \widetilde{Q}_{1t-1}, \widetilde{Q}_{2t-1}, \widetilde{Q}_{3t-1}, \widetilde{Q}_{4t-1}; \ldots)\end{aligned}.$$

Here we use $t+3'$ to denote Q_1 and Q_2 observed through $t+3$, and Q_3 and Q_4 observed through time t. This simply undoes the staggering so that the information set is appropriately aligned. (Additional details on the partial updating procedure are provided in the Appendix.)

These two regime assessments are plotted in Figure 2. The vertical axis is the probability of being in the high-growth regime. The retrospective assessment presents a very clear picture: The economy was in a high-growth state until the early 1970's, followed by a roughly 20-year low-growth regime, followed by a switch back to high-growth in the second half of the 1990's. Perhaps the only surprise here is how unambiguous the current assessment is. The probability that the economy was in the high-growth regime surpassed 0.95 in 1998:Q4, and has not subsequently fallen below 0.97. Given the 3-quarter lead for productivity, this dates the acceleration of productivity growth at 1998:Q1. If we use a more lenient 0.5 threshold, we would date the acceleration at 1997:Q3.

It is worth noting that the partial updating methodology described earlier can make a substantial difference in assessing current conditions. For example, without it (that is, ignoring the last three observations of productivity and real labor compensation per hour), the estimated probability that the economy is in the high-growth regime falls from 0.9997 in 2000:Q4 to 0.9057 in 2002:Q2. Adding back the information contained in those observations results in a 2002:Q2 estimate of 0.9740.

As the previous discussion makes clear, however, the retrospective estimates do not mean that as of 1997:Q3 we actually could have made such an optimistic assessment. With data only through 1998:Q4 the high-growth probability assessment would have been 0.69 rather than 0.95. The probability only surpassed 0.95 when we include data through 1999:Q4. Thus hindsight provides us with roughly a one-year jump on dating the productivity acceleration. And it is easy to understand why, looking at Figure 2. In real time there were a lot of "false alarms," post-1973 episodes when it appeared that productivity growth might shift into high gear. These tended to occur during recoveries, when productivity growth did in fact increase, and when it was too soon to tell—notwithstanding an effort to disentangle cycle from trend—whether the higher growth rate would be sustained.

Nonetheless, even in real time it would have been clear from these techniques with data through 1999:Q1 (when the probability assessment first topped 0.9) that there had been a change in regime back to stronger productivity growth beginning in 1998:Q2. While certainly the idea of a "new economy" with strong productivity growth had gained many adherents well before 1999, there were also plenty of nay-sayers, and few of the optimists would have ventured to base their views on objective statistical analysis.[13]

For completeness, we can also look at the assessment of the transitory regimes and the overall transitory and idiosyncratic components of the series. The (retrospective) probability of

[13] Indeed, optimistic views go back as early as 1997 (see, for example, *The New York Times*, August 2, 1997, "Measuring Productivity in the 90's: Optimists vs. Skeptics," by Louis Uchitelle). The optimism appears, however, to have been based on something other than the productivity data themselves, about which there was much skepticism (see Corrado and Slifman, 1999). Of course, pessimists have also based their views on skepticism about the data, e.g. Roach (1998).

being in the "plucked down" state is plotted in Figure 3. Here the probability assessments are a little more ambiguous, which is not surprising considering that the regimes themselves are relatively transitory. Nonetheless, the more prominent spikes all coincide with NBER-defined recessions, though several recessions (most notably the 1990-91 recession) are missed entirely, and several others show up with a relatively low probability. It is perhaps instructive that the 1990-91 recession does not register in this picture. The idea of a pluck is a sharp downturn followed by an equally sharp recovery sufficient to get the economy back to trend. The 1990-91 recession was characterized by a relatively mild downturn followed by an unusually slow and gradual recovery. If we instead look at the total common transitory component (which includes an autoregressive process as well as the pluck) in Figure 4, we see that this recession does in fact register (as do all of the postwar recessions), albeit very mildly.

If we examine the permanent and transitory components of productivity growth directly (Figure 5), we see that the permanent component clearly indicates changes in trend in the early 1970's and mid-1990's, with little apparent cyclical residue.[14] At the same time, however, the model still assigns a lot of variation to the "idiosyncratic" component (Figure 4). Presumably this reflects the high volatility of quarterly productivity growth, volatility which cannot be easily be explained away by cyclical factors. While inclusion of more cyclical variables might reduce the role of the idiosyncratic component somewhat, we did not pursue this as it is would be unlikely to alter significantly the assessment of permanent regimes that is our focus.

D. The Relative Importance of Additional Variables

Finally, from a methodological perspective one may ask how important is the use of additional series in helping to identify changes in trend growth. To answer this, we use the same

[14] This contrasts with Kim and Murray (2001), whose estimated permanent aggregate component shows substantial downward movement during recessions.

econometric model, but with only two series, nonfarm output per hour and detrended hours (variables 1 and 4 from the previous analysis). Thus we are looking to estimate trend growth with productivity data alone, using the detrended hours series to control for the business cycle. The result of this exercise is the second set of estimates in Table 1. Note first that the estimates of the transition probabilities are very similar to the earlier estimates, suggesting that the fundamental properties of the regime-switching dimension of the model will be similar. Secondly, while many other parameters are similar, it is clear that the nature of the permanent component is very different from the previous estimate. Not only is the factor loading much higher (.802 to .273)—not surprising since it is the only trending variable—but its dynamics are also different, as indicated by the AR coefficient (ϕ) of -0.096 and insignificant, versus -0.381 and significant in the first set of results.

These differences are reflected in the retrospective regime assessments (see Figure 6). Compared to the previous estimates, the estimated probability spends a lot more time in "gray areas" between transitions, and most important, ends at around 0.5, meaning that the 2-variable system still is undecided about which growth regime the economy resides in today or over the past six years. The difficulty in distinguishing trend from cycle is also manifested in the volatility of the estimated permanent component (Figure 7). Now most of the movements in productivity are viewed as permanent, because there is little other information in the 2-variable system to help filter out the noise. While the 2-variable system arguably does a better job of getting at the transitory component, as the remaining idiosyncratic term is smaller than in the 4-variable system, from the point of view of identifying changes in underlying trends, the simpler approach appears to fall short.

On the other hand, a look at the real-time probability assessments in Figure 6 shows (in comparison to Figure 2) that the probability assessments exhibit fewer "false alarms"—episodes where the probability that a regime shift has occurred jumps up, only to quickly reverse itself as more data come in. For example, in 1977 the probability of a regime shift back to higher productivity growth reached 0.72; in 1986 and 1992 it came to 0.82 and 0.67. In all three instances these assessments proved false, in the sense that the retrospective probabilities were substantially smaller: 0.10, 0.08, and 0.05 respectively. Perhaps the four-variable system is "overfitting" the data, i.e. it does better than the two-variable system in avoiding "type 2" errors (the error of failing to detect a regime change), but it makes more "type 1" errors (detecting a regime change that in fact never took place).

To address this question we use the two models to forecast productivity growth at one- and four-quarter horizons. The idea here is that if the four-variable system were overfitting the data, the root mean square error of its forecasts would be larger than that of the two-variable system. We constructed forecasts beginning in 1972:Q1 using the formulas outlined in Hamilton (1994). Consequently, the sample for the one-quarter forecast horizon covers the period 1971:Q2 - 2002:Q2, while the sample for the four-quarter forecast horizon covers the period 1972:Q1-2002:Q2. Table 2 provides the results of this exercise. For both forecast horizons, the root mean square error of the two-variable system is larger, suggesting that the four-variable system is doing a better job, notwithstanding the fact that its primary intention is not short-term forecasting.

Table 2 also provides the model's forecasts of productivity growth for 2002:Q3-2003:Q2. The four-variable system predicts substantially stronger productivity growth for all four quarters, primarily because of its assessment of a relatively high probability of being in a high-growth regime. Even four quarters out, the model forecasts growth substantially above the sample mean of 2.16 percent, notwithstanding that its built-in (albeit slow) mean reversion. In fact, just the possibility of a switch back to a lower-growth regime builds in approximately a 0.08 percent per quarter decline in growth. Thus conditional on no regime change, the forecasted growth rates would be about 4.1, 3.2, 3.2, and 3.1 percent for the four quarters. The long-run growth rate conditional on remaining in the high-growth regime is roughly 2.9 percent.

Of course productivity growth is a very noisy series, so a forecast for any given quarter cannot be made with much confidence. But the difference between the two sets of forecasts largely reflects different assessments of underlying trends, and provides some idea of the potential usefulness of the approach adopted in this paper.[15]

Can we say anything about which of the additional variables (consumption/hours or labor compensation per hour) is more important in helping to pick up the regime changes? To answer this question, we estimated two additional specifications, adding back alternately one variable or the other to the 2-equation system. The results are illustrated in Figure 8, in which all four specifications' retrospective regime assessments are plotted against each other. It turns out that there is a big impact from adding either variable to the system, with the labor compensation variable appearing to outperform consumption/hours in terms of approaching the 4-equation results. The change in going from three to four equations is relatively modest, especially if the fourth variable is consumption/hours. Note, however, that in the more recent data it is consumption/hours variable that is giving the stronger indication that the economy is in the high-growth regime, and comes closest to matching the assessment based on all four series.

E. Changing Volatility

Another fact that emerges from our analysis is a distinct decline in the volatility of productivity growth over time, particularly since the early 1980's. This parallels the findings of McConnell and Perez-Quiros (2000), who document a break in the volatility of GDP growth at

[15]The actual number for 2002:Q3, released after all of this analysis was done, turned out in fact to be 5.1 percent (in the report released on December 4, 2002; see http://www.bls.gov/schedule/archives/prod_nr.htm).

around 1984:Q1. In fact, while the overall unconditional standard deviation of productivity growth is 3.61 percent (relative to a mean of 2.16), between 1948:Q1 and 1973:Q4 it is 4.19 percent. From 1974:Q1 to 1995:Q4 it falls to 2.99 percent, and since 1996:Q1 it is 2.53 percent.

The natural place to look for declining volatility in our framework is in the idiosyncratic components z_i or the residuals η_i, i=1,…,4. We computed 10-year centered rolling variances of each of the estimated residuals, covering the period 1958:Q2 to 1997:Q2 (using data from 1953:Q2-2002:Q2).[16] The results (shown in Figure 9) show a striking decline in only the volatility of the productivity residual η_1, with no clear pattern for the other series.

One reason this finding is of interest is that if indeed productivity growth is less noisy than it once was, movements in the series may be more informative than they used to be about changes in the underlying trend. We estimated a model allowing for declining volatility of η_1, and found that the lower volatility did in fact lead to earlier inferences regarding the regime change, but only by about one quarter. It also resulted in an even higher assessment of the probability of being in the high-growth regime during 2002. We do not report these results in detail, as they do not change the overall picture, but allowing for heteroscedasticity could prove to be important for some applications, such as forecasting.

V. Conclusions

The view that higher productivity growth is likely to be sustained has only really gained something approaching a consensus with the recent recession. Prior to 2001, one could more easily argue that the increased growth rates experienced since 1995 were merely cyclical or otherwise ephemeral. That lack of agreement not only reflects the difficulty of separating a time series into its trend and cycle, but also the sensitivity of the results to various assumptions used in the decomposition. In the case of productivity, the problematic nature of the decomposition is only likely to be exacerbated by the inherent volatility of the series. Policymakers faced the same difficulty (albeit in the opposite direction) in the mid-1970's when the dramatic slowing of productivity growth coincided with a severe recession.

We explore the issue of the long-term trend in productivity by adopting a modeling strategy that integrates both theoretical considerations and recently developed statistical methods. In contrast to previous univariate analyses, we undertake a multivariate analysis in which we exploit information from additional variables that should be helpful in the identification of the trend and cycle in productivity. Specifically, we extend the data set to include consumption and labor compensation as well as detrended hours.

For our empirical framework we adopt the regime-switching dynamic factor model recently proposed by Kim and Murray (2002). This approach has a number of attractive features. First, it allows for the estimation of a common permanent component and a common transitory component, consistent with our interest in the trend and cycle in the productivity data. Second, the model allows for rich dynamics and can account for periodic changes in the underlying processes generating the common components. This latter consideration is not only important for providing a better characterization of the data, but is central to any discussion about a possible shift to the secular growth rate of productivity. Last, the nature and timing of the regime changes is determined as an outcome of the estimation procedure rather than imposed *a priori*. In fact, one could view the implied regime changes and their reasonableness as an additional metric by which to judge the adequacy of our approach.

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[16] We begin in 1953 so as to leave out the Korean War years, during which there was a brief episode of unusual volatility.

We find strong support in the data for the notion that the economy (and productivity growth in particular) switched from a relatively low-growth to a high-growth regime in the mid-1990's. The annualized difference between the mean growth rates in the two regimes is estimated to be approximately 1.5 percent. We also show that these techniques could have provided conclusive signals of the regime shift by the beginning of 1999. Finally, from a methodological standpoint we argue that the incorporation of additional information from other time series is crucial to the strength of our conclusions.

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Appendix

A1. State Space Model

We employ the following state-space representation for our model:

Measurement Equation: $\Delta Q_t = H'\xi_t\,,\ \Delta Q_t \equiv (\Delta Q_{1t},\ldots,\Delta Q_{4t})'$

Transition Equation: $\xi_t = \alpha(S_t)\ + F\xi_{t-1} + V_t\,,$

with $E(V_t V_t') = \Sigma\,,$

and where (after we restrict $\gamma_1 = \gamma_2 = \gamma_3 \equiv \gamma$, and set $\gamma_4 = 0$)

$$H' = \begin{bmatrix} \gamma & \lambda_1 & -\lambda_1 & 1 & 0 & 0 & 0 \\ \gamma & \lambda_2 & -\lambda_2 & 0 & 1 & 0 & 0 \\ \gamma & \lambda_3 & -\lambda_3 & 0 & 0 & 1 & 0 \\ 0 & \lambda_4 & -\lambda_4 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xi_t = \begin{bmatrix} \Delta\Pi_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{2t} \\ z_{3t} \\ z_{4t} \end{bmatrix}, \quad \alpha(S_t) \equiv \alpha(S_{1t}, S_{2t}) = \begin{bmatrix} \mu_0(1-S_{1t}) + \mu_1(S_{1t}) \\ \tau S_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad V_t = \begin{bmatrix} \upsilon_t \\ \varepsilon_t \\ 0 \\ \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \end{bmatrix}$$

$$F = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_1^* & \phi_2^* & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_{41} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 0 & & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}$$

We next provide a brief overview of a filter developed by Kim (1994) that can be used for approximate maximum likelihood estimation of the state-space model with Markov switching. We focus our attention on the issue of drawing inferences about the unobserved regimes. For further details, interested readers are referred to Kim and Murray (2002).

To facilitate the discussion, we will represent the two unobserved Markov-switching variables S_{1t} and S_{2t} by a single Markov-switching variable defined such that:

$$\begin{aligned}
S_t &= 1 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 0 \\
S_t &= 2 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 1 \\
S_t &= 3 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 0 \\
S_t &= 4 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 1
\end{aligned}$$

with

$$\Pr[S_t = j \mid S_{t-1} = i] = p_{ij}$$

and

$$\sum_{j=1}^{4} p_{ij} = 1$$

Conditional on $S_t = j$ and $S_{t-1} = i$, the Kalman filter equations are given by:

$$\begin{aligned}
\xi_{t|t-1}^{(i,j)} &= \alpha(S_j) + F\xi_{t-1|t-1}^{i} + V_t \\
P_{t|t-1}^{(i,j)} &= FP_{t-1|t-1}^{i}F' + Q \\
\eta_{t|t-1}^{(i,j)} &= \Delta Q - H'\xi_{t|t-1}^{(i,j)} \\
f_{t|t-1}^{(i,j)} &= HP_{t|t-1}^{(i,j)}H' \\
\xi_{t|t}^{(i,j)} &= \xi_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1}\eta_{t|t-1}^{(i,j)} \\
P_{t|t}^{(i,j)} &= (I - P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1})HP_{t|t-1}^{(i,j)}
\end{aligned}$$

where $\xi_{t|t}^{(i,j)}$ and $\xi_{t|t-1}^{((i,j)}$ are, respectively, an inference on ξ_t based on information through time period t (Ω_t) and t-1 (Ω_{t-1}), given $S_t = j$ and $S_{t-1} = i$; $P_{t|t}^{(i,j)}$ and $P_{t|t-1}^{(i,j)}$ are, respectively, the mean squared error matrix of $\xi_{t|t}^{(i,j)}$ and $\xi_{t|t-1}^{(i,j)}$, given $S_t = j$ and $S_{t-1} = i$; $\eta_{t|t-1}^{(i,j)}$ is the conditional forecast error of ΔQ_t based on information through time period t-1, given $S_t = j$ and $S_{t-1} = i$; and $f_{t|t-1}$ is the conditional variance of the forecast error $\eta_{t|t-1}^{(i,j)}$.

To keep the Kalman filter from becoming computationally infeasible, the following approximations are introduced to collapse the posteriors terms $\xi_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ into the posterior terms $\xi_{t|t}^{j}$ and $P_{t|t}^{j}$:

$$\xi_{t|t}^{j} = \frac{\sum_{i=1}^{4} \Pr[S_{t-1} = i, S_t = j \mid \Omega_t]\xi_{t|t}^{(i,j)}}{\Pr[S_t = j \mid \Omega_t]}$$

and

$$P_{t|t}^{j} = \frac{\sum \Pr[S_{t-1} = i, S_t = j \mid \Omega_t]\{P_{t|t}^{(i,j)} + (\xi_{t|t}^{j} - \xi_{t|t}^{(i,j)})(\xi_{t|t}^{j} - \xi_{t|t}^{j})'\}}{\Pr[S_t = j \mid \Omega_t]}$$

The approximations result from the fact that $\xi_{t|t}^{(i,j)}$ does not calculate $E[\xi_t \mid S_{t-1} = i, S_t = j, \Omega_t]$ and $P_{t|t}^{(i,j)}$ does not calculate $E[(\xi_t - \xi_{t|t}^{(i,j)})(\xi_t - \xi_{t|t}^{(i,j)})' \mid S_{t-1} = i, S_t = j, \Omega_t]$ exactly. This is because

ξ_t conditional on Ω_{t-1}, $S_t = j$, and $S_{t-1} = i$ is a mixture of normals for t > 2.

To obtain the probability terms necessary to construct the approximations, the following three-step procedure is employed.

Step 1:

At the beginning of the t^th iteration, given $\Pr[S_{t-1} = i \mid \Omega_{t-1}]$, we can calculate:

$$\Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}] = \Pr[S_t = j \mid S_{t-1} = i] \times \Pr[S_{t-1} = i \mid \Omega_{t-1}]$$

where $\Pr[S_t = j \mid S_{t-1} = i]$ is a transition probability.

Step 2:

We can then consider the joint density of $\Delta Q_t, S_t$ and S_{t-1}:

$$f(\Delta Q_t, S_t = j, S_{t-1} = i \mid \Omega_{t-1}) = f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]$$

and then obtain the marginal density of ΔQ_t as:

$$f(\Delta Q_t \mid \Omega_{t-1}) = \sum_{i=1}^{4} \sum_{i=1}^{4} f(\Delta Q_t, S_t = j, S_{t-1} = i \mid \Omega_{t-1})$$

$$= \sum_{i=1}^{4} \sum_{i=1}^{4} f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]$$

where the conditional density $f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1})$ is obtained using the prediction error decomposition:

$$f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) = (2\pi)^{\frac{T}{2}} \mid f_{t|t-1}^{(i,j)} \mid^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}\eta_{t|t-1}^{(i,j)}{}' f_{t|t-1}^{(i,j)-1} \eta_{t|t-1}^{(i,j)}\}$$

A byproduct of this step is that we can obtain the log likelihood function:

$$\ln L = \sum_{t=1}^{T} \ln(f(\Delta Q_t \mid \Omega_{t-1}))$$

which can be maximized with respect to the parameters of the model.

Step 3:

We can then update the probability terms after observing ΔQ_t and the end of period t:

$$\Pr[S_t = j, S_{t-1} = i \mid \Omega_t] = \Pr[S_t = j, S_{t-1} = i \mid \Delta Q_t, \Omega_{t-1}]$$

$$= \frac{f(S_t = j, S_{t-1} = i, \Delta Q_t \mid \Omega_{t-1})}{f(\Delta Q_t \mid \Omega_{t-1})}$$

$$= \frac{f(\Delta Q_t \mid S_t = j, S_{t-1} = i, \Omega_{t-1}) \times \Pr[S_t = j, S_{t-1} = i \mid \Omega_{t-1}]}{f(\Delta Q_t \mid \Omega_{t-1})}$$

with

$$\Pr[S_t = j \mid \Omega_t] = \sum_{i=1}^{4} \Pr[S_t = j, S_{t-1} = i \mid \Omega_t]$$

The last term provides the "real-time" inference about the unobserved regimes conditional on only contemporaneously available information.

We can also derive smoothed values of ξ_t and S_t using all available information through period T. That is, we can construct $\xi_{t|T}$ as well as $\Pr[S_t = j \mid \Omega_T]$ which represent the "retrospective" assessments of the state vector and unobserved regimes. Because the inferences about the unobserved regimes do not depend on the state vector, we can first calculate smoothed probabilities. The smoothed probabilities can then be used to generate the smoothed estimates of the state vector.

The smoothing algorithm for the probabilities will involve the application of approximations similar to those introduced in the basic filtering. The procedure can be understood by considering the following derivation of the joint probability that $S_{t+1} = k$ and $S_t = j$ conditional on full information:

$$\Pr[S_{t+1} = k, S_t = j \mid \Omega_T] = \Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_t = j \mid S_{t+1} = k, \Omega_T]$$

$$\approx \Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_t = j \mid S_{t+1} = k, \Omega_t]$$

$$= \frac{\Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_{t+1} = k, S_t = j \mid \Omega_t]}{\Pr[S_{t+1} = k \mid \Omega_t]}$$

$$= \frac{\Pr[S_{t+1} = k \mid \Omega_T] \times \Pr[S_t = j \mid \Omega_t] \times \Pr[S_{t+1} = k \mid S_t = j]}{\Pr[S_{t+1} = k \mid \Omega_t]}$$

and

$\Pr[S_t = j \mid \Omega_T] = \sum_{i=1}^{4} \Pr[S_{t+1} = k, S_t = j \mid \Omega_T]$

The actual construction of the smoothed probabilities requires running through the basic filter and then storing the sequences $P_{t|t-1}^{(i,j)}, P_{t|t}^{j}, \Pr[S_t = j \mid \Omega_{t-1}]$ and $\Pr[S_t = j \mid \Omega_t]$. For $t = T$ - 1, T - 2, . . . , 1, the above formulas define a backwards recursion that can be used to derive the full-sample smoothed probabilities. It should be noted that the starting value for the smoothing algorithm is $\Pr[S_t = j \mid \Omega_T]$, which is given by the final iteration of the basic filter.

A2. Partial Updating

Suppose that additional observations become available, but only for some subset of the four data series represented by Q. Specifically, suppose that for the subset Q^1, data are available for periods 1 through T+3, whereas for Q^2, observations are only available through T. Let $T+1'$ denote the augmented information available through $T+1$, i.e. including Q_{T+1}^1 but not Q_{T+1}^2.

Through T the standard Kalman updating algorithm applies (ignoring the regime-related term $\alpha(S_t)$ for brevity's sake), i.e.

$$\begin{aligned}\hat{\xi}_{t+1|t} &= F\hat{\xi}_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H)^{-1}(\Delta Q_t - H'\hat{\xi}_{t|t-1}) \\ P_{t+1|t} &= F[P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H)^{-1}H'P_{t|t-1}]F' + \Sigma\end{aligned}$$

for $t \leq T$.

To update further, we simply recognize that

$$\Delta Q_t^i = H^{i\prime}\xi_t, \qquad i = 1,2$$

where H^i is the appropriate submatrix of H. We then iterate beginning at T+1 according to

$$\begin{aligned}\hat{\xi}_{T+1|T+1'} &= \hat{\xi}_{T|T-1} + P_{T|T-1}H^1(H^{1\prime}P_{T+1|T}H^1)^{-1}(\Delta Q_{T+1}^1 - H^{1\prime}\hat{\xi}_{T+1|T}) \\ P_{T+1|T+1'} &= P_{T+1|T} - P_{T+1|T}H^1(H^{1\prime}P_{T+1|T}H^1)^{-1}H^{1\prime}P_{T+1|T}\end{aligned}$$

and then

$$\begin{aligned}\hat{\xi}_{T+2|T+1'} &= F\hat{\xi}_{T+1|T+1'} \\ P_{T+2|T+1'} &= FP_{T+1|T+1'}F' + \Sigma\end{aligned}.$$

The iteration can then proceed forward if subsequent observations on Q^1 become available.

In our case we essentially have three additional observations on two of the variables, since they appear lagged by three quarters. To take them into account at any point in time t, we can compute $\hat{\xi}_{t|t+3'}$, that is, the assessment of the state vector given the three additional observations of the two series that would otherwise be ignored because they are lagged by three quarters. To obtain $\hat{\xi}_{t|t+3'}$, we iterate forward to get $\hat{\xi}_{t+3|t+3'}$ and $P_{t+3|t+3'}$, and then iteratively "smooth" backwards using, e.g.

$$\hat{\xi}_{t+2|t+3'} = \hat{\xi}_{t+2|t+2'} + P_{t+2|t+2'} H^1 (H^{1\prime} P_{t+3|t+2'} H^1)^{-1} (\hat{\xi}_{t+3|t+3'} - \hat{\xi}_{t+3|t+2'}) .$$

The result is an improved estimate of the state vector, one that incorporates the most recent values of all of the variables.

Table 1: Estimation of Model

Coefficient	4 equation system	2 equation system
p_1	0.989 (0.012)	0.992 (0.011)
q_1	0.984 (0.014)	0.992 (0.011)
p_2	0.982 (0.016)	0.952 (0.038)
q_2	0.555 (0.280)	0.509 (0.153)
ϕ	-0.381 (0.147)	-0.096 (0.162)
ϕ_1^*	1.424 (0.097)	1.439 (0.075)
ϕ_2^*	-0.576 (0.084)	-0.565 (0.072)
ψ_{11}	-0.246 (0.079)	-0.710 (0.233)
ψ_{21}	-0.011 (0.087)	—
ψ_{31}	-0.636 (0.087)	—
ψ_{41}	0.494 (0.070)	-0.026 (0.279)
γ	0.273 (0.041)	0.802 (0.123)
λ_1	0.212 (0.041)	0.156 (0.038)
λ_2	0.118 (0.031)	—
λ_3	-0.444 (0.049)	—
λ_4	0.434 (0.047)	0.581 (0.058)
μ_0	0.822 (0.228)	0.206 ((0.126)
μ_1	-1.035 (0.271)	-0.173 (0.118)
τ	-2.926 (0.769)	-2.128 (0.458)
logL	-353.744	-315.956

Note: The estimation also produces estimates of the variances of the idiosyncratic errors, not reported here.

Table 2: Forecast Performance

	2-variable system	4-variable system
	Root Mean Square Errors	
1-quarter horizon	3.024	2.966
1-year horizon	3.000	2.992
	Forecast (1 quarter ahead)	
2002:Q3	3.14%	4.03%
2002:Q4	2.28	3.04
2003:Q1	2.58	2.99
2003:Q2	2.16	2.75

Note: All figures are at annualized percentage rates. The average annualized growth rate of productivity (log differences) over the full sample is 2.16 percent, with a standard deviation of 3.61.

Figure 1: Evidence on the Level Shift
of the Capital-Labor Ratio

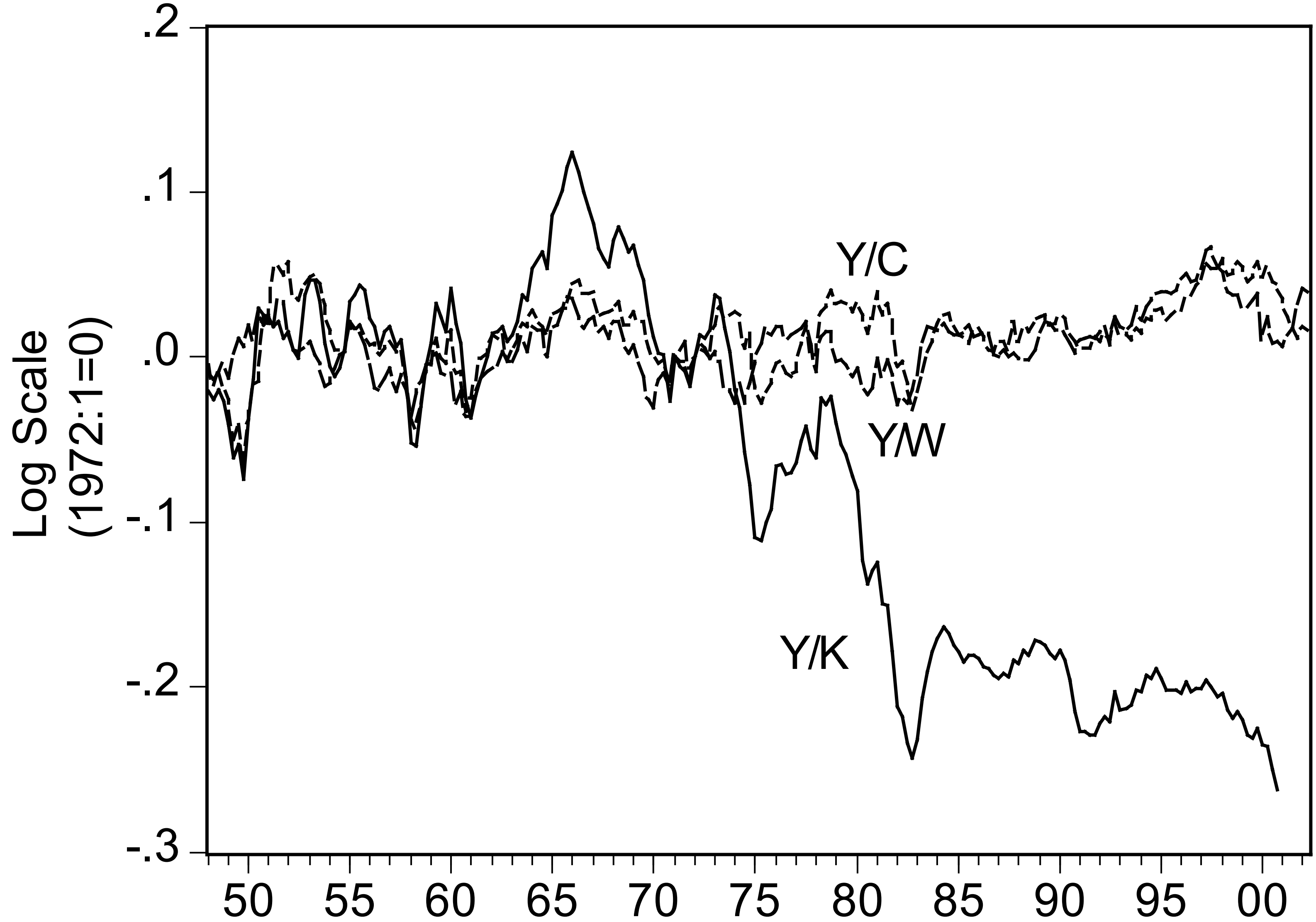


Figure 2: Assessment of Growth Regimes

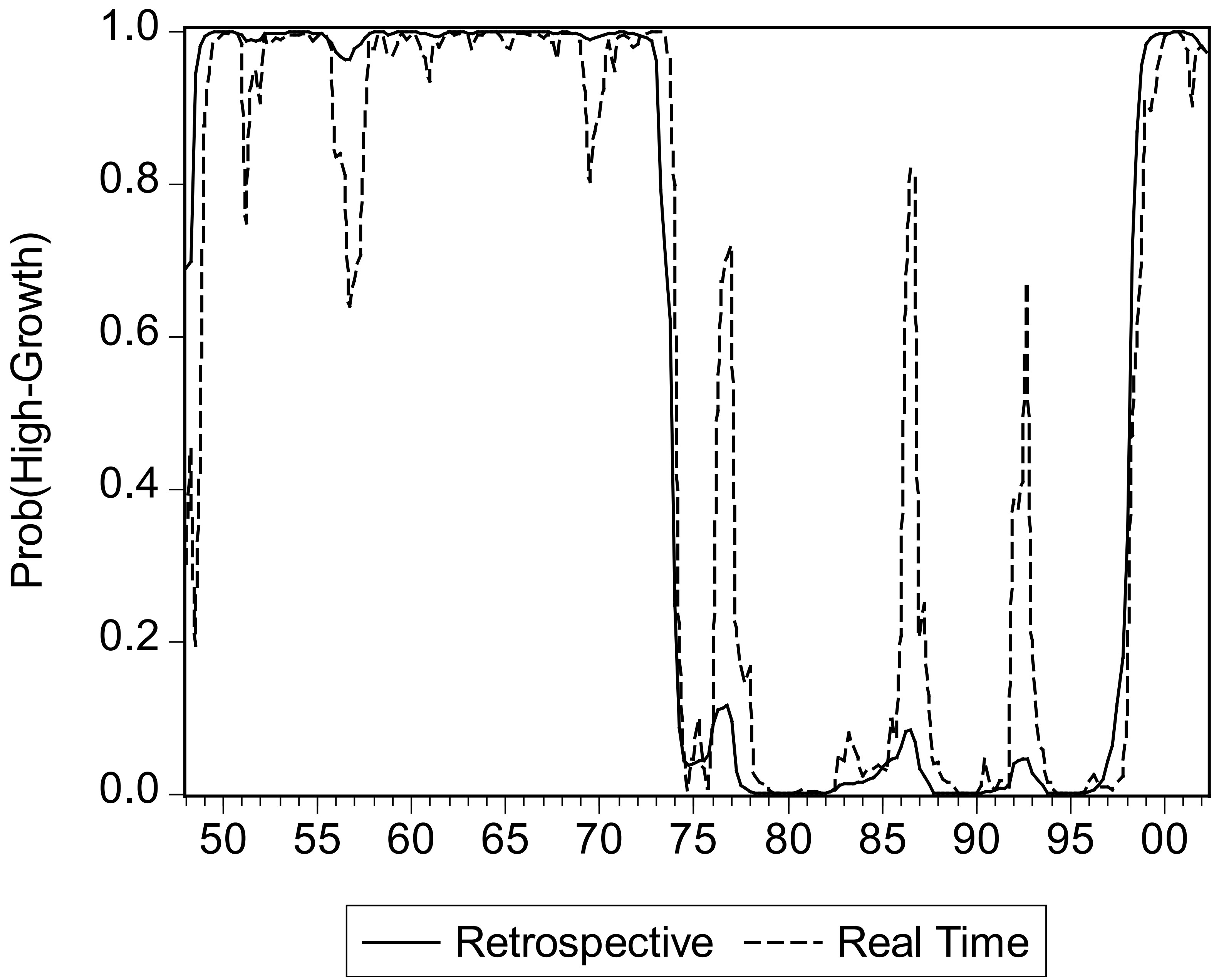


Figure 3: Probability of a Negative Pluck

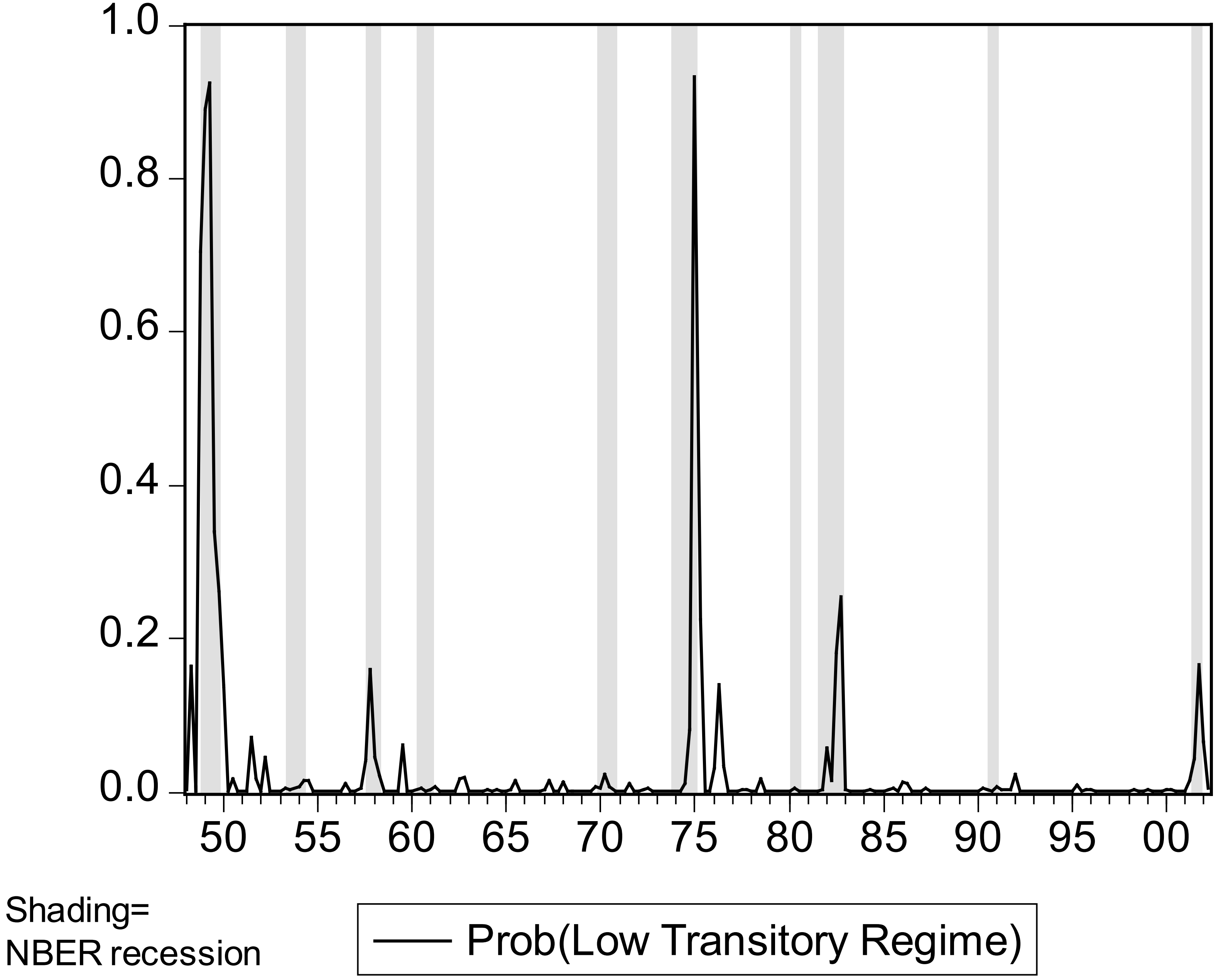


Figure 4: Transitory and Idiosyncratic Components of Productivity

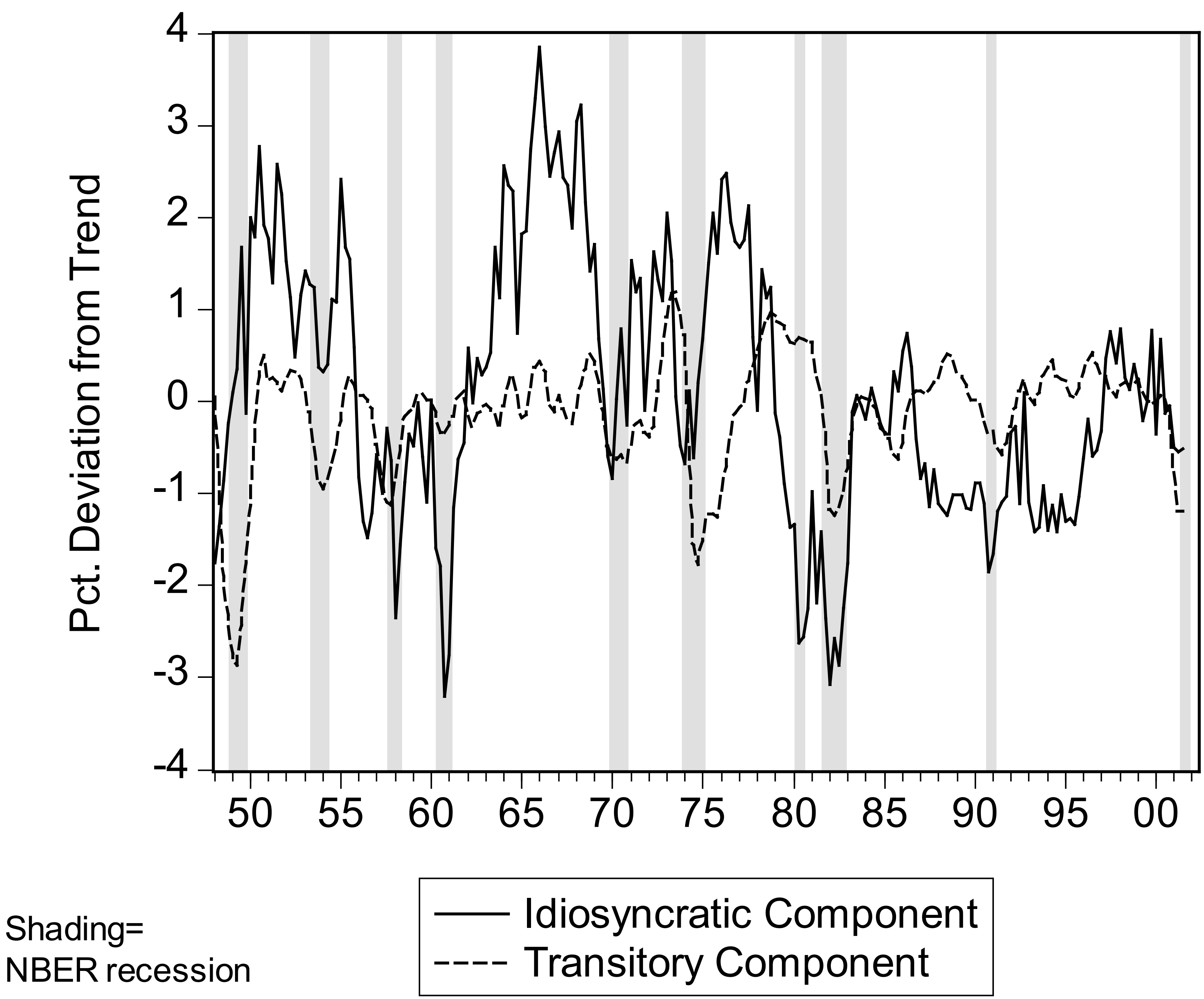
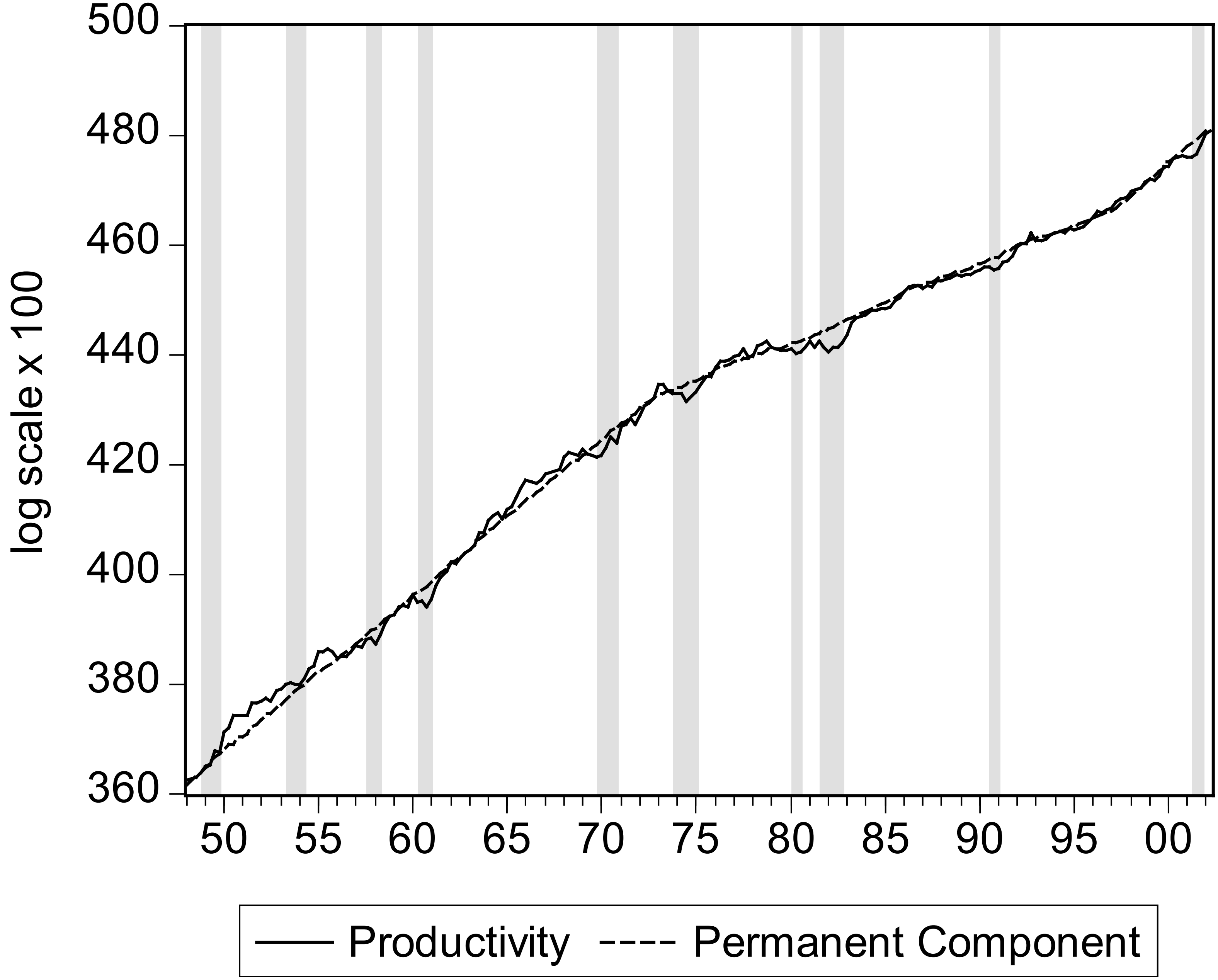


Figure 5: Trend Productivity



Note: This is a plot of $\gamma\Pi_{t+3}$ against the logarithm of productivity growth at date t.

Figure 6: Regime Assessment without Other Trending Variables

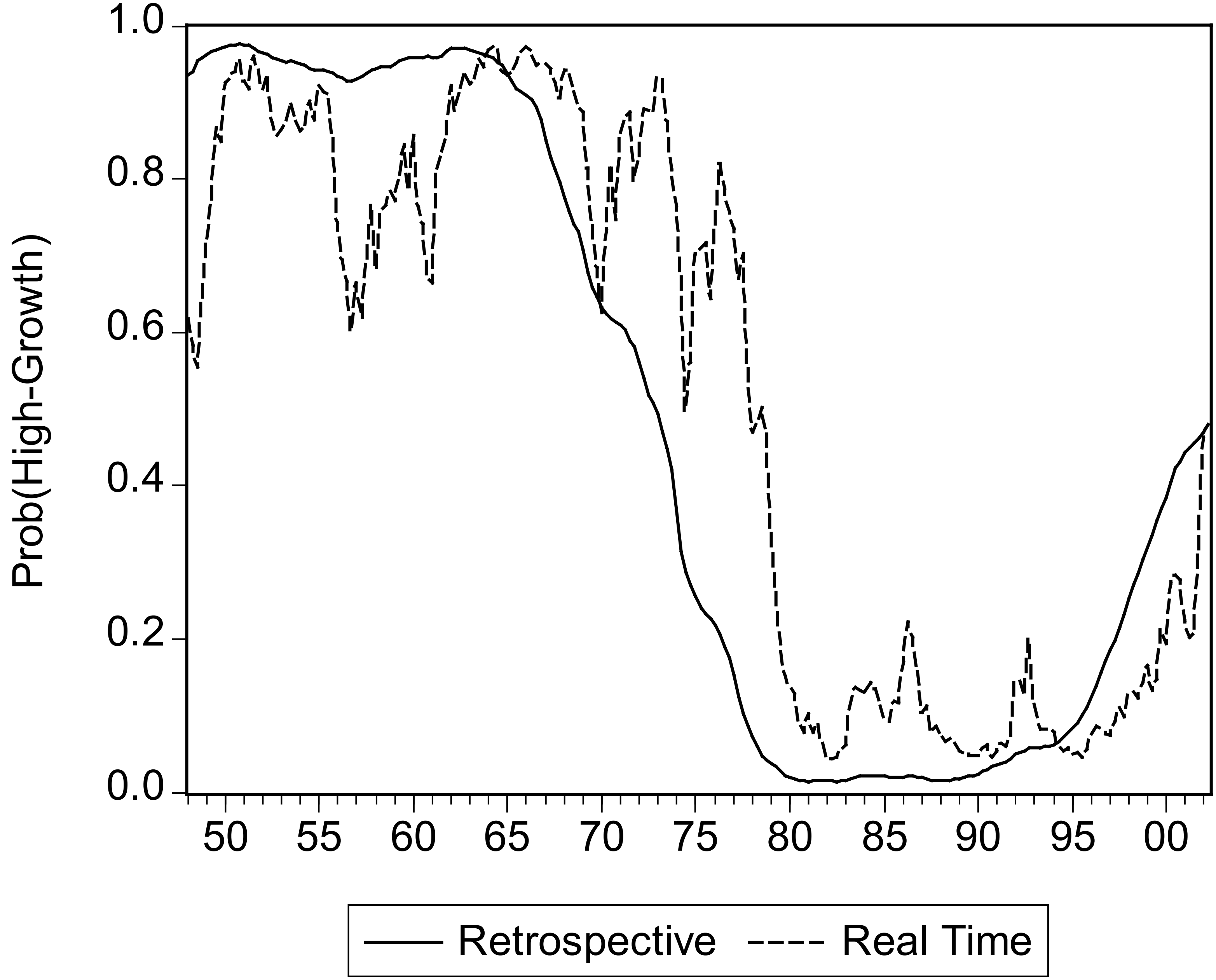


Figure 7: Productivity and its Permanent Component
(Two-Series System)

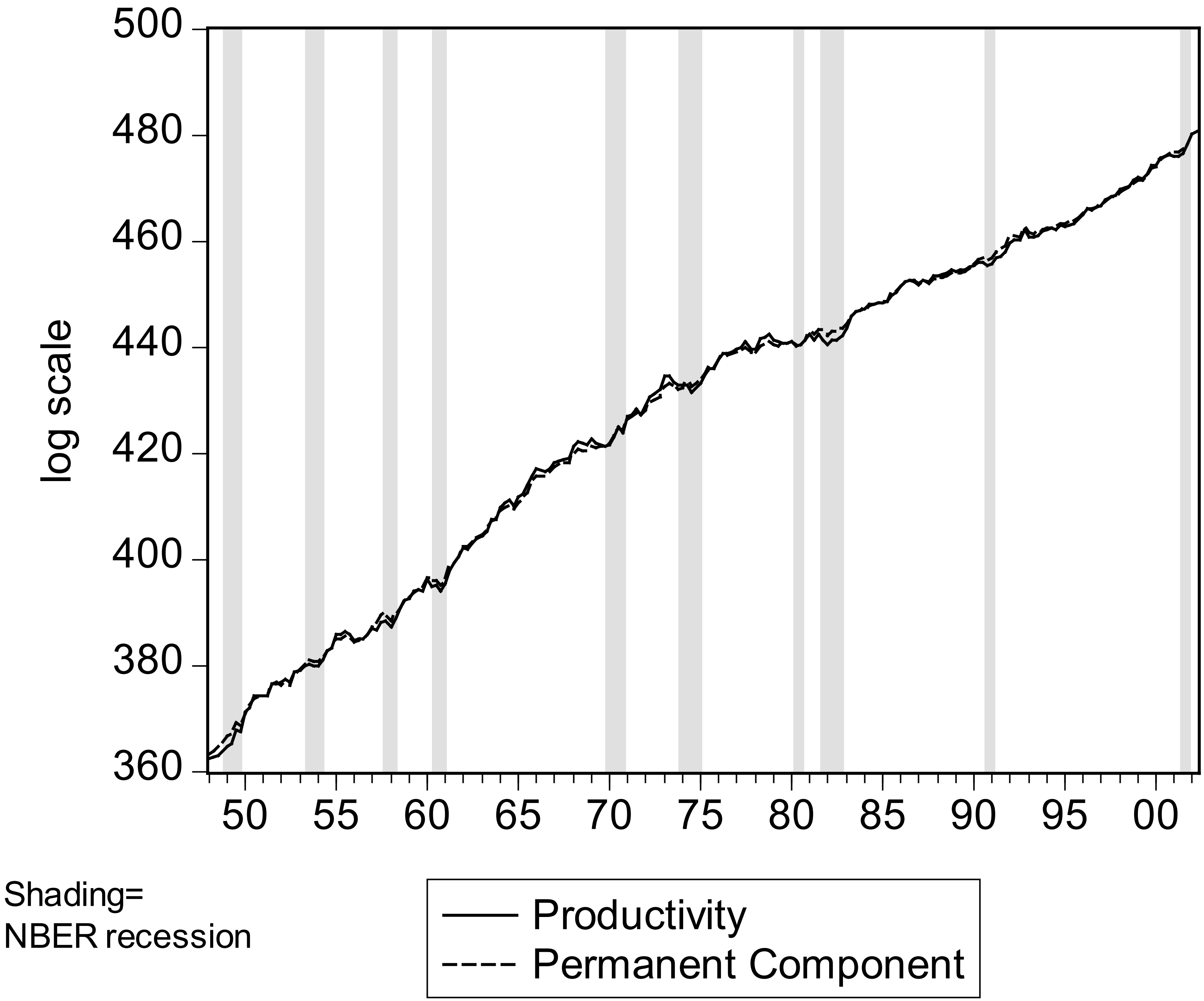


Figure 8: Relative Importance of Variables
for Regime Assessment

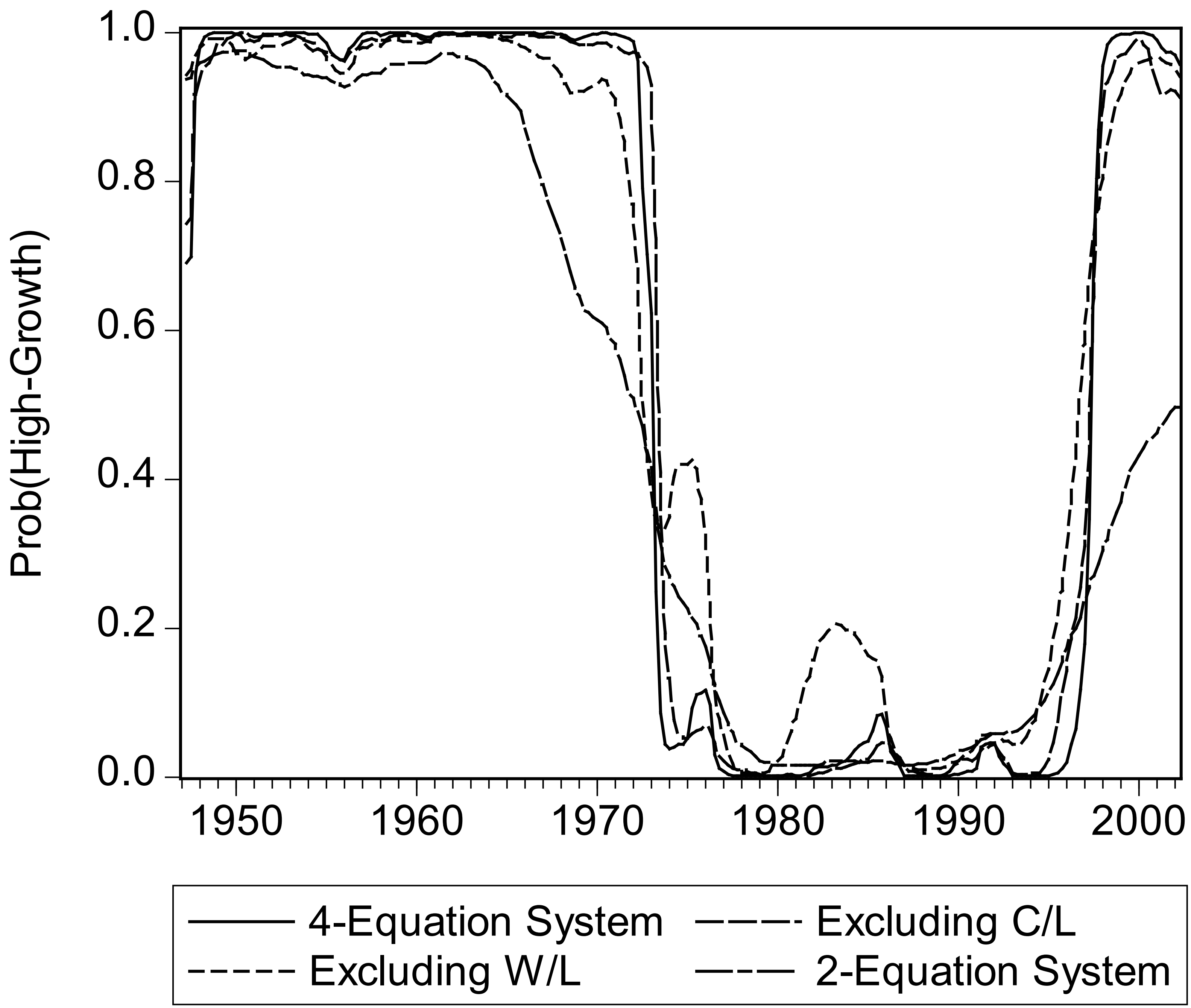


Figure 9: Variance of Idiosyncratic Residuals
(10-year Rolling Estimates)