Federal Reserve Bank of New York
Staff Reports

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Staff Report no. 455
June 2010
Revised May 2011

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JEL classification: E00, E31

Abstract

We characterize optimal state-dependent pricing rules under various forms of infrequent information. In all models, infrequent price changes arise from the existence of a lump-sum "menu cost." We entertain various alternatives for the source and nature of infrequent information. In two benchmark cases with continuously available information, optimal pricing rules are purely state-dependent. In contrast, in all environments with infrequent information, optimal pricing rules are both time- and state-dependent, characterized by "trigger strategies" that depend on the time elapsed since the last date when information was fully factored into the pricing decision. After considering the case in which information arrives infrequently for exogenous reasons, we address pricing problems in which gathering and processing information also entails a lump-sum cost. When the information and adjustment costs must be incurred simultaneously, the optimal pricing policy is a fixed-price time-dependent rule. When the costs are dissociated, the optimal rule features price stickiness and inattentiveness. Finally, we consider versions of the price-setting problems in which firms continuously entertain partial information. We characterize the optimal pricing rules and provide numerical solution algorithms and examples in a unified framework.

Key words: menu costs, information costs, infrequent information, sticky information, inattentiveness, optimal price setting, state-dependent pricing, time-dependent pricing

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1 Introduction

The recent availability of vast amounts of micro price data has generated renewed interest in price setting among macroeconomists, especially since the seminal work of Bils and Klenow (2004). It has also fostered the development of new microfounded models with explicit price-setting frictions. While most such papers focus on expanding the frontier of so-called menu-cost models,[1] some recent work analyzes the implications of explicit informational frictions for price setting behavior.[2]

In this paper we make a contribution to the literature on microfounded models of price setting in the presence of frictions, and characterize optimal state-dependent pricing rules under infrequent information. As in the menu-cost literature, infrequent price adjustments arise due to the existence of a lump-sum cost to changing prices. We entertain various alternatives for the source and nature of infrequent information, and solve for the resulting optimal price-setting rules using a unified framework that can be applied to most of the price-setting problems analyzed previously in the literature.

We first assume that information gathering and processing is costless, and analyze two benchmark cases with promptly available information: one with continuous disturbances to the economic environment, as in most of the menu-cost literature, and another with sporadic disturbances, as in the work of Danziger (1999) and Gertler and Leahy (2008). In both cases optimal pricing rules are purely state-dependent, characterized by an inaction region which is time invariant.

In contrast, in environments with infrequent information, optimal pricing rules are both time- and state-dependent, characterized by "trigger strategies" that depend on the time elapsed since information was last fully factored into the pricing decision. We start by analyzing cases in which information arrives infrequently for reasons that are outside the control of firms. We consider both random arrival of information, and information releases at deterministic times (as in Bonomo and Garcia 2001). A common feature of both specifications is that the inaction region widens as time passes. In the case of deterministic information arrival times, the inaction region becomes arbitrarily wide just prior to the information release, as the option value of waiting to make an informed decision about price adjustment becomes arbitrarily large. These specifications correspond to Knotek (2010) and Klenow and Willis (2007), who analyze menu-cost pricing in a sticky-information environment.

We then assume that gathering and processing information also entails a fixed cost. In the absence of menu-costs, this leads to a purely time-dependent policy of price plan revisions, as in

[1] Some examples are Almeida and Bonomo (2002), Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2010), Nakamura and Steinsson (2010).

[2] For example, Reis (2006), Woodford (2009), Maćkowiak and Wiederholt (2009).

Caballero (1989) and Reis (2006).[3] Optimal plans might entail changing prices continuously.[4] If instead the information and adjustment costs are borne out together, the optimal policy is a fixed-price time-dependent pricing rule, as in Bonomo and Carvalho (2004, 2010).[5] The assumption of a single adjustment/information cost is also consistent with Woodford (2009), who in addition assumes that firms are subject to an information-processing constraint in periods in which they choose not to incur the lump-sum adjustment/information cost.[6]

When adjustment and information gathering/processing costs are dissociated, firms have the option to adjust without information (what we refer to as a "uninformed adjustment") and to entertain information but choose not to adjust prices. As a result, the optimal policy features both infrequent price changes and infrequent incorporation of information into prices (or "inattentiveness"). It is characterized by an inaction region for price adjustment without new information, the borders of which depend on the time elapsed since the last date when the firm gathered and processed information. In addition, the policy specifies the next time for information gathering and processing as a function of the estimated price discrepancy relative to the firm's so-called frictionless optimal price. As in the case of deterministic information arrival times, firms never choose to adjust prices shortly before entertaining information. Gorodnichenko (2008), Abel, Eberly, and Panageas (2009), Alvarez, Guiso, and Lippi (2010), and Alvarez, Lippi, and Paciello (2010) also analyze models in which agents face information and adjustment costs. We defer a more detailed description of the relationship between our two-cost model and their work to the end of this Introduction.

A noticeable general feature of optimal pricing rules under infrequent information is the possibility of uninformed adjustments, based only on the trend of the firm's frictionless optimal price. This is most obvious in the case with an information cost but no adjustment cost, as in Caballero (1989) and Reis (2006): the only price changes that incorporate new information are the ones that occur infrequently, when the firm incurs the information gathering and processing cost. All the other price changes are uninformed, based on the drift of the frictionless optimal price. Uninformed adjustments may also be optimal in the presence of menu costs, both with exogenously infrequent information, and with optimally chosen information gathering and processing dates. Such adjustments are more likely if the trend of the frictionless optimal price is large relative to the variance of its innovations.

[3] Moscarini (2004) obtains infrequent information sampling as the optimal policy under limited information-processing capacity.

[4] In contrast, Burstein (2006) derives state-dependent pricing plans in a framework in which firms face a fixed cost of changing their price paths.

[5] In earlier work, Ball, Mankiw, and Romer (1988) solve for such a fixed-price time-dependent rule as an approximation to the optimal pricing policy in a menu-cost model.

[6] Woodford (2009) states explicitly that he prefers to interpret the lump-sum cost is his model as being purely an information cost, and to add the assumption that firms set a fixed price once they have factored in new information (as opposed to choosing a price plan). As he acknowledges, this is equivalent to assuming a single adjustment/information cost.

These pricing rules can thus rationalize adjustment based on the trend growth of prices between infrequent "informed" adjustments.

Finally, we analyze an arguably more realistic setup in which part of the relevant information is continuously observed and processed by firms, while another part is only entertained infrequently.[7] In this situation, some adjustments may occur given the partially available information. If such information is the aggregate price level, this pricing rule might lead to partially informed adjustments based on realized aggregate inflation. This amounts to an indexation rule that is arguably a realistic representation of price and wage setting rules during periods of very high inflation, as witnessed in Brazil, Israel and Chile in the 80s.[8]

For all the economic environments analyzed in this paper, we derive the conditions that characterize the optimal pricing rule and provide numerical solution algorithms and examples within a common framework. Each pricing problem is cast as an optimal stopping problem, the solution of which can be characterized by dynamic programming with two conveniently chosen state variables: the time elapsed since the last date when information was fully factored into pricing decisions, and the firm's conditional expectation (given its information) of the price discrepancy relative to its frictionless optimal price. For each problem, the Bellman equation that characterizes the firm's value function in the inaction region is rewritten as either an ordinary differential equation in one of those state variables, or as a partial differential equation in both variables. Boundary conditions dictated by the nature of the problem pin down the solution, which in some cases can be written in (almost) closed form, or is otherwise obtained numerically through algorithms that make use of finite-difference methods.

1.1 Other papers with information and adjustment costs

Abel, Eberly, and Panageas (2009) and Alvarez, Guiso, and Lippi (2010) concern consumption-portfolio problems under observation and adjustment (transaction) costs. Abel, Eberly, and Panageas (2009) characterize the inaction region - due to transaction costs - that applies on dates in which the agent has full information about the underlying state ("observation dates"). In general there might be observation without a subsequent portfolio adjustment. The authors then provide sufficient conditions under which the problem eventually converges to one in which the consumer chooses equally-spaced observation dates and on each observation date she adjusts her portfolio by transferring resources to the liquid asset irrespective of the state that she observes - that is, the behavior becomes purely time-dependent. Whenever this convergence occurs, observations and adjustments

[7] Klenow and Willis (2007), Gorodnichenko (2008), and Knotek (2010) also allow for continuous incorporation of partial information into pricing decisions, in the context of menu-cost models.

[8] This type of price-setting policies have also been assumed in the literature on the consequences of indexation for the cost of disinflation (see Bonomo and Garcia 1994 for price setting, and Jadresic 2002 for wage setting).

always coincide. Alvarez, Guiso, and Lippi (2010) analyze a similar consumption-portfolio problem, but focus on the consumption of durable goods while Abel, Eberly and Panageas (2009) concentrate on non-durable goods. They show that in their model there can always be instances of observation without a subsequent portfolio adjustment. In those circumstances, the optimal time until the next observation date depends on the observed state.

Alvarez, Lippi, and Paciello (2010) study price setting under observation and menu costs. In analogy with Alvarez, Guiso, and Lippi (2010), the optimal pricing rule that they derive consists of an inaction range that applies on observation dates, and an optimally chosen time interval until the next observation date, which depends on the observed price discrepancy. They compare implications from their two-cost model with a menu-cost model and an observation-cost model (which in their case turns out to be equivalent to the model with a joint information/adjustment cost of Bonomo and Carvalho 2004, 2010 - more on this in Subsection 5.3.2). They show that the model with two costs matches the observed frequencies of price adjustment and observation, and the distribution of the size and the hazard of price changes better than the alternative models.

None of the three papers with information and adjustment costs discussed above provide a solution to the general unrestricted problem in which agents have the option to adjust without information, and indeed choose to do so occasionally. These uninformed price adjustments become optimal when the drift in the process of the frictionless optimal price is large enough relative to the variance of its innovations. In fact, under some circumstances it may be optimal to conduct several uninformed adjustments between dates of information gathering and processing.[9] Alvarez, Guiso, and Lippi (2010) and Alvarez, Lippi, and Paciello (2010) provide sufficient conditions under which such uninformed adjustments are not optimal and focus on parameterizations that satisfy those conditions.[10] They otherwise prevent agents from making uninformed adjustments by imposing the restriction that adjustment requires observation. In the cases in which this restriction applies, they do not solve for how the boundaries of the inaction region vary as a function of the time elapsed since the last observation date (we detail this comparison in Subsection 5.3.2).

Finally, Gorodnichenko (2008) develops a model in which firms face a menu cost to change prices, and have partial and imperfect information about the underlying state of the economy. The aggregate price level serves as a free source of (partial) information. Moreover, firms can acquire

[9] This is more likely to be the case if the price discrepancy observed upon gathering and processing information falls within the inaction region, but close to an appropriate boundary.

[10] In section 7.3 of Alvarez, Lippi, and Paciello (2010) and Appendix AA-3 of Alvarez, Guiso, and Lippi (2010) the authors discuss the case of adjustment without information. Rather than providing a solution to the problem, they provide sufficient conditions under which such uninformed adjustments are not optimal. For the price-setting problem, Alvarez, Lippi, and Paciello (2010) show that this is the case for a sufficiently small rate of inflation. For the problem of asset managemnt with consumption of durables, Alvarez, Guiso, and Lippi (2010) show that this is the case when there is no uncertainty in asset returns.

private signals that provide noisy (imperfect) information about the state of the economy. He focuses on the externality induced by the structure of information that he assumes, and how its effects interact with infrequent price changes to produce inflation inertia.[11] The main difference relative to our two-cost model is that our information cost is in the spirit of Reis (2006) - an information gathering and processing cost. As a result our model has no room for information externalities.

The rest of the paper is organized as follows. Section 2 formulates the basic firm problem under infrequent information. Section 3 presents two benchmark cases with continuously available information, which help build intuition for the nature of the optimal pricing policies analyzed subsequently. Section 4 analyzes price-setting under exogenously infrequent information, whereas in Section 5 infrequent information arises endogenously, due to the existence of information gathering and processing costs. Section 6 extends the analyses to an environment with continuous information about one of two components of firms' frictionless optimal prices, and infrequent information about the other component. The last section concludes with a discussion of other applications of our framework, and of directions for future research.

2 The basic problem under infrequent information

We start by setting up the firm's problem under infrequent information. To help build intuition we rely on an heuristic characterization of its essential features. In Appendices A, B, and C, we present all the results that are not essential for the exposition of the paper, but which formalize arguments and provide microfoundations for assumptions made in the main text.

The general idea behind the price-setting problems we analyze is that, in the absence of frictions (and thus under full information), a firm would set its price equal to the so-called frictionless optimal price - which is the instantaneously profit-maximizing price. In the presence of impediments to such "ideal" price setting, firms choose the optimal pricing policy subject to adjustment costs and exogenously infrequent or costly information gathering and processing.

In Appendix A we present a simple general equilibrium model that yields an expression for the (logarithm of the) frictionless optimal price for a firm, p_t^*, as the sum of two components - an aggregate (nominal aggregate demand) and an idiosyncratic (productivity) component. For Sections 2-5, the distinction between the sources of variation in the frictionless optimal price is a potential distraction, and for expositional simplicity we refer to a single source of uncertainty. In contrast, in Section 6 we allow for partial information about the frictionless optimal price, and thus distinguish between those two components.[12]

[11] Information externalities are also the focus of Caballero (1989) and Kryvtsov (2009).

[12] While in Appendices A, and B we differentiate among firms and index them by i, throughout the paper we focus on the problem of an individual firm, and omit the i subscripts for notational simplicity.

Any deviation between a firm's actual price, p_t, and p_t^* - what we refer to as a *price discrepancy* - entails an instantaneous flow "cost" in the form of foregone (potential) profits. In Appendix B we use the same simple model from which p_t^* is derived to show that, after a normalization, the profit loss due to a non-zero price discrepancy can be taken as being approximately equal to the square of the discrepancy: $(p_t - p_t^*)^2$. The objective of firms is to minimize the present discounted value of expected total costs, which comprise the (integral of) flow deviation costs plus any other costs that prevent the firm from charging the frictionless optimal price continuously. Changing the price to reduce the gap relative to the frictionless optimal price entails a lump-sum adjustment cost. In subsequent sections, whenever gathering and processing information entails a cost, the latter is also factored into the intertemporal price-setting problem. We formalize the firm's intertemporal optimization problem in Appendix C.

Under infrequent information about p_t^*, in order to evaluate the expected flow cost due to price discrepancies the firm must form a probabilistic assessment of p_t^* given its information. Let $t_0 < t$ denote the last time when the firm had access to and processed full information about p_t^*. We can then decompose the instantaneous expected flow cost due to a price discrepancy at time t as:

$$\begin{aligned} E_{t_0}(p_t - p_t^*)^2 &= (p_t - E_{t_0}p_t^*)^2 + E_{t_0}(p_t^* - E_{t_0}p_t^*)^2 \\ &= (p_t - E_{t_0}p_t^*)^2 + Var_{t_0}(p_t^*), \end{aligned} \tag{1}$$

where E_{t_0} and Var_{t_0} denote, respectively, the conditional expectation and conditional variance given time t_0 information. The first term in the right-hand side of (1) represents the flow cost of deviating from the *expected* level of the frictionless optimal price, and the second term represents the expected flow cost from not continuously entertaining information about the latter. In the absence of adjustment costs, p_t would be set equal to $E_{t_0}p_t^*$, reducing the first part of the discrepancy cost to zero. Otherwise the firm must optimally solve the trade-off between letting p_t drift away from $E_{t_0}p_t^*$, and paying the cost to adjust.

As for the second term in (1), it is zero when information can be freely and continuously incorporated into the pricing decision. If information gathering and processing is costly, the firm can reduce the second term at the expense of incurring the information cost. Finally, if information is exogenously infrequent, the firm cannot take interim actions to reduce the second term, but it will affect the price adjustment decision, as will become clear. In what follows we refer to times when the firm gathers and processes relevant information about p_t^* as *information dates.*

We assume throughout that p_t^* follows a Markovian stochastic process, and that for any $\Delta t > 0$ the distribution of $p_{t+\Delta t}^* - p_t^*$ depends only on Δt. From this assumption and the structure of the firm's problem, given an information date t_0, the value function at a time $t > t_0$ - the optimized value of the firm's dynamic cost-minimization problem, denoted by V - is determined by two state variables: the time elapsed since the last information date, denoted by $\tau \equiv t - t_0$, and the deviation of p_t from its expected frictionless optimal level (which we refer to as the *expected discrepancy*), defined as:

$$z_t \equiv p_t - E_{t-\tau} p_t^*. \tag{2}$$

We thus write the firm's (normalized) expected profit loss as a function of τ and z:

$$\begin{aligned} E_{t-\tau}(p_t - p_t^*)^2 &= z_t^2 + Var_{t_0}(p_{it_0+\tau}^*) \\ &\equiv f(z_t, \tau). \end{aligned} \tag{3}$$

With lump-sum menu costs, price changes will be infrequent. In the absence of price changes and information updates, the value function V obeys the following Bellman equation:

$$V(z_t, \tau) = f(z_t, \tau)dt + e^{-\rho dt} E_t V(z_{t+dt}, \tau + dt), \tag{4}$$

where ρ is the time discount rate. Equation (4) is valid for all environments considered in this paper, including the standard full-information case. For each pricing problem that we consider, this Bellman equation is rewritten as either an ordinary differential equation in one of the two aforementioned state variables, or as a partial differential equation in both of them. The boundary conditions that pin down the solution to the firm's pricing problem vary depending on the nature of informational frictions, and will be introduced subsequently for each of the cases that we analyze.

3 Continuous information

In the next subsections we present two cases with continuous (as opposed to infrequent) information that will serve as benchmarks and help build intuition for the analysis of the cases with infrequent information. When information is continuous, the setup is a particular case of our general framework with t_0 always equal to t (i.e. τ is always equal to zero). Hence, $E_{t_0} p_t^* = p_t^*$ and there is only one state variable: the perfectly and continuously observed price discrepancy $z_t = p_t - p_t^*$. The deviation-cost function simplifies to:

$$f(z_t, 0) = z_t^2,$$

and the general Bellman equation (4) reduces to:

$$V(z_t) = z_t^2 dt + e^{-\rho dt} E_t \left[V\left(z_{t+dt} \right) \right]. \tag{5}$$

As a result, optimal pricing rules turn out to be purely state dependent.[13]

3.1 Continuous innovations

This is the canonical menu-cost model, and we report it for comparison purposes (for details, see, e.g., Dixit 1993). Changing prices entails a lump-sum cost K.[14] We assume, for analytical convenience, that p_t^* follows a Brownian motion with drift:

$$dp_t^* = \mu dt - \sigma dW_t, \tag{6}$$

where W_t is a standard Wiener process. Observe that when no control is exerted z_t follows the process:

$$dz_t = -\mu dt + \sigma dW_t.$$

Applying Ito's lemma we can rewrite the Bellman equation (5) as the following ordinary stochastic differential equation:

$$\frac{1}{2}\sigma^2 V''(z) - V'(z)\mu - \rho V(z) + z^2 = 0. \tag{7}$$

This is a well-known problem, and the general solution is given by:

$$V(z) = \frac{z^2}{\rho} + \frac{-2z\mu}{\rho^2} + \frac{\sigma^2}{\rho^2} + \frac{2\mu^2}{\rho^3} + Ae^{\alpha z} + Be^{\beta z}, \tag{8}$$

where:

$$\begin{aligned} \alpha &= \frac{\mu - \sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2}, \text{ and} \\ \beta &= \frac{\mu + \sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2}. \end{aligned}$$

The optimal pricing rule is characterized by (l, c, u), where l and u are, respectively, the lower and upper levels of the price discrepancy (sometimes referred to as "barriers") which trigger price adjustments, and c is the optimal ("target") discrepancy. The value function should satisfy several conditions. An optimality condition:

$$V_z(c) = 0, \tag{9}$$

two indifference ("value-matching") conditions between l and c and between u and c, which state that the difference between the value function at the barriers and at the optimal target point should

[13] By purely state dependent we mean that they only depend on the (perfectly and continuously observed) price discrepancy. Of course pricing rules are always "purely state dependent" relative to the full set of relevant state variables.

[14] As shown in Appendix C, this parameter can be interpreted as the (normalized) menu cost as a fraction of steady-state profits.

be equal to the adjustment cost:

$$V(l) = V(c) + K, \tag{10}$$

$$V(u) = V(c) + K, \tag{11}$$

and the so-called "smooth-pasting" optimality conditions:

$$V_z(l) = 0, \tag{12}$$

$$V_z(u) = 0. \tag{13}$$

Conditions (9)-(13) allow us to determine the constants A, B and the policy parameters l, u and c numerically. Figure 1 illustrates a trajectory of the state variable z_t with the following values of the parameters: $\mu = 0.1$, $\sigma = 0.1$, $K = 0.01$, $\rho = 0.025$.[15] Whenever z_t hits the lower or the upper barrier the firm incurs the menu cost K and reduces the price discrepancy to c. It should also be noted that $\mu > (<) 0 \Longrightarrow c > (<) 0$, since the drift in the frictionless optimal price tends to push the discrepancy in the opposite direction when there is no adjustment. The upper and lower barriers u and l are not symmetric with respect to c either.

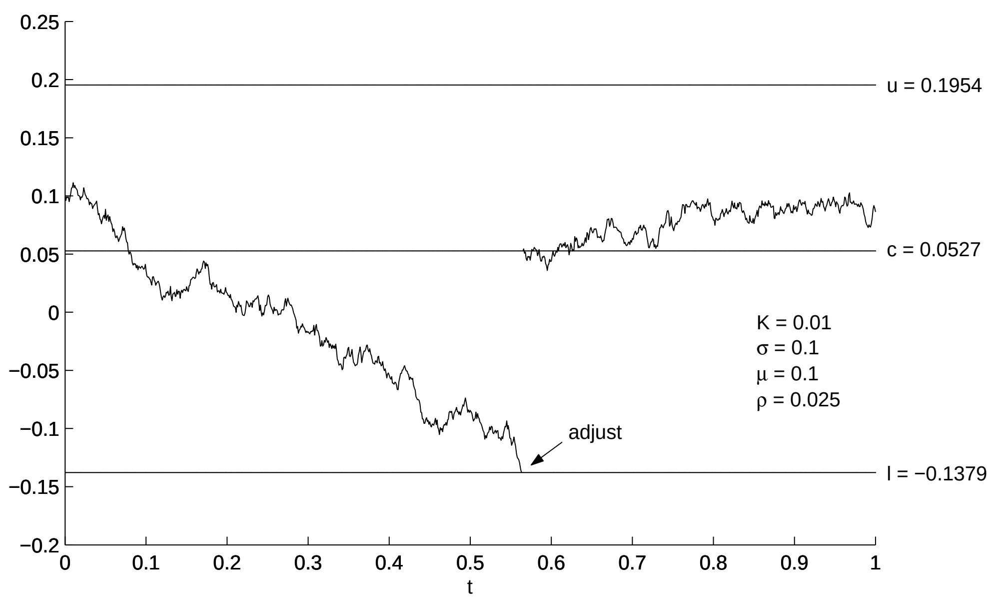

Figure 1: Standard state-dependent pricing policy under continuous innovations

[15]The time unit corresponds to one year (this normalization is used throughout the paper).

3.2 Infrequent innovations

We assume that the frictionless optimal price is perfectly and freely observable, but is subject to infrequent innovations.[16] Information is still continuously available, in the sense that firms can costlessly entertain news as they arrive.

Specifically, we assume that innovations to p_t^* occur according to the realization of a Poisson process with constant arrival rate λ, and that, conditional on an arrival, the innovation is a zero-mean Gaussian random variable. Then:

$$dp_t^* = \mu dt - \sigma\varepsilon dq_t,$$

where q_t is a Poisson arrival process with intensity λ, and ε is a standard normal random variable. In the absence of price changes the control z_t will evolve according to the following stochastic differential equation:

$$dz_t = -\mu dt + \varepsilon\sigma dq_t.$$

The differential form of the Bellman equation (5) can transformed into:

$$-V_z(z)\mu - (\rho+\lambda)V(z) + \lambda E\left[V(z+\sigma\varepsilon)\right] + z^2 = 0.$$

The general solution to the previous equation is:

$$V(z) = Ae^{\beta_1 z} + Be^{\beta_2 z} + V_p(z), \tag{14}$$

where $V_p(z)$ denotes a particular solution, and $e^{\beta z}$ solves the homogeneous equation - which implies that the βs are the solutions to:

$$\frac{1}{2}\beta^2\sigma^2 = \ln\left(1 + \frac{\rho}{\lambda} + \frac{\mu}{\lambda}\beta\right).$$

One particular solution to the differential equation corresponds to the case of never adjusting. This is given by:

$$V_p(z) = \int_0^\infty \lambda e^{-\lambda\tau}\left(\int_0^\tau e^{-\rho s}\left(z^2 - 2z\mu s + \mu^2 s^2\right)ds + e^{-\rho\tau}E\left[V_p(z-\mu\tau+\sigma\varepsilon)\right]\right)d\tau. \tag{15}$$

The solution to the above equation can be obtained by the method of undetermined coefficients. We

[16] This specification can be seen as the pricing problem underlying Danziger (1999) and Gertler and Leahy (2008).

guess (and verify) that $V_p(z) = az^2 + bz + g$, and substitute it into equation (15) to find:[17]

$$a = \frac{1}{\rho}, \tag{16}$$

$$b = \frac{-2\mu}{\rho^2}. \tag{17}$$

As in the continuous-innovation case of Subsection 3.1, the smooth-pasting and value-matching boundary conditions (9), (12), (13), (10), and (11) must be satisfied, determining the constants A and B, and the policy parameters l, c, and u.

Figure 2 below illustrates various possibilities for adjustment of the price discrepancy. First, a jump (denoted by a small circle in the figure) caused by the arrival of a shock brings the price discrepancy outside the lower barrier, triggering an adjustment to c. In the second instance of the Poisson arrival of a shock the price discrepancy jumps but stays within the barriers: there is no adjustment. There is also the possibility of adjustments in the absence of innovations, if the drift causes the discrepancy to reach its lower level before another innovation arrives. Henceforth we refer to the latter type of adjustments - made despite the absence of "news" - as *uninformed adjustments*.

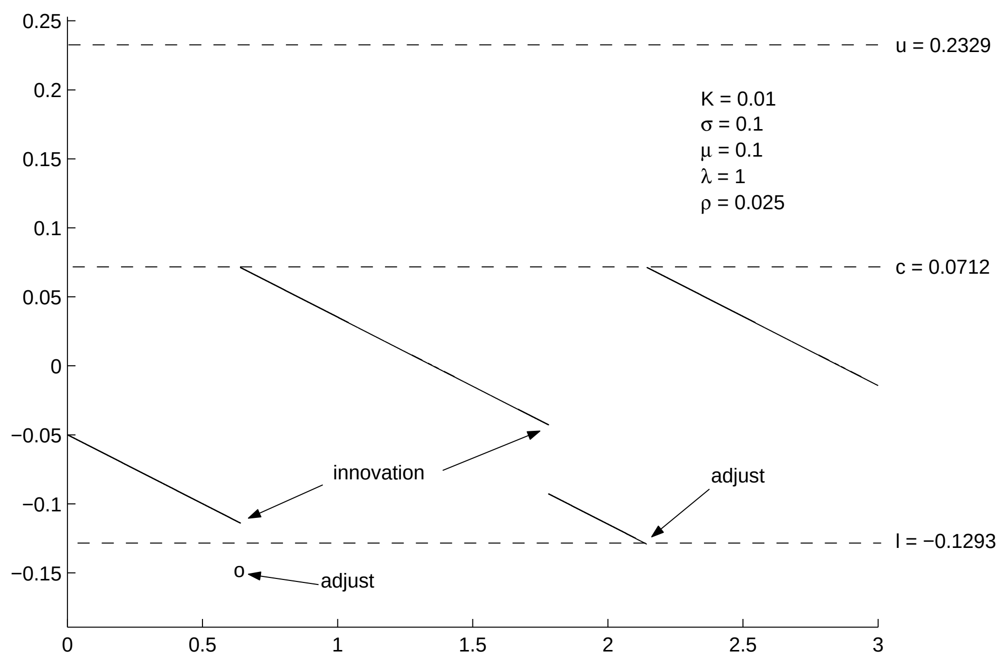

Figure 2: State-dependent pricing policy under infrequent innovations

Although we characterize the optimal pricing policy as a two-sided Ss rule in both the infrequent-innovation and in the standard continuous-innovation case, there are qualitative differences between the controlled processes. With continuous innovations, adjustments in a given direction always have

[17] The constant g is not used in the solution for the optimal pricing policy.

the same size, while in the infrequent-innovation case this does not happen - adjustments may be triggered by large enough innovations or by the deterministic trend.

4 Optimal pricing rules under exogenous infrequent information

In this section we assume that information arrives infrequently for reasons that are outside the control of firms. We start with a case of random information arrival. It differs from the continuous-information infrequent-innovation specification of the last section in that, between information dates, innovations to the frictionless optimal price do take place, but are unobserved. Thus they accumulate over time until they are fully revealed on the subsequent information date. We then consider the case of deterministic information dates. We show that in both cases the optimal pricing rule becomes time-and-state dependent.

4.1 Random information arrival

We assume that p_t^* follows a Brownian motion as in (6), but it is only observed at a random time, which has a negative exponential distribution. Then, in the absence of price changes the expected price discrepancy z_t will have a trend given by $-\mu$. Upon an information arrival, the observed innovation has zero mean and variance proportional to the time elapsed since the last information date. Formally, while there is no price change the expected discrepancy evolves according to the following stochastic differential equation:

$$dz_t = -\mu dt + \varepsilon\sigma\sqrt{\tau}dq_t, \tag{18}$$

where q_t is a Poisson arrival process with intensity λ, and ε is a standard normal random variable. This is essentially the sticky-information assumption of Mankiw and Reis (2002). Models with menu costs and sticky information have been analyzed by Knotek (2010) and Klenow and Willis (2007).

Notice the similarity of this case with the continuous-information infrequent-innovation case of the previous section. However, in the two cases the jumps in the state variable z_t have different sources: under continuous information, $z_t = p_t - p_t^*$ jumps whenever p_t^* jumps, while under infrequent information $z_t = p_t - E_{t_0}p_t^*$ jumps when a new information date arrives, bringing updated information about p_t^*.

An important difference relative to the continuous-information infrequent-innovation case is that the time since the last information date, τ, now matters. The reason is that, although the probability of an information arrival does not depend on τ, the amount of information at each arrival (as measured by the variance of the accumulated innovation in p_t^* during this period) is proportional to τ. Thus, the flow deviation costs of being uninformed about the optimal level p_t^* are also increasing in τ:

$$f(z_t, \tau) = z_t^2 + \sigma^2 \tau. \tag{19}$$

As a result, the barriers that determine the inaction region now depend on τ.

We now look for a policy $\{l(\tau), c(\tau), u(\tau)\}_{0 \leq \tau < \infty}$ where $l(\tau)$, $u(\tau)$, $c(\tau)$ represent, respectively, lower and upper trigger points and target point for the price discrepancy as a function of the time elapsed since the last information date. Using (18) and (19), we can write the differential form of the Bellman equation (4) as:

$$V_\tau(z, \tau) - \mu V_z(z, \tau) - (\rho + \lambda) V(z, \tau) + \lambda E\left[V\left(z + \sigma\varepsilon\sqrt{\tau}, 0\right)\right] + z^2 + \sigma^2\tau = 0. \tag{20}$$

Since adjustment costs are lump-sum, at any point in time when an adjustment happens, it is made to the point that minimizes the value function:

$$c(\tau) = \arg\min_z V(z, \tau). \tag{21}$$

Since it is always possible to pay the adjustment cost K and adjust to $c(\tau)$, the value function must satisfy:

$$V(z(\tau), \tau) \leq V(c(\tau), \tau) + K. \tag{22}$$

The trigger points $l(\tau)$ and $u(\tau)$ satisfy:

$$\begin{aligned} V(l(\tau), \tau) &= V(c(\tau), \tau) + K, \\ V(u(\tau), \tau) &= V(c(\tau), \tau) + K. \end{aligned} \tag{23}$$

The time variable τ can take any positive value. Hence, we need to find the value function and the trigger and target points for each positive time. In Appendix E we provide an algorithm for solving the optimal pricing problem numerically, using a finite-difference method. With the solution in hand we can study the properties of the optimal rule, which we illustrate in the next subsection.

4.1.1 The optimal rule

Figure 3 shows the functions $l(\tau), c(\tau), u(\tau)$, which characterize the optimal pricing rule. Since in this parameterization the process for z_t has a negative drift, uninformed adjustments are always upwards. As a consequence, only the lower barrier is (potentially) binding between information dates. The upper barrier matters only on information dates, when $\tau = 0$. The lower bound function, $l(\tau)$, is slightly decreasing. The reason is that the option value of waiting for an information arrival increases with τ due to the higher amount of information produced by the cumulation of underlying innovations. We also depict a sample trajectory for z_t, with two information arrivals. Whenever there is an information arrival, τ is reset to zero. In the first information arrival, z_t jumps to a point below the lower barrier $l(0)$ triggering adjustment to c. In the second arrival, z_t jumps upwards to a point inside the inaction range $(l(0), u(0))$. Thus, there is no adjustment.

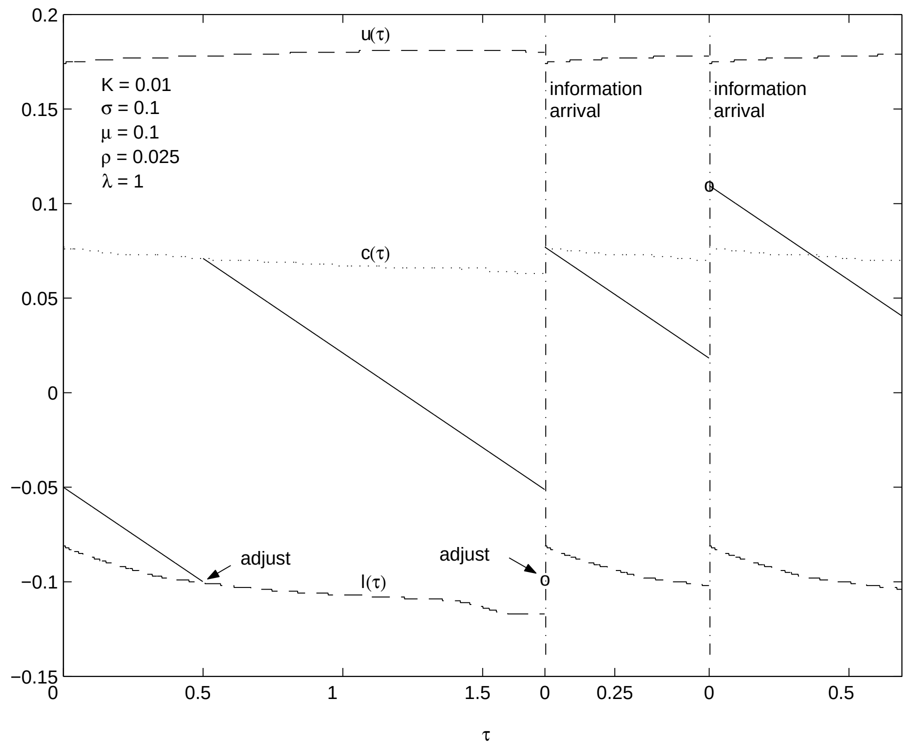

Figure 3: Optimal pricing policy under exogenous and randomly infrequent information

4.2 Deterministic information arrival

While the timing of some information releases is unknown, important pieces of information become public periodically. The release of economic statistics and monetary policy decisions are noteworthy examples. How does the previous pricing problem change if information arrives at regular intervals of time T?

In the absence of price changes and between information dates (for $0 < \tau < T$), the expected discrepancy z_t has a deterministic trend $-\mu$ and no innovation:

$$dz_t = -\mu dt.$$

The flow cost, $f(z_t, \tau)$, is still given by (19). As a consequence, the general Bellman equation (4) yields the following partial differential equation:

$$-\mu V_z(z,\tau) + V_\tau(z,\tau) - \rho V(z,\tau) + z^2 + \sigma^2\tau = 0. \tag{24}$$

The general solution to (24) is:[18]

$$V(z,\tau) = \frac{2\mu^2}{\rho^3} - \frac{2z\mu}{\rho^2} + \frac{z^2}{\rho} + \frac{\sigma^2}{\rho^2} + \frac{\sigma^2\tau}{\rho} + e^{-\frac{\rho z}{\mu}} G\left(\frac{z+\mu\tau}{\mu}\right), \tag{25}$$

where $G(\cdot)$ is a function to be determined by the nature of the firm's optimization problem.

Again we look for rules $\{l(\tau), c(\tau), u(\tau)\}_{0\leq\tau\leq T}$. Observe that now τ is bounded by T. Since adjustment costs are lump-sum, we know that price changes will be made to the point that minimizes the value function. Conditions (21), (22), and (23) are still valid. Since information arrives deterministically, we need to tie the value function just before the information arrival to the value function after the arrival. When information arrives, the expected discrepancy receives a shock with distribution $N(0,\sigma^2 T)$, and τ is reset to zero. We thus have the following additional condition:

$$V(z,T) = E\left[V\left(z+\sigma\sqrt{T}\varepsilon, 0\right)\right], \tag{26}$$

where ε is a random variable with distribution $N(0,1)$. This problem can be solved numerically as described in Appendix E, where we also provide an alternative solution approach for the no-drift case ($\mu = 0$).

4.2.1 The optimal rule

Figure 4 illustrates the optimal pricing rule under deterministic information arrival, assuming $T = 1$. Once more, observe that, due to the positive drift, between information dates there are only upward adjustments. The upper barrier might only be binding at times of information arrival. We illustrate a sample path for z_t. Initially z_t is close to zero, and arrives at time 1 inside the inaction region for T^-, but outside the inaction region for $\tau = 0$. Then, the accumulated shock is revealed and z_t jumps to the position marked with o - outside the time-zero inaction range. An immediate adjustment is triggered to $c(0)$. Then, with no information, z_t decreases at a constant rate from $c(0)$, and so on.

While the pricing rules in both the random and deterministic information arrival cases have some similarities, there is an obvious distinguishing feature. In the deterministic case the inaction range becomes arbitrarily large just before information arrivals. The intuition is clear: the option value of waiting becomes very large when information is about to be released. This is a stark testable implication of this specification: one should see fewer adjustments when potentially important information

[18] We obtain this solution directly with Wolfram's Mathematica.

is about to be released.

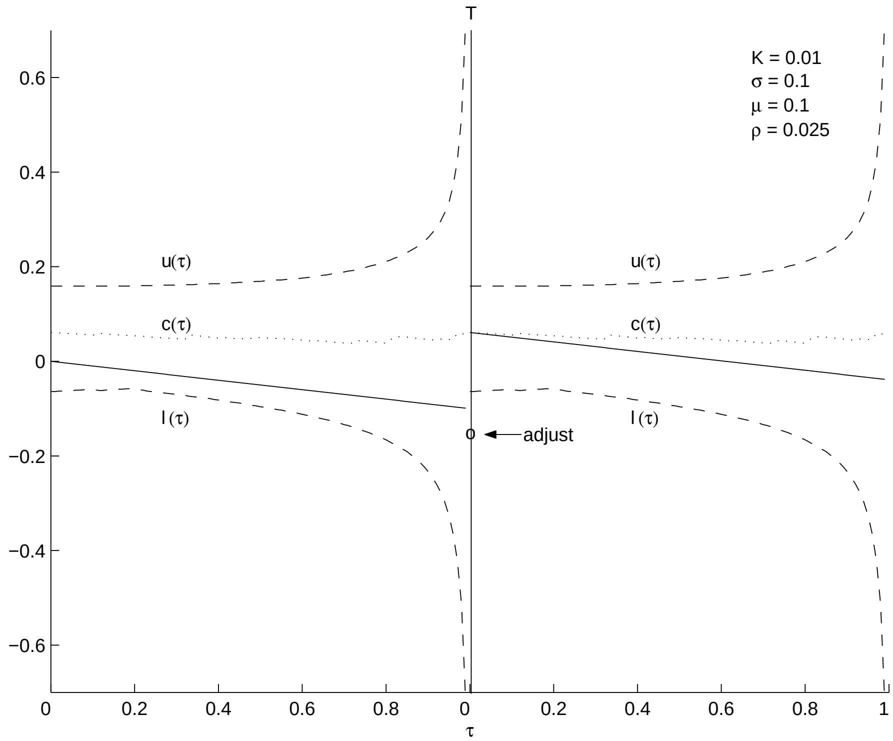

Figure 4: Optimal pricing policy under exogenous and deterministically infrequent information

5 Costly information gathering and processing

In this section we analyze environments in which information gathering and processing is costly - and this becomes the source of infrequent information. We start with a benchmark case without adjustment costs, as in Caballero (1989) and Reis (2006). The optimal rule implies infrequent information gathering and processing, but (potentially) continuous price adjustments. We then consider a case where information gathering and processing and price adjustments are tied to the same lump-sum cost. This implies a fixed-price time-dependent pricing rule with optimally chosen information/adjustment dates, as in Bonomo and Carvalho (2004, 2010). Finally, we analyze the case where information and adjustment costs are dissociated. In this case the optimal pricing rule features both infrequent price changes and infrequent incorporation of information into prices. It is characterized by an inaction region for price adjustment without new information, the borders of which depend on the time elapsed since the last information date. In addition, it specifies the next information date as a function of the expected price discrepancy.

5.1 No adjustment cost

We start with a simple case of costly information gathering and processing, but no adjustment cost, as in Caballero (1989) and Reis (2006). The optimal frictionless price evolves according to (6) but now there is a lump-sum cost F for gathering and processing information.[19]

Since there are no adjustment costs, the expected deviation z_t is always kept equal to zero - i.e., the firm always charges the expected value of its frictionless optimal price given its information. As a result, the flow deviation cost depends only on the time elapsed since the last information date:

$$f(\tau) = \sigma^2 \tau.$$

Thus, between information dates the Bellman equation (4) simplifies to:

$$V(\tau) = \sigma^2 \tau dt + e^{-\rho dt} V(\tau + dt),$$

which is equivalent to the following ordinary differential equation:

$$V'(\tau) - \rho V(\tau) + \sigma^2 \tau = 0.$$

The solution to the homogeneous equation is given by:

$$V_{\text{hom}}(\tau) = A e^{\rho \tau},$$

where A is a constant to be determined. A particular solution is given by:

$$V_{part}(\tau) = \frac{\sigma^2 \tau}{\rho} + \frac{\sigma^2}{\rho^2},$$

which leads to the general solution:

$$V(\tau) = A e^{\rho \tau} + \frac{\sigma^2 \tau}{\rho} + \frac{\sigma^2}{\rho^2}. \tag{27}$$

Take a policy that specifies intervals of length τ between information dates. The resulting value function must satisfy the following value-matching condition:

$$V(\tau) = V(0) + F,$$

which implies that:

$$A = \frac{e^{-\rho \tau}}{1 - e^{-\rho \tau}} \left(F - \frac{\sigma^2 \tau}{\rho} \right). \tag{28}$$

[19] As shown in Appendix C, this parameter can be interpreted as the (normalized) cost of information gathering and processing as a fraction of steady-state profits.

After plugging (28) into (27), the second condition is an optimality condition for the choice of information dates:

$$V'(\tau^*) = 0,$$

which yields:

$$\rho\tau^* + e^{-\rho\tau^*} = 1 + \rho^2 \frac{F}{\sigma^2}.$$

The latter equation implicitly defines the optimal time interval between information dates. The solution can be expressed in "quasi-closed-form" as:

$$\tau^* = \frac{1}{\rho} + \rho\frac{F}{\sigma^2} + \frac{1}{\rho}h^{-1}\left(-e^{-\left(1+\rho^2 F/\sigma^2\right)}\right),$$

where $h(x) = xe^x$.[20]

Notice that this pricing rule prescribes continuous uninformed price adjustment between information dates whenever $\mu \neq 0$, and a (possible) jump to the level of the frictionless optimal price on information dates, in order to keep the process z_t always at zero. As a consequence it does not prescribe price inaction, although the adjustment to the frictionless optimal level comes with a lag.

5.2 Single information/adjustment cost

Here we follow Bonomo and Carvalho (2004, 2010) and develop an alternative to the previous setup that entails microeconomic inaction. This follows from the assumption that the information gathering and processing cost and a menu cost are borne out together. We denote this single cost by Q.

Between information dates (and thus also in the absence of price changes) the expected discrepancy z_t has a deterministic trend $-\mu$ and no innovation:

$$dz_t = -\mu dt,$$

and the value function satisfies the partial differential equation (24).

It is easy to confirm that, along the curves $\tau = t$ and $z = z_0 - \mu t$, the function V satisfies the ordinary differential equation:

$$V'(t) - \rho V(t) + \sigma^2 t + (z_0 - \mu t)^2 = 0.$$

A particular solution to the latter equation is:

$$V_p(t) = \frac{\mu^2}{\rho}t^2 + \left(\frac{2\mu^2}{\rho^2} + \frac{\sigma^2 - 2z_0\mu}{\rho}\right)t + \frac{2\mu^2}{\rho^3} + \frac{\sigma^2 - 2z_0\mu}{\rho^2} + \frac{z_0^2}{\rho},$$

[20] In the Appendix we obtain the solution to this price-setting problem through an alternative approach.

which combined with the solution to the homogeneous equation $V'(t) - \rho V(t) = 0$ yields the general solution:

$$V(t) = Ae^{\rho t} + \frac{\mu^2}{\rho}t^2 + \left(\frac{2\mu^2}{\rho^2} + \frac{\sigma^2 - 2z_0\mu}{\rho}\right)t + \frac{2\mu^2}{\rho^3} + \frac{\sigma^2 - 2z_0\mu}{\rho^2} + \frac{z_0^2}{\rho},$$

where A is to be determined from the optimality conditions of the problem. Relying on the previous change of variables, the solution can be rewritten as:

$$V(z,\tau) = Ae^{\rho\tau} + \frac{\mu^2}{\rho}\tau^2 + \left(\frac{2\mu^2}{\rho^2} + \frac{\sigma^2 - 2(z+\mu\tau)\mu}{\rho}\right)\tau + \frac{2\mu^2}{\rho^3} + \frac{\sigma^2 - 2(z+\mu\tau)\mu}{\rho^2} + \frac{(z+\mu\tau)^2}{\rho}. \quad (29)$$

For an arbitrary policy that specifies intervals of length τ between information-adjustment dates, and a discrepancy c on such dates, the value function must satisfy the following value-matching condition:

$$V(c - \mu\tau, \tau) = V(c, 0) + Q,$$

which implies:

$$A = \frac{Q\rho^2 - 2\mu^2\tau + 2c\mu\rho\tau - \rho\sigma^2\tau - \mu^2\rho\tau^2}{(e^{\rho\tau} - 1)\rho^2}. \quad (30)$$

After plugging (30) into (29), the final conditions are first-order conditions for the choice of the optimal policy τ^* and c^*:

$$\begin{aligned} V_\tau(c^*, \tau^*) &= 0, \\ V_z(c^*, \tau^*) &= 0, \end{aligned}$$

which yield:

$$\begin{aligned} c^* &= \mu\left(\frac{1}{\rho} - \frac{e^{-\rho\tau^*}}{1 - e^{-\rho\tau^*}}\tau^*\right), \\ 0 &= \frac{\left(\sigma^2 - 2c^*\mu + \mu^2\tau^*\right)\tau^* e^{\rho\tau^*} + 2\frac{\mu^2}{\rho}\tau^* - \rho Q e^{\rho\tau^*}}{\left(1 - e^{\rho\tau^*}\right)^2} - 2\frac{\frac{\mu}{\rho}c^* - \frac{\mu^2+\sigma^2}{\rho^2}}{1 - e^{\rho\tau^*}}. \end{aligned}$$

This system of two equations can be solved numerically for τ^* and c^*.[21]

Under this pricing policy there are no uninformed price adjustments. If there is a positive drift in the frictionless optimal price, the expected price discrepancy decreases until the next information date. At that point information is factored into the pricing decision, and z_t jumps and is immediately adjusted to c^*. Notice that adjustment happens independently of the size of the surprise due to new information, and that the size of the adjustment is stochastic. This model can be seen as providing microfoundation for the pricing policy in the seminal work of Phelps (1978) and Taylor (1979, 1980). It implies that "contract lengths", rather than being exogenously given, are chosen optimally.

[21] In Appendix D we obtain the solution to this price-setting problem through an alternative approach.

5.3 Dissociated information and adjustment costs

We now tackle the pricing problem for a firm that faces both adjustment and information gathering and processing costs. In contrast to the previous subsection, here these costs are dissociated. With separate lump-sum costs for adjustment (K) and information gathering/processing (F) it is possible to adjust without information, and to entertain information but choose not to adjust.

Between information dates and in the absence of price adjustments, the differential equation which characterizes the evolution of the value function is still given by (24):

$$-\mu V_z(z,\tau) + V_\tau(z,\tau) - \rho V(z,\tau) + z^2 + \sigma^2\tau = 0,$$

with general solution given by (25):

$$V(z,\tau) = \frac{2\mu^2}{\rho^3} - \frac{2z\mu}{\rho^2} + \frac{z^2}{\rho} + \frac{\sigma^2}{\rho^2} + \frac{\sigma^2\tau}{\rho} + e^{-\frac{\rho z}{\mu}} G\left(\frac{z+\mu\tau}{\mu}\right),$$

where $G(\cdot)$ is a function to be determined by the nature of the firm's optimization problem.

The option to incur F and entertain information implies that:

$$\forall (z,\tau), \quad V(z,\tau) \leq E\left[V\left(z+\sigma\sqrt{\tau}\varepsilon, 0\right)\right] + F. \tag{31}$$

As before, it is also possible to pay the adjustment cost K and reset z to a chosen expected discrepancy $c(\tau)$. As a result:

$$\forall (z,\tau), \quad V(z,\tau) \leq V(c(\tau),\tau) + K, \tag{32}$$

where:

$$c(\tau) = \arg\min_z V(z,\tau).$$

The adjustment barriers at each point in (elapsed) time $l(\tau), u(\tau)$ satisfy:

$$V(l(\tau),\tau) = V(c(\tau),\tau) + K,$$
$$V(u(\tau),\tau) = V(c(\tau),\tau) + K.$$

As in the case of exogenous deterministic information, the value function just before an information date is tied to the value function on such a date, at which time the expected discrepancy receives a shock with distribution $N(0,\sigma^2\tau)$ - where τ denotes the time elapsed since the previous information date - and τ is reset to zero. However, because information dates are now determined by firms' decisions to incur the cost F and entertain information, the equivalent of (26) is now given by:

$$V\left(z,\tau^*\left(z\right)\right)=E\left[V\left(z+\sigma\sqrt{\tau^*\left(z\right)}\varepsilon,0\right)\right]+F. \tag{33}$$

where ε is a random variable with distribution $N(0,1)$, and where $\tau^*(z)$ denotes the (optimally chosen) time between information dates as a function of the expected discrepancy. In Appendix D we provide a solution for the no-drift case ($\mu = 0$), and in Appendix E we provide an algorithm for solving the problem when $\mu \neq 0$ numerically using a finite-difference method.

5.3.1 The optimal rule

Figure 5 illustrates the optimal pricing rule under adjustment and information gathering/processing costs. The dashed (red) lines define the limits of the no–price-adjustment region, and the discrepancy to which firms revert when they choose to adjust. The solid (blue) line provides the limits for the region in which the firm chooses not to incur the information gathering/processing cost.

Due to the positive drift in the frictionless optimal price, between information dates there can only be upward adjustments. The upper barrier might only be binding on information dates. We illustrate a sample path in which z_t is initially close to zero. Due to the high enough drift μ, the expected discrepancy hits the lower boundary $l(\tau)$, leading to an uninformed adjustment to $c(\tau)$. After that, the expected discrepancy drifts down until it touches the information boundary $\tau^*(z)$ at a point where $z \approx -0.04$ and $\tau \approx 0.73$. At that point the firm incorporates information into the pricing decision, as the expected discrepancy receives a shock with distribution $N\left(0, \sigma^2 \times 0.73\right)$. The time-elapsed variable τ is reset to zero, and the firm decides whether or not to pay the menu cost and change its price, depending on whether the just-learned price discrepancy is inside or outside the inaction region defined by $(l(0), u(0))$.

As in the case with deterministically exogenous information, notice that it is never optimal to make an uninformed price adjustment just before incurring the information gathering and processing cost. This result can be seen visually in the example depicted in Figure 5 once one recalls the crucial distinction between the (red) dashed and the (blue) solid curves that define the inaction region: for $\tau > 0$ the (red) dashed $l(\tau)$ and $u(\tau)$ lines trigger adjustment without information. Such an adjustment brings the expected discrepancy to $c(\tau)$, which is always "distant" from the $\tau^*(z)$ curve that dictates information gathering/processing. The intuition for the result just discussed is slightly different from the case with deterministic information arrival: rather than incurring the menu cost to make an uninformed adjustment and then immediately incurring the information gathering/processing cost, it is always better to reverse the order of these actions and keep adjustment as an option, to be exercised or not depending on the new information.

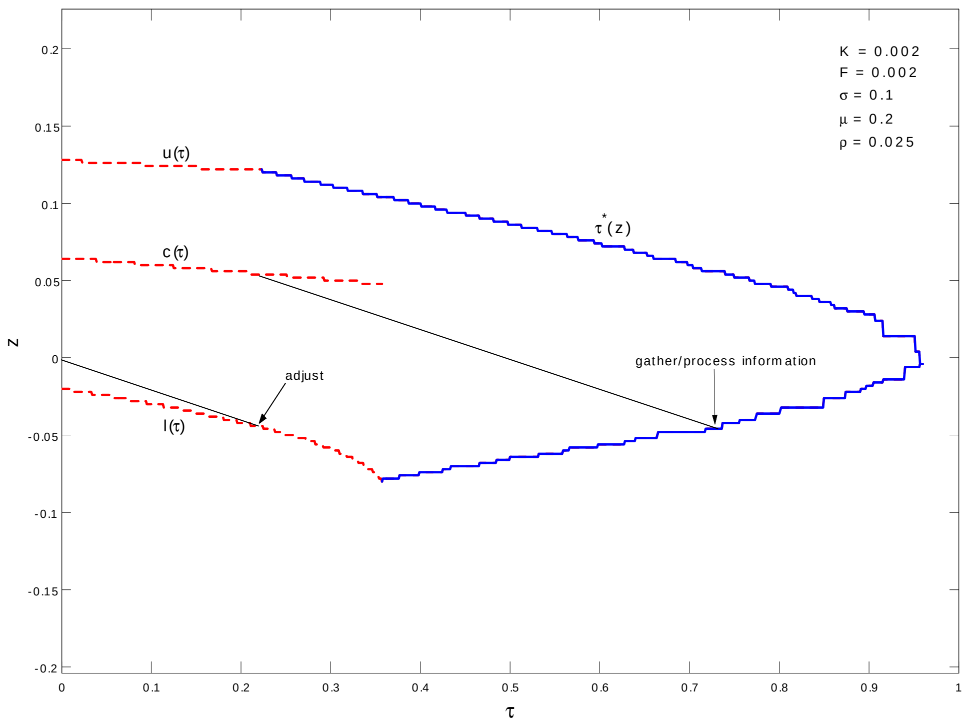

Figure 5: Optimal pricing policy under adjustment and information gathering/processing costs

5.3.2 Relation to other papers

As we noticed in the Introduction, Abel, Eberly, and Panageas (2009), Alvarez, Guiso, and Lippi (2010), and Alvarez, Lippi, and Paciello (2010) (from here on ALP) also formulate problems with both adjustment and information costs. As Abel, Eberly, and Panageas' (2009) complete characterization focuses on the case in which behavior eventually becomes purely time-dependent, we focus on the other two papers, where the optimal observation time depends on the state of the economy. Here we extend the discussion in Subsection 1.1.

As we commented in that subsection, Alvarez, Guiso, and Lippi (2010) and ALP focus on parameterizations that satisfy sufficient conditions under which uninformed adjustments are not optimal. They otherwise prevent agents from making such adjustments by imposing the restriction that adjustment requires observation. In contrast, we never impose such a restriction, and use our approach to provide a solution that encompasses cases in which uninformed adjustments are optimal.

A key difference between our approach and the one followed by Alvarez, Guiso, and Lippi (2010), and ALP is the choice of state variables. Our state variables at a time t when the last information date occurred τ "periods" ago are elapsed time, and the expected price discrepancy at time t given the information at time $t-\tau$. This formulation automatically takes into account the expected increase in the frictionless optimal price between $t-\tau$ and t, which is given by the drift of the frictionless optimal process. In the case where firms continuously entertain partial information, which we analyze in the next section, the state also changes to reflect this information. Thus, our information boundary ($\tau^*(z)$) should be seen as applying to the state at that same time, as this state evolves during periods of no-information. In contrast, in ALP their observation time is a function of the state defined as the known discrepancy at the last observation date.

In the unrestricted problem that we analyze, in which uninformed adjustments may be optimal, an increase in the drift of the frictionless optimal price relative to the variance of its innovations need not affect the optimal choice of the time interval between information dates. The reason is that the firm has the option to undertake uninformed adjustments. In contrast, whenever the restriction that firms need to observe the state before deciding on a price change binds in ALP's model, anything that affects the incentives to change prices will also affect the incentives to gather and process information - simply because price changes require observation. This point becomes clear in Figure 6 of ALP, as their observation functions are tilted when inflation increases, becoming essentially an upward sloping straight line when the annual inflation rate is 60%. This does not happen in our case, as is clear from Figure 6 below. This figure depicts the optimal unrestricted policy for two values of the drift in the frictionless optimal price process. The curves subscripted with the number one $(l_1(\tau), u_1(\tau), c_1(\tau), \tau_1^*(z))$ correspond to the case of $\mu = 0$, whereas the curves subscripted with two $(l_2(\tau), u_2(\tau), c_2(\tau), \tau_2^*(z))$ correspond to $\mu = 0.60$. The remaining parameter values are the same as in Figure 5. It is apparent from the comparison between the two policy rules that the general shape of the $\tau^*(z)$ does not change much beyond the asymmetry induced by the higher drift. The pattern is different in Figure 6 of ALP where the information boundary tends to shift down asymmetrically with a larger inflation drift. These changes in the shape and location of the function that specifies the time until the next observation date are due to the restriction imposed by ALP.

A simple and extreme example of the possible implications of ruling out uninformed adjustments is provided when there are no adjustment costs. If the firm has the option to make uninformed adjustments, as in our paper, the optimal choice of time interval between information dates is independent of the drift in the frictionless optimal price (see Subsection 5.1). The reason is that the firm can adjust the path for its price between information dates to fully "neutralize" the effect of any drift in the frictionless optimal price (this result accords with Caballero 1989 and Reis 2006). Price adjustments along the path implemented between information dates are simply uninformed adjustments.

In sharp contrast, if uninformed adjustments are ruled out (as in section 4.1 of ALP) , an increase in the drift of the frictionless optimal price leads the firm to shorten the time interval between

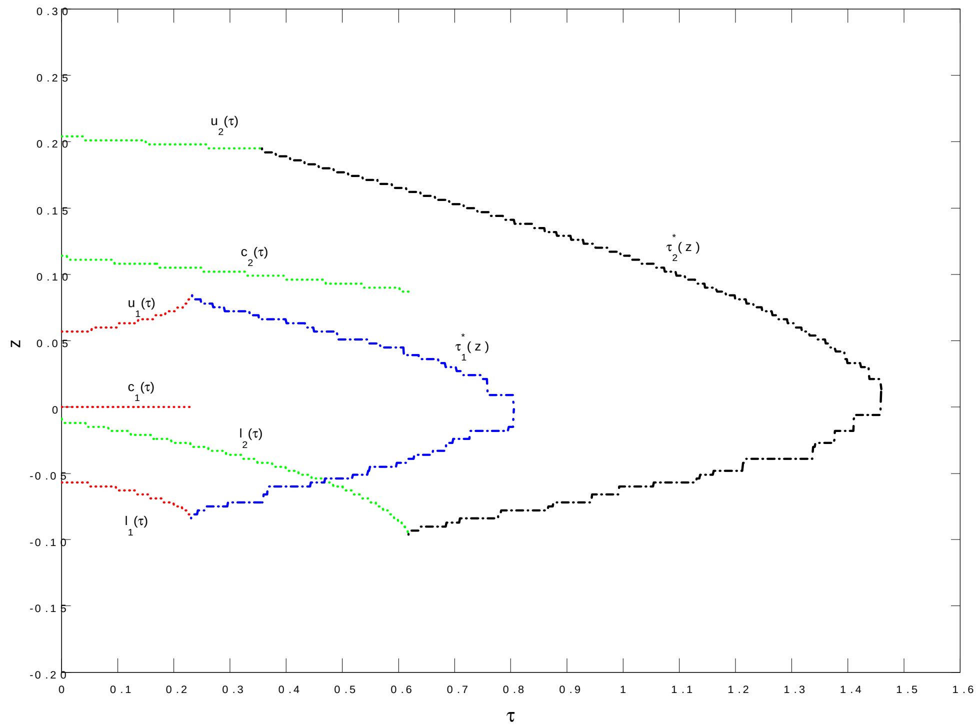

Figure 6: Comparative statics of the optimal pricing policy under adjustment and information gathering/processing costs - changes in μ

information dates (Proposition 1 in ALP). This reaction is not dictated by a desire to entertain information more frequently per se, but is rather a by-product of the need to update information in order to be able to change prices, even if only to "keep up" with the known drift in the frictionless optimal price. As a matter of fact, because of the restriction that firms have to observe the state when making a price change, the model with no adjustment costs in section 4.1 of ALP is mathematically equivalent to the model with a single information/adjustment cost as in Bonomo and Carvalho (2004, 2010), for which we provide a novel solution in Subsection 5.2. This is a side-effect of the aforementioned restriction, which carries over to the case with a strictly positive adjustment cost. As a result, that restriction might induce a higher frequency of information gathering and processing than would otherwise be optimal. This side-effect obviously disappears when firms have the option to undertake uninformed adjustments.

6 Partial continuous information

In the previous sections we assumed either entirely continuous or infrequent information (for exogenous or endogenous reasons). In reality, it is more reasonable to assume that there is always some continuous flow of information that can be factored into pricing decisions somewhat costlessly, and some information that is only incorporated infrequently - either for exogenous reasons, or due to information gathering and processing costs.[22]

In this section we extend our previous results by assuming that one of the two components of the frictionless optimal price is continuously and freely observable, and that processing it entails no costs.[23] In particular, we assume that:

$$dp_t^* = \mu dt - \sigma_i dW_{it} - \sigma_a dW_{at},$$

where W_{it} and W_{at} are independent standard Wiener processes. Information about W_{it} is continuously and freely available, and costless to process. In contrast, firms face infrequent information about W_{at}. From here on we thus refer to information dates as "W_{at}-information dates."

In the next subsections we analyze optimal pricing policies in two environments with partial continuous information. In the first one, information about W_{at} arrives infrequently for reasons that are outside the control of the firm. In the second, this component of the frictionless optimal price is subject to costly information gathering and processing.

6.1 Exogenous infrequent W_{at}-information

For simplicity we present only the case with deterministic arrival of information about W_{at}. Between W_{at}-information dates and in the absence of price adjustments, z_t changes continuously because of both the drift and the W_{it} process:

$$dz_t = -\mu dt + \sigma_i dW_{it}.$$

W_{at}-driven uncertainty also impacts the expected costs of deviating from the frictionless optimal price. The instantaneous flow-cost function is given by:

$$f(z_t, \tau) = z_t^2 + \sigma_a^2 \tau.$$

[22] Klenow and Willis (2007), Gorodnichenko (2008), and Knotek (2010) propose menu-cost models in which firms continuously incorporate partial information into pricing decisions.

[23] The two-component representation of p_t^* can be justified from first principles, as in Appendix A.

Hence, the differential form of the Bellman equation (4) is now written as:

$$\frac{1}{2}\sigma_i^2 V_{zz}(z,\tau) - V_z(z,\tau)\mu + V_\tau(z,\tau) - \rho V(z,\tau) + z^2 + \sigma_a^2\tau = 0. \tag{34}$$

The condition that determines $c(\tau)$, (21), the adjustment-option condition, (22), and the conditions that determine $l(\tau)$ and $u(\tau)$, (23), remain the same. However, the condition that ties the value function at time 0 to the value function at the subsequent W_{at}-information date changes to incorporate only uncertainty due to the unobserved component of p_t^*:

$$V(z,T) = E\left[V\left(z + \sigma_a\sqrt{T}\varepsilon, 0\right)\right].$$

The numerical solution algorithm used to solve this problem is essentially the same as the one used for solving the problem with deterministic information arrivals and no interim information about W_{it}. We present it in Appendix E.

6.1.1 The optimal rule

Figure 7 shows the optimal pricing rule and a sample path for z_t, assuming $T = 1$. For τ between zero and one, adjustment is dictated by the evolution of the expected discrepancy, which depends on μ and on realizations of W_{it}. When z_t reaches the lower barrier, adjustment is triggered to $c(.)$ of the respective τ. These adjustments take into consideration only the continuously and freely available W_{it}-information. When τ reaches time $T = 1$, W_{at}-information arrives and z_t jumps. If it falls outside the inaction range at zero, an adjustment is triggered to $c(0)$.

In this environment there are no totally-uniformed adjustments. The firm uses the W_{it}-information between W_{at}-information dates, and adjusts if the expected price discrepancy becomes large enough. Despite the continuous flow of W_{it}-information, the inaction range still becomes very wide before the deterministic times of W_{at}-information arrival, and the implication that one should not see adjustments before an important information announcement still holds.

6.2 Costly W_{at}-information gathering and processing

The setting is identical to the previous subsection, but now W_{at}-information can be factored into pricing decisions at any time, at a lump-sum cost F. The differential equation for the value function in the absence of adjustment and W_{at}-information is still:

$$\frac{1}{2}\sigma_i^2 V_{zz}(z,\tau) - V_z(z,\tau)\mu + V_\tau(z,\tau) - \rho V(z,\tau) + z^2 + \sigma_a^2\tau = 0, \tag{35}$$

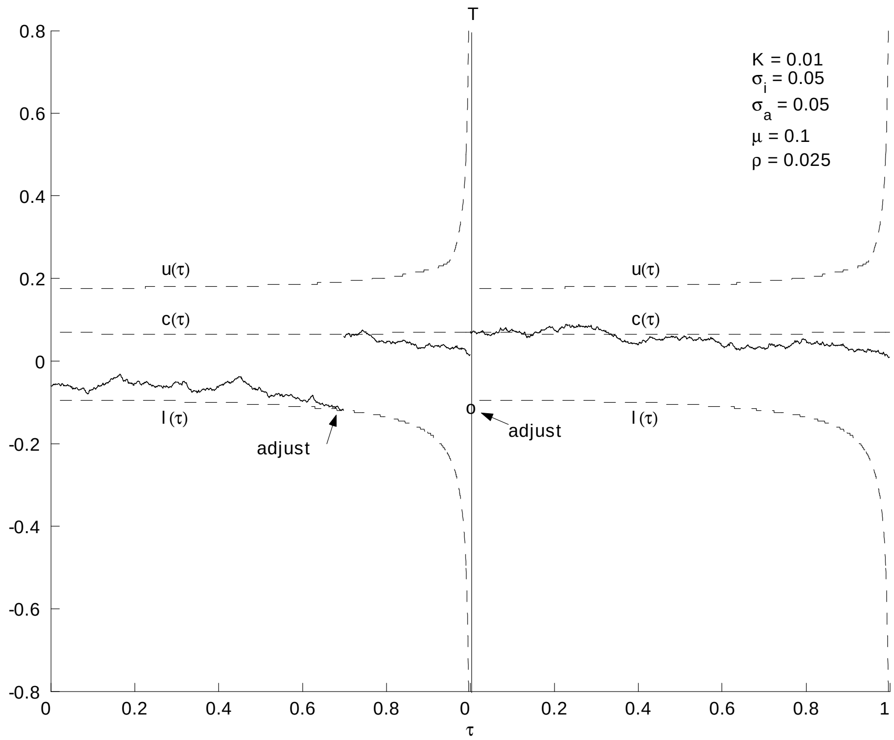

Figure 7: Optimal pricing policy under partial continuous information, and exogenous and deterministically infrequent information

and the optimality conditions are now:

$$\forall (z,\tau), \quad V(z,\tau) \leq E\left[V\left(z + \sigma_a \sqrt{\tau}\varepsilon, 0\right)\right] + F \text{ and}$$

$$\forall (z,\tau), \quad V(z,\tau) \leq V(c(\tau),\tau) + K,$$

with:

$$c(\tau) = \arg\min_z V(z,\tau).$$

Furthermore, the adjustment barriers satisfy:

$$V(l(\tau),\tau) = V(c(\tau),\tau) + K,$$
$$V(u(\tau),\tau) = V(c(\tau),\tau) + K,$$

and the function $\tau^*(z)$ that specifies the time of the next information date is defined implicitly by:

$$V(z,\tau^*(z)) = E\left[V\left(z + \sigma_a \sqrt{\tau^*(z)}\varepsilon, 0\right)\right] + F.$$

As in all previous cases, Appendix E provides a numerical solution algorithm for this price-setting problem.

6.2.1 The optimal rule

Figure 8 illustrates the optimal pricing rule under adjustment and information gathering/processing costs, and partial information. As in the previous case, in this environment there are no totally-uniformed adjustments. The firm uses the W_{it}-information between W_{at}-information dates, and adjusts if the expected price discrepancy hits the boundaries of the no-price-adjustment region. The latter is defined by the outer dashed (red) lines. The inner such line gives the discrepancy to which firms revert when they choose to adjust. The solid (blue) line provides the limits for the region in which the firm chooses not to incur the information gathering/processing cost.

In the sample path realization for z_t that we depict in Figure 8, there are two partially-informed adjustments before the firm decides to incur the cost to entertain information about W_{at}. At that point the time-elapsed variable τ is reset to zero, and the firm decides whether or not to pay the menu cost and change its price, depending on whether the price discrepancy is inside or outside the inaction region defined by $(l(0), u(0))$. As in the case with no interim information about W_{it}, notice that it is never optimal to make a partially uninformed price adjustment just before incurring the information gathering and processing cost.

In this example we assume the same parameter values as in Subsection 5.3.1, splitting the sources of variation of the p_t^* process evenly between the W_{it} and W_{at} processes (i.e. $\sigma_i = \sigma_a = \sigma/\sqrt{2}$). This leads to a quite dramatic change in the optimal pricing policy, in that the firm is now willing to wait longer until the subsequent information date than in the case without interim information. This is quite intuitive, since the expected flow deviation cost due to unobserved variation in p_t^* is now smaller.

7 Conclusion

In this paper we study optimal price setting under adjustment costs and infrequent information arising from various sources. Pricing rules are more complex than the usual purely state-dependent strategies. In general, the inaction ranges depend on the time elapsed since the last information date. There is scope for uninformed adjustments. When some important determinant of the frictionless optimal price can be freely and continuously factored into pricing decisions, there can be partially-informed adjustments. There should be no adjustment just prior to the release of important information, if the release date is known. Likewise, it is never optimal to make an uninformed price adjustment just before incurring the information gathering and processing cost.

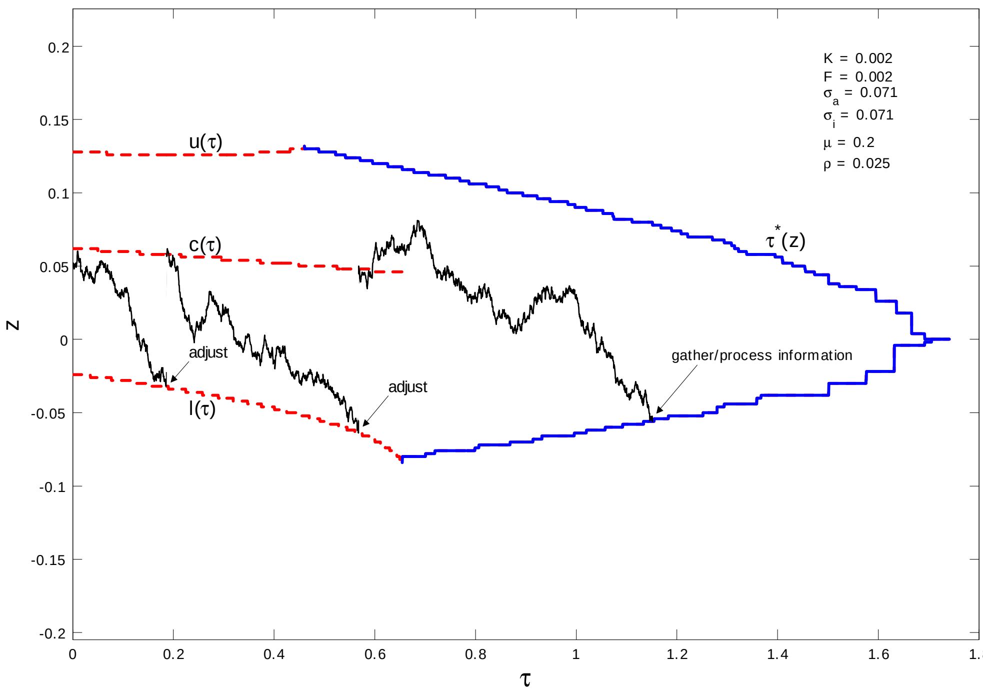

Figure 8: Optimal pricing policy under partial continuous information, and adjustment and information gathering/processing costs

While in this paper we focus on price setting, as emphasized in the early Bonomo and Garcia (2003) paper, our framework is, more generally, suitable for studying optimal decision-making under adjustment costs and infrequent information. For instance, our results might be of interest in the context of employment adjustment, inventory management and investment problems.

In its current form, our framework has the big advantage that the optimal policies can be solved for independently of equilibrium considerations. This makes the various models that we entertain relatively cheap to solve computationally, and thus allows for a relatively straightforward attack on their quantitative micro and macro implications. Of course such simplicity, which is afforded by the nature of the underlying economic environment, also has some costs. Importantly, it precludes interactions of agents' decisions through general-equilibrium effects. While our formulations can be extended to allow for such interactions, they have to be handled with methods for solving (infinitely-dimensional) heterogeneous agents models, which typically make computational solutions more costly. Despite this complication, solutions are feasible, and should open the possibility of addressing important research questions.

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8 Appendix A

We derive the frictionless optimal price in a simple general equilibrium framework. A representative consumer maximizes expected discounted utility:

$$E_{t_0} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left[\log(C_t) - H_t\right] dt,$$

subject to the budget constraints:

$$B_t = B_0 + \int_0^t W_r H_r dr - \int_0^t \left(\int_0^1 P_{ir} C_{ir} di \right) dr + \int_0^t T_r dr + \int_0^t \Lambda_r dQ_r + \int_0^t \Lambda_r dD_r, \text{ for } t \geq 0.$$

Utility is defined over the composite consumption good $C_t \equiv \left[\int_0^1 (C_{it}/A_{it})^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ with $\theta > 1$, where C_{it} is the consumption of variety i, and A_{it} is a relative-preference shock. P_{it} is the price of variety i, H_t is the supply of labor, which commands a wage W_t, B_t is total financial wealth, T_t are total net transfers, including any lump-sum flow transfer from the government, and profits received from the firms owned by the representative consumer. Q_r is the vector of prices of traded assets, D_r is the corresponding vector of cumulative dividend processes, and Λ_r is the trading strategy, which satisfies conditions that preclude Ponzi schemes. The associated consumption price index, P_t, is given by:

$$P_t = \left[\int_0^1 P_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{36}$$

The demand for an individual variety is:

$$C_{it} = A_{it}^{1-\theta} \left(\frac{P_{it}}{P_t} \right)^{-\theta} C_t. \tag{37}$$

Firms hire labor to produce according to the following production function:

$$Y_{it} = A_{it} H_{it}.$$

Note that we assume that the productivity shock is perfectly correlated with the relative-preference shock in the consumption aggregator. This has precedence in the sticky-price literature (for instance, King and Wolman 1999 and Woodford 2009).[24] Our specific assumption follows Woodford (2009), and aims to produce a tractable profit-maximization problem that can be written as a price-setting "tracking problem" in which the firm only cares about the ratio of the two stochastic processes driving profits, which will be specified below.

[24] More generally, assumptions relating preference and technology processes have been used previously in the literature on "balanced growth" in multi-sector models (e.g. Kongsamut, Rebelo, and Xie 2001).

The static profit-maximizing price for firm i, P_{it}^* (also referred to as its *frictionless optimal price*), is given by the usual markup rule:

$$P_{it}^* = \frac{\theta}{\theta - 1} \frac{W_t}{A_{it}}. \tag{38}$$

From the representative household's labor supply:

$$\frac{W_t}{P_t} = C_t,$$

which leads to:

$$P_{it}^* = \frac{\theta}{\theta - 1} \frac{P_t C_t}{A_{it}}.$$

In logarithms (lowercase variables denote logarithms throughout), this reads:

$$p_{it}^* = \log\left(\frac{\theta}{\theta - 1}\right) + \log\left(P_t C_t\right) - \log\left(A_{it}\right).$$

Ignoring the unimportant constant and assuming appropriate exogenous stochastic processes for nominal aggregate demand and for idiosyncratic productivity yields the specifications used throughout the main text.

9 Appendix B

Here we derive the quadratic approximation to the static profit-maximization problem used in the main text. Write real flow profits as:

$$\Pi\left(\frac{P_i}{P}, C, A_i\right) = A_i^{1-\theta} \frac{P_i}{P} \left(\frac{P_i}{P}\right)^{-\theta} C - \frac{W}{P A_i} A_i^{1-\theta} \left(\frac{P_i}{P}\right)^{-\theta} C,$$

where P_i is the price *charged* by firm i. We can use the labor supply equation to express the real wage as a function of aggregate consumption ($\frac{W}{P} = C$), and rewrite the expression for real flow profits as:

$$\Pi\left(\frac{P_i}{P}, C, A_i\right) = A_i^{1-\theta} \left(\frac{P_i}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta} \left(\frac{P_i}{P}\right)^{-\theta}.$$

Let $\overline{\Pi}$ be the steady-state level of real profits in a frictionless economy (upper bars denote steady-state values):[25]

$$\overline{\Pi} \equiv \Pi\left(\frac{P_i^*}{P}, \overline{C}, A_i\right).$$

[25] A constant level of aggregate consumption requires the restriction $\left[\int_0^1 A_{it}^{\theta-1} di\right]^{\frac{1}{1-\theta}} = 1$, which we assume holds throughout the paper.

We want to approximate the loss function $\overline{\overline{L}}$ defined as:

$$\overline{\overline{L}}\left(\frac{P_i^*}{P}, \frac{P_i}{P}, C, A_i\right) = \frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\overline{\overline{\Pi}}}$$
$$= \frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)} \frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)}{\overline{\overline{\Pi}}}, \tag{39}$$

The second ratio in (39) can be written as:

$$\frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)}{\overline{\overline{\Pi}}} = \frac{A_i^{1-\theta}\left(\frac{P_i^*}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta}\left(\frac{P_i^*}{P}\right)^{-\theta}}{\overline{\overline{C}} - \overline{\overline{C}}^2} \tag{40}$$
$$= \frac{A_i^{1-\theta}\left(\frac{P_i^*}{P}\right)^{1-\theta} C - \frac{\theta-1}{\theta} C A_i^{1-\theta}\left(\frac{P_i^*}{P}\right)^{1-\theta}}{\overline{\overline{C}} - \frac{\theta-1}{\theta}\overline{\overline{C}}}$$
$$= A_i^{1-\theta}\frac{C}{\overline{\overline{C}}}\left(\frac{P_i^*}{P}\right)^{1-\theta}$$
$$= \left(\frac{C}{\overline{\overline{C}}}\right)^{2-\theta},$$

where we use the facts that $\frac{P_i^*}{P} = \frac{\theta}{\theta-1}\frac{C}{A_i}$ and $\overline{\overline{C}} = \frac{\theta-1}{\theta}$. Note how the link between preferences and technology eliminates the idiosyncratic shock from the expression for maximized profits.

The first ratio in (39) is the proportional profit loss due to the "suboptimal" price P_i. It is convenient to rewrite it as:

$$\frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)} = 1 - \frac{\Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)}.$$

The profit ratio in the above expression can be written as:

$$\frac{\Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)} = \frac{A_i^{1-\theta}\left(\frac{P_i}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta}\left(\frac{P_i}{P}\right)^{-\theta}}{A_i^{1-\theta}\left(\frac{P_i^*}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta}\left(\frac{P_i^*}{P}\right)^{-\theta}}$$
$$= \frac{A_i^{1-\theta}\left(\frac{P_i}{P}\right)^{1-\theta} - \frac{\theta-1}{\theta}\frac{P_i^*}{P} A_i^{1-\theta}\left(\frac{P_i}{P}\right)^{-\theta}}{A_i^{1-\theta}\left(\frac{P_i^*}{P}\right)^{1-\theta} - \frac{\theta-1}{\theta}\frac{P_i^*}{P} A_i^{1-\theta}\left(\frac{P_i^*}{P}\right)^{-\theta}}$$
$$= \theta\frac{\left(\frac{P_i}{P}\right)^{1-\theta} - \frac{\theta-1}{\theta}\frac{P_i^*}{P}\left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P_i^*}{P}\right)^{1-\theta}}$$
$$= \theta\left(\frac{P_i^*}{P_i}\right)^{\theta-1} - (\theta-1)\left(\frac{P_i^*}{P_i}\right)^{\theta},$$

so that:

$$\frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)} = 1 - \theta\left(\frac{P_i^*}{P_i}\right)^{\theta-1} + (\theta - 1)\left(\frac{P_i^*}{P_i}\right)^{\theta}. \tag{41}$$

As before, note how the link between preference and technology eliminates the idiosyncratic shock from the expression above.

Combining (40) and (41), and keeping the relevant arguments of the loss function, we obtain:

$$\overline{\overline{L}}\left(\frac{P_i^*}{P}, \frac{P_i}{P}, C, A_i\right) = \overline{L}\left(\frac{P_i^*}{P_i}, C\right) = \left(\frac{C}{\overline{\overline{C}}}\right)^{2-\theta}\left[1 - \theta\left(\frac{P_i^*}{P_i}\right)^{\theta-1} + (\theta - 1)\left(\frac{P_i^*}{P_i}\right)^{\theta}\right]. \tag{42}$$

We can rewrite the loss function $\overline{L}$ in terms of logarithms:

$$G\left(p_i^* - p_i, c\right) = e^{(2-\theta)c}\left[\left(1 - \theta e^{(\theta-1)\left(p_i^* - p_i\right)}\right) + (\theta - 1)\, e^{\theta\left(p_i^* - p_i\right)}\right].$$

The exact loss function $G\left(p_i^* - p_i, c\right)$ can be used in the optimal price-setting problems. However, the presence of aggregate consumption in the expression implies that solving for the optimal pricing rule in the presence of pricing frictions involves a fixed-point problem, even in the absence of strategic complementarity or substitutability in price setting. To make the optimal pricing problem more tractable, we eliminate the effect of aggregate output by assuming $\theta = 2$ (as in Danziger 1999 and Bonomo and Carvalho 2010). In addition, for analytical convenience we take a second-order Taylor expansion of flow profit losses around the frictionless optimal price, based on which we analyze the price-setting problems discussed in the paper:

$$flow\ profit\ losses\,(p_{it}) \propto \left(p_{it} - p_{it}^*\right)^2.$$

10 Appendix C

In this appendix we show how we formalize firms' intertemporal optimization problems in the presence of frictions. For brevity we focus on the case with dissociated information gathering/processing and adjustment costs, which is more involved. The other cases are simpler and can be formalized analogously. Let $\widehat{F}$ and $\widehat{K}$ denote the levels of, respectively, the information and adjustment costs.

Formally, the pricing problem of a firm may be written as:[26]

$$\widetilde{V}\left(s_{t_0}\right)=\max_{\left\{\left\{t_j,t_{jn},X_{t_{jn}}\right\}_{n=0}^{N_j}\right\}_{j=1}^{\infty}} E_{t_0}\sum_{j=0}^{\infty}e^{-\rho(t_j-t_0)}E_{t_j}\left[\begin{array}{c} -e^{-\rho(t_{j+1}-t_j)}\widehat{F}+\int\limits_{t_j}^{t_{j0}}e^{-\rho r}\Pi\left(\frac{X_r}{P_r},C_r,A_r\right)dr \\ +e^{-\rho\left(t_{j0}-t_j\right)}\sum\limits_{n=0}^{N_j}\left[\begin{array}{c}\int\limits_{t_{jn}}^{t_{jn+1}}e^{-\rho r}\Pi\left(\frac{X_r}{P_r},C_r,A_r\right)dr \\ -e^{-\rho\left(t_{jn}-t_{j0}\right)}\widehat{K}\end{array}\right]\end{array}\right],$$

where for all $r\notin\left\{t_{0_0},...t_{0_1},t_{0_{N_0}},...t_{1_0},...\right\}$ $X_r=X_{r^-}$. $\widetilde{V}\left(s_{t_0}\right)$ denotes the expected present value of real profits Π, net of adjustment and information costs, when the state of the economy is s_{t_0}. The sequence t_j denotes the information dates - dates in which the firm incurs the information gathering and processing cost. The sequence t_{j_n} denotes the dates in which the firm incurs the menu cost and changes its price, and the sequence $X_{t_{j_n}}$ denotes the prices chosen on these dates.

Let $V^*\left(s_{t_0}\right)$ denote the expected present value of profits of a hypothetical identical firm in the same economy that does not face any pricing friction. Then,

$$V^*\left(s_{t_0}\right)=E_{t_0}\left[\int\limits_{t_0}^{\infty}e^{-\rho r}\Pi\left(\frac{P_r^*}{P_r},C_r,A_r\right)dr\right],$$

where P_r^* is the individual price that maximizes real profits at time r, i.e. the frictionless optimal price of the firm. With this auxiliary value function, $\widehat{V}\left(s_{t_0}\right)\equiv V^*\left(s_{t_0}\right)-\widetilde{V}\left(s_{t_0}\right)$ is the minimized expected present value of the real profit losses due to the existence of information and adjustment costs, and our problem can be equivalently rewritten in terms of $\widehat{V}\left(s_{t_0}\right)$.

Defining $\widehat{L}\left(\frac{P^*}{P},\frac{P_i}{P},C,A\right)\equiv\Pi\left(\frac{P^*}{P},C,A\right)-\Pi\left(\frac{P_i}{P},C,A\right)$ to be the instantaneous real profit loss due to a "suboptimal" price P_i, and normalizing the pricing problem by the steady-state level of real profits in a frictionless economy, $\overline{\Pi}$, we can rewrite the firm's program as:

$$\overline{V}\left(s_{t_0}\right)=\min_{\left\{\left\{t_j,t_{jn},X_{t_{jn}}\right\}_{n=0}^{N_j}\right\}_{j=1}^{\infty}} E_{t_0}\sum_{j=0}^{\infty}e^{-\rho(t_j-t_0)}E_{t_j}\left[\begin{array}{c} e^{-\rho(t_{j+1}-t_j)}\overline{F}+\int\limits_{t_j}^{t_{j0}}e^{-\rho r}\overline{L}\left(\frac{P_r^*}{X_r},C_r\right)dr \\ +e^{-\rho\left(t_{j0}-t_j\right)}\sum\limits_{n=0}^{N_j}\left[\begin{array}{c}\int\limits_{t_{jn}}^{t_{jn+1}}e^{-\rho r}\overline{L}\left(\frac{P_r^*}{X_r},C_r\right)dr \\ +e^{-\rho\left(t_{jn}-t_{j0}\right)}\overline{K}\end{array}\right]\end{array}\right],$$

where $\overline{V}\left(s_{t_0}\right)\equiv\frac{\widehat{V}\left(s_{t_0}\right)}{\overline{\Pi}}$, $\overline{L}\left(\frac{P_r^*}{X_r},C_r\right)\equiv\frac{\widehat{L}\left(\frac{P_r^*}{P_r},\frac{X_r}{P_r},C_r,A_r\right)}{\overline{\Pi}}$, $\overline{F}\equiv\frac{\widehat{F}}{\overline{\Pi}}$, $\overline{K}\equiv\frac{\widehat{K}}{\overline{\Pi}}$. Note that from (42), $\overline{L}\left(.,.\right)$ only depends on the ratio of the frictionless optimal price to the charged price, and on aggregate consumption.

Finally, assuming $\theta=2$, and relying on the same second-order Taylor approximation of flow

[26] Initially we drop the i subscripts in order to simplify the notation.

profit losses used in Appendix B, the firm's (approximate) pricing problem can be written as:

$$V\left(s_{t_0}\right) = \min_{\left\{\left\{t_j, t_{jn}, x_{t_{jn}}\right\}_{n=0}^{N_j}\right\}_{j=1}^{\infty}} E_{t_0} \sum_{j=0}^{\infty} e^{-\rho\left(t_j - t_0\right)} E_{t_j} \left[\begin{array}{c} e^{-\rho\left(t_{j+1}-t_j\right)} F + \int\limits_{t_j}^{t_{j0}} e^{-\rho r} \left(x_r - p_r^*\right)^2 dr \\ +e^{-\rho\left(t_{j0}-t_j\right)} \sum\limits_{n=0}^{N_j} \left[\begin{array}{c} \int\limits_{t_{jn}}^{t_{jn+1}} e^{-\rho r} \left(x_r - p_r^*\right)^2 dr \\ +e^{-\rho\left(t_{jn}-t_{j0}\right)} K \end{array} \right] \end{array} \right],$$

where $V\left(s_{t_0}\right) \equiv \frac{\overline{V}\left(s_{t_0}\right)}{2}$, $F \equiv \frac{\overline{F}}{2}$ and $K \equiv \frac{\overline{K}}{2}$.

11 Appendix D

Here we provide alternative solution approaches to some of the problems analyzed in the main text.

11.1 Deterministic information arrival, no drift

We provide an alternative solution to the case of zero drift ($\mu = 0$), which has been analyzed by Bonomo and Garcia (2001). Then, there are no uninformed adjustments: a firm either adjusts on an information date or waits for the next such date. Suppose the firm has just learned the discrepancy z_{t_0} on an information date t_0. It has the option of incurring the menu cost K and adjusting the discrepancy to 0. As a result, the optimal policy can be characterized by the barrier parameter S, which gives the (symmetric) trigger points for the discrepancy on information dates. After that, the firm will incur the expected flow deviation cost starting from the appropriate initial discrepancy (z_{t_0} or 0) until the next information date.

To introduce the appropriate notation, let S be such that:

$$V(S,0) = V(0,0) + K. \tag{43}$$

For $z_{t_0} > S$ or $z_{t_0} < -S$:

$$V(z_{t_0},0) = V(0,0) + K. \tag{44}$$

For $-S < z_{t_0} < S$:

$$V(z_{t_0},0) = B\left(z_{t_0},T\right) + e^{-\rho T} V\left(z_{t_0},T\right), \tag{45}$$

where the function $B\left(z,\tau\right)$ corresponds to the expected flow deviation cost over a period of length τ starting from the initial discrepancy z, and is given by:

$$B\left(z,\tau\right) = \int_0^{\tau} e^{-\rho s}\left(z^2 + \sigma^2 s\right) ds = z^2 \left(\frac{1-e^{-\rho\tau}}{\rho}\right) + \sigma^2 \left(\frac{-\tau e^{-\rho\tau}}{\rho} + \frac{1-e^{-\rho\tau}}{\rho^2}\right). \tag{46}$$

As in the general case analyzed in the main text, the solution must also satisfy condition (26), which can be used to rewrite equation (45):

$$V(z_{t_0}, 0) = B(z_{t_0}, T) + e^{-\rho T} E\left[V\left(z_{t_0} + \sigma\sqrt{T}\varepsilon, 0\right)\right]. \tag{47}$$

This problem can be solved numerically as described in the Appendix E.

11.2 No adjustment cost

This solution is similar in spirit to Reis (2006). Since the control problem between information dates is trivial (changing p_i to keep $z_i = 0$), we can use an alternative Bellman equation to solve for the optimal choice of information dates:

$$V(0) = \min_{\tau} \left\{B(\tau) + e^{-\rho\tau}(V(0) + F)\right\}, \tag{48}$$

where $B(\tau) = B(0, \tau)$ represents the expected cost due to lack of information over a time interval of length τ, and is given by:

$$B(\tau) = \int_0^{\tau} e^{-\rho s}\left(\sigma^2 s\right) ds = \sigma^2\left(\frac{1 - e^{-\rho\tau}}{\rho^2} - \frac{\tau e^{-\rho\tau}}{\rho}\right).$$

The first-order condition with respect to τ yields:

$$\sigma^2\tau^* = \rho(V(0) + F), \tag{49}$$

which states that the marginal cost of postponing an information date ($\sigma^2\tau^*$) must equal the marginal benefit of doing so ($\rho(V(0) + F)$). Solving for $V(0)$ in (48) and combining with (49) delivers the solution to the problem.

11.3 Single information/adjustment cost

This solution approach is the one employed by Bonomo and Carvalho (2004, 2010). Given that there is no price adjustment between information dates, let us concentrate on these dates. We can characterize the optimal pricing rule with the following Bellman equation:

$$\begin{aligned} V^* &= \min_{\tau, z} \int_0^{\tau} e^{-\rho s} f(z - \mu s, s) ds + e^{-\rho\tau}(Q + V^*) \\ &= \min_{\tau, z} \int_0^{\tau} e^{-\rho s}\left((z - \mu s)^2 + \sigma^2 s\right) ds + e^{-\rho\tau}(Q + V^*), \end{aligned} \tag{50}$$

where $V^* = V(c, 0)$. The first-order conditions are:

$$\int_0^{\tau^*} e^{-\rho s}(c - \mu s)ds = 0,$$
$$(c - \mu\tau^*)^2 + \sigma^2\tau^* - \rho(V^* + Q) = 0.$$

Rearranging the first condition yields:

$$c = \mu\left(\frac{1}{\rho} - \frac{e^{-\rho\tau^*}}{1 - e^{-\rho\tau^*}}\tau^*\right).$$

To get the second equation, solve the Bellman equation (50) for V^* to obtain:

$$V^* = \frac{\int_0^{\tau^*}\left((c - \mu s)^2 + \sigma^2 s\right)ds + e^{-\rho\tau^*}}{1 - e^{\rho\tau^*}}.$$

Plugging this expression for V^* into the second first-order condition and simplifying yields the solution to the problem.

11.4 Dissociated information and adjustment costs, no drift

In the particular case of $\mu = 0$, the logic behind the solution is similar to the case of exogenous deterministic information and no drift. There are no uninformed adjustments, and the optimal price-change decision can be characterized by a barrier parameter S, which gives the (symmetric) trigger-points for the discrepancy on information dates. In addition, and differently from the case with exogenous information arrival, the optimal policy specifies the time until the next information date as a function of the discrepancy as of the previous such date (here denoted $\widetilde{\tau}^*(z_{t_0})$).

Suppose the firm has just learned the discrepancy z_{t_0}. It has the option of paying the menu cost K and adjusting the discrepancy to zero:

$$V\left(z_{t_0}, 0\right) \leq K + V\left(0, 0\right). \tag{51}$$

As in the case of deterministic information and no drift, $V(S, 0) = V(0, 0) + K$. After choosing whether or not to adjust the firm will incur the expected deviation cost of being away from the frictionless optimal price starting from the appropriate initial discrepancy (z or 0) until the next optimally chosen information date, and the discounted value of the information cost F plus the value function at that time:

$$V(z, 0) = \min_{\tau}\left[B\left(z, \tau\right) + e^{-\rho\tau}\left(F + V\left(z, \tau\right)\right)\right], \tag{52}$$

where the function $B(.,.)$ is again given by (46). Using the notation for the function which gives the optimal time until the next information date for each initial discrepancy z, (52) can be rewritten as:

$$V(z,0) = B\left(z, \widetilde{\tau}^*(z)\right) + e^{-\rho\widetilde{\tau}^*(z)}\left(F + V\left(z, \widetilde{\tau}^*(z)\right)\right). \tag{53}$$

Condition (33) still holds, and as a result (52) and (53) become, respectively:

$$\begin{aligned} V(z,0) &= \min_{\tau}\left[B(z,\tau) + e^{-\rho\tau}E\left[V\left(z+\sigma\sqrt{\tau}\varepsilon, 0\right)\right]\right], \text{ and} \qquad (54) \\ V(z,0) &= B\left(z,\widetilde{\tau}^*(z)\right) + e^{-\rho\widetilde{\tau}^*(z)}E\left[V\left(z+\sigma\sqrt{\widetilde{\tau}^*(z)}\varepsilon, 0\right)\right]. \qquad (55) \end{aligned}$$

This problem can be solved numerically as described in the Appendix E.

12 Appendix E

In this appendix we provide numerical solution algorithms for the optimal price-setting problems.

12.1 Random information arrival

In order to find the optimal rule $\{l(\tau), c(\tau), u(\tau)\}$, we need to find the value function. We start by discretizing the partial differential equation (20) over a grid with time-increments Δt and discrepancy-increments Δz, using an explicit finite-difference method. We make the following approximations:

$$\begin{aligned} z &\approx n \bigtriangleup z, \qquad (56) \\ \tau &\approx m \bigtriangleup t, \\ V_\tau &\approx \frac{v_{n,m+1} - v_{n,m}}{\bigtriangleup t}, \\ V_z &\approx \frac{v_{n,m+1} - v_{n-1,m+1}}{\bigtriangleup z}, \end{aligned}$$

and obtain:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + \frac{\lambda}{\rho + \frac{1}{\bigtriangleup t}} \sum_{j=-J}^{J} \pi(j) v_{n+j,0} + \left(\frac{1}{\rho + \frac{1}{\bigtriangleup t}}\right)\left[(n \bigtriangleup z)^2 + \sigma^2 m \bigtriangleup t\right], \tag{57}$$

where $\pi(.)$ is a discretization of the normal distribution,[27] and:

$$\begin{aligned} p^0 &= \left(\frac{1}{\rho + \frac{1}{\bigtriangleup t}}\right)\left(\frac{-\mu}{\bigtriangleup z} + \frac{1}{\bigtriangleup t}\right), \\ p^- &= \left(\frac{1}{\rho + \frac{1}{\bigtriangleup t}}\right)\frac{\mu}{\bigtriangleup z}. \end{aligned}$$

[27] In all solutions presented in this paper we use a 120-mass-point discretization of the normal distribution.

If we have the value function for all states at time $m+1$, we can use equation (57) to find the value function at time m. We start with an arbitrary value function for very large τ and proceed backwards using the difference equation until arriving at $\tau = 0$. If τ is large enough, the resulting value function should be a good approximation for small τ, even though the initial guess for the value function is arbitrary. The reason is that if τ is large enough (relative to λ and ρ) distant points in the grid have little importance for the value function evaluated at a small τ. In the end we use conditions (21), and (23) for each τ to find $c(\tau)$, $u(\tau)$, and $l(\tau)$.

12.2 Deterministic information arrival

12.2.1 No-drift case

Start with a guess for $V(.,0)$. Impose (44) to get a new V, replacing $V(z,0)$ by $V(0,0)+K$ whenever $V(z,0) > V(0,0)+K$. Use the right-hand-side of (47) to obtain a new value for $V(z,0)$, and iterate to convergence. At the end, find S according to (43).

An alternative solution method explores the structure of the general solution of the differential equation (24) when $\mu = 0$. In this case, the value function takes the form:

$$V(z,\tau) = \frac{z^2}{\rho} + \frac{\sigma^2\tau}{\rho} + \frac{\sigma^2}{\rho^2} + e^{\rho\tau} H(z),$$

with the $H(z)$ function being determined by the specific boundary conditions of the problem. It turns out that the previous method, which iterates directly on V, is more stable than methods that use the above equation and iteration on $H(z)$. The reason is that while V must necessarily be constant at the boundaries of the z-grid, H must not be constant, and thus the unavoidable truncation imposed by the grid generates more problems for the numerical integration required for taking the expectation under the Gaussian distribution in (47).

12.2.2 General case

Making the same approximations as in (56) we discretize (24) as:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left[(n \triangle z)^2 + \sigma^2 m \triangle t\right], \tag{58}$$

where:

$$p^0 = \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\frac{-\mu}{\triangle z} + \frac{1}{\triangle t}\right),$$

$$p^- = \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\frac{\mu}{\triangle z}.$$

As before, if we have the value function for all states at time $m+1$, we can use equation (58) to find the value function at time m. We use the following algorithm. We guess a value function at time zero and impose condition (22) as follows: find $c(0)$ and impose $V(z,0)$ by $V(c(0),0)+K$ whenever $V(z,0) > V(c(0),0)+K$. We then compute the expectation in (26) to find the value function at time T, and the discrepancy z that minimizes the function at T, $c(T)$. Imposing condition (22) determines the new value at T. We then use the difference equation (58) to find the value function at time $T-\triangle t$, and solve for the value of z that minimizes the function at $T-\triangle t$, and so on, until we arrive at time zero. At that point we test if the value function at time zero is close enough (according to a convergence criterion set a priori) to the value we had at the previous iteration; otherwise we continue until convergence. In the end we use conditions (21), and (23) for each τ to find $c(\tau)$, $u(\tau)$, and $l(\tau)$.

12.3 Dissociated information and adjustment costs

12.3.1 No-drift case

Start with a guess for $\tau^*(.)$ and $V(.,0)$. Impose (51) to get the new V, replacing $V(z,0)$ by $V(0,0)+K$ whenever $V(z,0) > V(0,0)+K$. Use the right-hand-side of (55) to obtain a new value for $V(z,0)$, and then use this new $V(.,0)$ to find a new $\tau^*(.)$ by solving for τ in (54). Repeat the process until convergence, and at the end find S such that $V(S,0) = V(0,0)+K$.

12.3.2 General case

Because we start from the same partial differential equation (24), we use the same finite-difference discretization scheme as in Subsection 12.2.2:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left[(n \triangle z)^2 + \sigma^2 m \triangle t\right], \tag{59}$$

where:

$$p^0 = \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\frac{-\mu}{\triangle z} + \frac{1}{\triangle t}\right)$$
$$p^- = \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\frac{\mu}{\triangle z}.$$

We then apply the following algorithm. We guess values for the function $v_{n,m}$ for a large grid of times and discrepancies. It is important to impose conditions (31) and (32), which state that at any time and discrepancy the price setter will incur the information and/or the adjustment cost, if it is advantageous for her to do so. We then choose a time T large enough to exceed the optimal time interval between information dates for any initial discrepancy z_{t_0}. For such time T, find the z that minimizes $V(z,T)$, denoted $c(T)$, and impose conditions (31) and (32) to determine the new value at T. Then use the difference equation (59) to find the value function at time $T-\triangle t$. Next, impose conditions (31) and (32) to determine the new value at $T-\triangle t$ and so on, until time zero. At that point test if the value function at each time and discrepancy is close enough (according to some convergence criterion set a priori) to the value function at the previous iteration. Otherwise begin another iteration. After convergence, use conditions (21), and (23) for each τ to find $c(\tau)$, $u(\tau)$, and $l(\tau)$ and condition (33) to determine $\tau^*(z)$ for any given discrepancy.

12.4 Exogenous infrequent W_{at}-information

Discretizing the partial differential equation (34) using the explicit difference method, and making the same approximations as in (56), we arrive at:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + p^+ v_{n+1,m+1} + \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left[(n \triangle z)^2 + \sigma_a^2 m \triangle t\right], \tag{60}$$

where:

$$\begin{aligned} p^0 &= \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(-\left(\frac{\sigma_i}{\triangle z}\right)^2 + \frac{1}{\triangle t}\right) \\ p^- &= 0.5\left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\left(\frac{\sigma_i}{\triangle z}\right)^2 + \frac{\mu}{\triangle z}\right) \\ p^+ &= 0.5\left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\left(\frac{\sigma_i}{\triangle z}\right)^2 - \frac{\mu}{\triangle z}\right) \end{aligned}$$

To find the solution we apply the same algorithm as in Subsection 12.2.2.

12.5 Costly W_{at}-information gathering and processing

The numerical solution to this case can be obtained by applying the algorithm described in Subsection 12.3.2, but with the finite-difference scheme of Subsection 12.4 that provides the discretization (60) for the partial differential equation (35).