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Abstract

Long-term nominal interest rates are surprisingly sensitive to high-frequency (daily or monthly) movements in short-term rates. Since 2000, this high-frequency sensitivity has grown even stronger in U.S. data. By contrast, the association between low-frequency changes (at six- or twelve-month horizons) in long- and short-term rates, which was also strong before 2000, has weakened substantially. This puzzling post-2000 pattern arises because increases in short rates temporarily raise the term premium component of long-term yields, leading long rates to *temporarily* overreact to changes in short rates. The frequency-dependent excess sensitivity of long-term rates that we observe in recent years is best understood using a model in which (i) declines in short rates trigger "rate-amplifying" shifts in investor demand for long-term bonds and (ii) the arbitrage response to these demand shifts is both limited and slow. We study, both theoretically and empirically, how such rate-amplifying demand can be traced to mortgage refinancing activity, investors who extrapolate recent changes in short rates, and investors who "reach for yield" when short rates fall. We discuss the implications of our findings for the validity of event-study methodologies and the transmission of monetary policy.

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The sensitivity of long-term interest rates to movements in short-term rates is a central feature of the term structure and plays a crucial role in the transmission of monetary policy. Short-term nominal interest rates are determined by current monetary policy and its near-term expected path. Shocks to real interest rates are generally thought to be short-lived, so long-term nominal rates should not be highly sensitive to changes in nominal short rates if long-run inflation expectations are well anchored and the expectations hypothesis holds (Shiller, 1979). In fact, long-term nominal rates are surprisingly sensitive to high-frequency changes in short rates (Shiller et al., 1983; Cochrane and Piazzesi, 2002; Gürkaynak et al., 2005; Giglio and Kelly, 2018). The deeper forces underpinning this puzzling degree of high-frequency sensitivity, and its evolution, remain poorly understood.

We provide evidence that, in the past two decades, the sensitivity of long rates has grown even stronger at high frequencies (daily or monthly), but has weakened substantially at lower frequencies (6 or 12 month horizons)[1]. As a result, the sensitivity of long rates to changes in short rates has become highly frequency-dependent since 2000. Between 1971 and 1999, a daily regression of changes in 10-year U.S. Treasury yields on changes in 1-year yields delivers a coefficient of 0.56; and the analogous regression using 12-month changes gives nearly the same coefficient. Between 2000 and 2019, the coefficient from the daily regression jumps to 0.87, while the coefficient from the corresponding 12-month regression drops to 0.23. Figure 1, which plots the sensitivity of 10-year yields to changes in 1-year yields as a function of horizon in both the pre-2000 and post-2000 samples, summarizes this key finding. This pattern is not specific to the U.S.: we find similar results for Canada, Germany, and the U.K.

What explains this puzzling post-2000 tendency of short- and long-term rates to move together at high but not low frequencies? Statistically, this pattern arises because recent increases in short rates predict a subsequent flattening of the yield curve, as well as declines in long-term yields and forward rates, in the post-2000 data. These predictable reversals in long rates are linked to a new form of short-lived bond return predictability: since 2000, the expected returns on long-term bonds (in excess of those on short-term bonds) are *temporarily* elevated following increases in short rates. Thus, relative to an expectations-hypothesis baseline, long rates temporarily overreact to changes in short rates, exhibiting what Mankiw and Summers (1984) dubbed "excess sensitivity." In the post-2000 data, we estimate that 10-year yields rise by 66 basis points in response to a 100 bps monthly increase in 1-year yields. Over the next 6 months, 10-year yields are expected to fall by 36 bps, reversing over half of the initial response.

What deeper forces underpin the evolving sensitivity of long rates to movements in short rates? Gürkaynak et al. (2005) note that the strong sensitivity of long-term nominal rates could be consistent with the expectations hypothesis if one adopts the view that long-run inflation expectations are unanchored and are being continuously updated—i.e., if there are large shocks to trend inflation as in Stock and Watson (2007). This narrative is a good explanation for the high degree of sensitivity observed before 2000. Consistent with the expectations-hypothesis logic of this explanation, in the pre-2000 data, we find no evidence that the reaction of long yields to movements in short rates tends to predictably reverse.

However, in the post-2000 period, the high-frequency sensitivity of long-term nominal rates primarily reflects the sensitivity of long-term *real* rates to nominal short rates, rather than the sensitivity of break-even inflation (Beechey and Wright, 2009; Hanson and Stein, 2015). To the extent that one shares the

[1] We do not mean to argue that there was a *discrete* change in the underlying data-generating process around 2000. Instead, our reading of the evidence is that the underlying data-generating process has changed *gradually* over time.

widespread view that expected future real rates at distant horizons do not fluctuate meaningfully at high frequencies (see Gürkaynak et al., 2005), this makes it hard to square the strong post-2000 sensitivity at high frequencies with expectations-hypothesis logic. Hanson and Stein (2015) argue that the strong post-2000 sensitivity works through the term premium component of long-term yields: shocks to short rates move term premia in the same direction. Consistent with this view, we find that the reaction of long yields to movements in short rates tends to predictably reverse in the post-2000 data, giving rise to short-lived shifts in the expected returns to holding long-term bonds.

How can we best understand our finding that, in recent decades, the sensitivity of long rates to changes in short rates declines steeply with horizon? Because it reflects a form of short-lived return predictability, the most natural explanations involve temporary supply-and-demand imbalances in financial markets (De Long et al. (1990), Shleifer and Vishny (1997), and Duffie (2010)). We develop a model of these imbalances that emphasizes the role of what we call "rate-amplifying" shocks to the supply-and-demand for long-term bonds. Our model builds on Greenwood and Vayanos (2014) and Vayanos and Vila (2020) who stress the limited risk-bearing capacity of the specialized fixed-income arbitrageurs who must absorb shocks to the net supply of long-term bonds. In our model, risk-averse bond arbitrageurs can invest in either short- or long-term nominal bonds. While monetary policy pins down the interest rate on short-term bonds, long-term bonds are available in a net supply that varies over time. This net supply, which arbitrageurs must hold in equilibrium, equals the gross supply of long-term bonds net of the amount inelastically demanded by other, non-arbitrageur investors. To induce risk-averse arbitrageurs to absorb an increase in net supply of long-term bonds, the expected return on long-term bonds in excess of that on short-term bonds must rise, lifting the term premium component of long-term yields.

Our explanation adds two novel ingredients to this familiar setup: (i) "rate-amplifying" shifts in the supply or demand for long-term bonds and (ii) a slow-moving arbitrage response. First, we assume that shocks to the net supply of long-term bonds are *positively* correlated with shocks to short rates. This positive correlation can obtain either because increases in short rates are associated with increases in the gross supply of long-term bonds or with non-standard reductions in the demand of other, non-arbitrageur investors. Since arbitrageurs' risk-bearing capacity is limited, this implies that increases in short rates are associated with increases in the term premium component of long rates, generating "excess sensitivity" relative to the expectations hypothesis. This reduced-form assumption is consistent with several distinct rate-amplification mechanisms that we detail below, each rooted in well-known institutional frictions and facets of investor psychology, that have arguably grown in importance recent decades.

The second ingredient is that arbitrage capital is slow-moving as in Duffie (2010). As a result, these rate-amplifying demand shocks encounter a short-run arbitrage demand curve that is steeper than the long-run arbitrage demand curve, generating a short-lived imbalance in the market for long-term bonds. This slow-moving capital dynamic implies that the shifts in term premia triggered by movements in short rates are transitory. As a result, the excess sensitivity of long rates is greatest when measured at high frequencies. Furthermore, we show that frequency-dependent excess sensitivity is most pronounced when the underlying rate-amplifying demand shocks are themselves short-lived. In summary, the combination of rate-amplifying demand shocks and slow-moving arbitrage capital enables our model to match the frequency-dependent sensitivity of long rates observed since 2000.

We explore three rate-amplification channels: (i) shifts in the effective gross supply of long-term bonds due to mortgage refinancing waves (Hanson, 2014; Malkhozov et al., 2016), (ii) shifts in the demand for long-term bonds from biased investors who extrapolate recent changes in short rates (Giglio and Kelly, 2018; D'Arienzo, 2020), and (iii) shifts in the demand from investors who "reach for yield" when short rates fall (Hanson and Stein, 2015). For each channel, we first show how it can be used to microfound rate-amplifying shocks to the net supply of long-term bonds similar to those we previously assumed in reduced-form. Next, we discuss why the strength of each channel may have grown in recent decades: the key underlying trend here is the increasing financialization of interest-rate risk. Finally, by looking at the relationship between bond yields and different financial quantities, we empirically assess the extent to which each channel contributes to the frequency-dependent sensitivity of long-term rates we observe since 2000. We find evidence that mortgage refinancing and investor extrapolation both help explain why long yields rates have temporarily overreacted to short rates since 2000 in the U.S. By contrast, we find less evidence that reaching-for-yield plays an important role in driving our key empirical findings.

A vast literature demonstrates that, contrary to the expectations hypothesis, the expected excess returns on long-term bonds vary meaningfully over time (Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005). In contrast to most of the existing literature on bond return predictability which focuses on business-cycle frequency variation in expected returns, our findings point to a new, short-lived form of return predictability that has emerged in recent decades.

Our findings have important implications for how economists should interpret event-study evidence based on high-frequency changes in long-term bond yields. Macroeconomic news, including news about monetary policy, is lumpy, and the short-run change in long-term rates around news announcements is often used as a measure of the expected longer-run impact of news shocks on short rates. Nakamura and Steinsson (2018) is a prominent recent example of this increasingly popular approach to identification in macroeconomics. But, if some of the impact of a news shock on long-term rates wears off quickly over time, then a shock's short- and long-run impact are quite different. And event-studies only captures the short-run impact. For instance, it is common for news announcements to cause large jumps in 10-year forward rates, but since a large portion of these jumps are due to *transient* shifts in term premia, event-studies are likely to provide biased estimates of the longer-run impact on short rates.

Our results also have implications for monetary policy transmission. In the textbook New Keynesian view (Gali, 2008), the central bank adjusts short-term nominal rates. This affects long-term rates via the expectations hypothesis, which in turn influences aggregate demand. Stein (2013) points out that the excess sensitivity of long-term yields, whereby shocks to short rates move term premia in the same direction, should strengthen the effects of monetary policy relative to the textbook view. Stein (2013) refers to this as the "recruitment" channel of monetary transmission. We find that the behavior of interest rates does not conform to the textbook New Keynesian view in which term premia are constant. Nonetheless, our findings suggest that the recruitment channel may not be as strong as Stein (2013) speculates since a portion of the resulting shifts in term premia are transitory and, thus, likely to have only modest effects on aggregate demand. We do not argue that there is no recruitment channel, just that it is smaller than one might conclude based on the high-frequency response of term premia to policy shocks documented in Hanson and Stein (2015), Gertler and Karadi (2015), and Gilchrist et al. (2015).

In Section 1, we document our key stylized facts about the changing high- and low-frequency sensitivity of long-term interest rates. In Section 2, we show that past increases in short rates predict a future reversals in long-term yields in the post-2000 data, reflecting a new form of bond return predictability. Section 3 develops the model we use to interpret our findings. We build on this framework in Section 4 where we explicitly microfound three specific rate-amplification mechanisms—mortgage refinancing, extrapolation, and reaching-for-yield—and then assess empirically the extent to which each mechanism helps explain our key findings. Section 5 discusses the implications of our findings for event-study identification strategies that exploit high-frequency movements in long-term yields, the transmission of monetary policy, bond market "conundrums," and affine term structure models. Section 6 concludes.

1 The sensitivity of long-term rates to short-term rates

Between 1971 and 2000, the sensitivity of long-term rates to changes in short-term rates was similarly strong at both high- and low-frequencies. Since 2000, the association between high-frequency changes in short- and long-term interest rates has grown even stronger. By contrast, the association between low-frequency changes in short- and long-term rates has weakened substantially. As a result, the sensitivity of long-term rates has become surprisingly frequency-dependent since 2000. We first document these basic facts for the U.S. We then contrast the patterns we see in the post-2000 data with those observed in the U.S. prior to the 1970s. Finally, we show that the sensitivity of long-term rates has evolved in a similar fashion in Canada, Germany, and the U.K.

Baseline findings for the U.S. We begin by regressing changes in 10-year Treasury yields or forward rates on changes in 1-year nominal Treasury yields. Specifically, we estimate regressions of the form:

$$y_{t+h}^{(10)} - y_t^{(10)} = \alpha_h + \beta_h(y_{t+h}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+h} \tag{1.1}$$

and

$$f_{t+h}^{(10)} - f_t^{(10)} = \alpha_h + \beta_h(y_{t+h}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+h}, \tag{1.2}$$

where $y_t^{(n)}$ is the continuously compounded n-year zero-coupon yield in period t and $f_t^{(n)}$ is the n-year-ahead instantaneous forward rate. We obtain data on the nominal and real U.S. Treasury yield curve from Gürkaynak et al. (2007) and Gürkaynak et al. (2010). We decompose nominal yields into real yields and inflation compensation, defined as the difference between nominal and real yields derived from Treasury Inflation-Protected Securities (TIPS). Our sample begins in August 1971, which is when reliable data on 10-year nominal yields first become available and ends in December 2019. For real yields and inflation compensation, we only study the post-2000 sample, since data on TIPS are not available until 1999. All data are measured as of the end of the relevant period—e.g., the last trading day of each month.

In standard monetary economics models, the central bank sets overnight nominal interest rates, and other rates are influenced by the expected path of overnight rates. A large literature argues that central banks in the U.S. and abroad have increasingly relied on communication—implicit or explicit signaling about the future path of overnight rates—as an active policy instrument (Gurkaynak et al., 2005). To capture news about the near-term path of policy that would not impact the current overnight rate, we take the short rate to be the 1-year nominal Treasury rate which follows approaches in the recent literature (Gertler and Karadi, 2015; Gilchrist et al., 2015; Hanson and Stein, 2015).

Panel A in Table 1 reports estimated coefficients β_h in equation (1.1) for zero-coupon nominal yields, real yields, and inflation compensation using daily data and end-of-month data with h = 1, 3, 6, 12 months—i.e., for daily, monthly, quarterly, semi-annual, and annual changes in yields.[2] The results are shown for the pre-2000 and post-2000 samples separately. We base this sample split on a number of break-date tests that we will discuss shortly. Figure 1 plots the estimated coefficients β_h in equation (1.1) for nominal yields versus monthly horizon h for the pre-2000 and post-2000 samples.

Since we use overlapping h-month changes in equation (1.1) when $h > 1$, we report Newey and West (1987) standard errors using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; when $h = 1$, we report heteroskedasticity-robust standard errors. To address the tendency for statistical tests based on Newey and West (1987) standard errors to over-reject in finite samples, we compute p-values using the asymptotic theory of Kiefer and Vogelsang (2005) which gives more conservative p-values and has better finite-sample properties than traditional Gaussian asymptotic theory.

Panel A shows that, prior to 2000, there was a strong tendency for short- and long-term yields to rise and fall together at both high- and low-frequencies. While the high-frequency relationship has grown even stronger since 2000, the low-frequency relationship has weakened significantly. Specifically, the daily coefficient for 10-year yields has risen from $\beta_{day} = 0.56$ in the pre-2000 sample to $\beta_{day} = 0.87$ in the post-2000 sample and this increase is statistically significant (p-val< 0.001). By contrast, the coefficient for $h = 12$-month changes in 10-year yields has fallen from $\beta_{12} = 0.56$ before 2000 to $\beta_{12} = 0.23$ in the post-2000 sample and this decline is also highly significant (p-val < 0.001).

Combining these observations, Figure 1 shows our main finding: in the post-2000 sample, the β_h coefficients decline steeply with the horizon h. By contrast, β_h is a relatively constant function of h in the pre-2000 sample. Furthermore, Table 1 shows that the majority of the decline in β_h as a function of h in the post-2000 sample is due to the real component of long-term yields.

This is a surprising result: one would not expect β_h to vary strongly with monthly horizon h as in the post-2000 data. In a standard term-structure models with a single factor, we have $y_t^{(10)} = \alpha + \beta \cdot y_t^{(1)}$ for some $\beta \in (0, 1)$, implying that $\beta_h = \beta$ for all h, regardless of whether the expectations hypothesis holds. More generally, even accounting for multiple risk factors, term premia only fluctuate at business-cycle frequencies in conventional asset-pricing models, implying that β_h should be quite stable across monthly horizons h. And, as detailed in Section 3 below, if there are both persistent and transient shocks to short rates, the expectations hypothesis implies that β_h should be slightly increasing in h as it was in the pre-2000 data. Thus, our finding that β_h is a steeply decreasing function of horizon h since 2000 suggests that term structure dynamics have shifted in an important way.

Panel B of Table 1 reports the corresponding β_h coefficients in equation (1.2) using changes in instantaneous forwards as the dependent variable. Like 10-year yields, the sensitivity of 10-year forward rates to changes in short-term rates has risen at high frequencies, but has declined markedly at low frequencies.

We use two approaches to date the timing of the break and both approaches suggest that there was a break around 2000. First, we estimate equations (1.1) and (1.2) using 10-year rolling windows. The estimated coefficients for $h = 12$-month changes are shown in Figure 2 for 10-year yields and forwards. These β_{12} coefficients decline substantially in more recent windows. The second approach is to test for a structural break in equations (1.1) and (1.2) for $h = 12$-month changes, allowing for an unknown break date. We use the test of Andrews (1993) who conducts a Chow (1960) test at all possible break dates, and then takes the maximum of the Wald test statistics. Figure 3 plots the Wald test statistic for each

[2] Bond maturities are in years and time periods are in months, except when we estimate regressions at a daily frequency.

possible break date in equations (1.1) and (1.2) along with the Cho and Vogelsang (2017) critical values for a null of no structural break. The strongest evidence for a break is in 1999 or 2000 in both equations (1.1) and (1.2) and the break is highly statistically significant.[3]

To clarify, we do not intend to argue that there was a *discrete* change in the underlying data-generating process in 2000. Instead, consistent with the rolling-window regressions shown in Figure 2, our reading of the data is that the underlying data-generating process has changed gradually over time—a gradual change which then becomes discernible when we compare the behavior of yields in across different samples. Nonetheless, throughout the remainder of the paper, we will adopt the heuristic of simply splitting the data into two samples: pre- and post-2000.

Robustness. In the Internet Appendix, we conduct a battery of robustness checks on our key findings. First, we show that similar results obtain if we use long-term private yields as the dependent variable in equation (1.1). We examine long-term corporate bond yields with Moody's ratings of Aaa and Baa, the 10-year swap yield, and the yield on mortgage-backed-securities. For all of these long rates, the sensitivity to changes in 1-year Treasury rates was similar irrespective of frequency before 2000. After 2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines significantly.

Second, we obtain similar results using different proxies for the short-term rate—i.e., using changes in 3-month, 6-month, or 2-year Treasury yields—as the independent variable in equation (1.1).

Third, one might be concerned about our use of overlapping changes in equations (1.1) and (1.2) when $h > 1$. Instead of computing Newey and West (1987) standard errors with a lag truncation parameter of $\lceil 1.5 \times h \rceil$, we find that one would draw almost identical inferences using Hansen and Hodrick (1980) standard errors with a lag truncation parameter equal to h. Going further, we show the estimates and our inferences are similar if we simply use non-overlapping h-month changes.

Finally, one might wonder if our dating of this break is due to distortions stemming from the 2009–2015 period when overnight nominal rates were stuck at the zero lower bound. Our use of 1-year rates as the independent variables in equations (1.1) and (1.2) limits any distortions since 1-year yields continued to fluctuate from 2009 to 2015. Indeed, even if we end our sample in 2007 or 2008, we still detect a break around 2000. For instance, if the post-2000 sample ends in December 2008, we find a daily $\beta_{day} = 0.77$ and a yearly $\beta_{12} = 0.20$, which are essentially indistinguishable from the numbers in Table 1.

U.S. evidence prior to the Great Inflation. A natural explanation for the strong sensitivity of long-term nominal rates during the 1971-1999 sample is that this was a period when long-run inflation expectations became unanchored and were being continuously revised (Gürkaynak et al., 2005). Since inflation expectations have become firmly moored in recent decades, it is useful to compare the post-2000 patterns to the those pre-dating the Great Inflation, which ran from the late 1960s to the early 1980s. In the Internet Appendix, we examine the sensitivity of long-term Treasury yields to changes in short-term yields from 1953 (when the data become available) to 1968 (when inflation expectations began to drift up). Consistent with the view that inflation expectations were better-anchored prior to the Great Inflation, the 1953-1968 coefficients are lower than the 1971-1999 coefficients. However, while the level of β_h coefficients is lower in the 1953-1968 sample, we do not see the strong dependence on horizon h

[3]In the Internet Appendix, we date the emergence of the frequency-dependent sensitivity of long rates. We report 10-year rolling estimates of $\beta_1 - \beta_{12}$ for 10-year yields and find that $\beta_1 - \beta_{12}$ turns significantly negative around 2000.

that is so evident in the post-2000 data. In summary, while the unanchoring and reanchoring of long-run inflation expectations may help explain shifts in the *level* of β_h over time, the strongly *frequency-dependent* sensitivity of long-term rates that we see since 2000 appears to be something new under the sun.

International evidence. Our focus is on the U.S., but it is useful to consider whether these same patterns are also observed in other large, highly-developed economies. In the Internet Appendix, we report estimates of equation (1.1) for the U.K., Germany, and Canada for both pre-2000 and post-2000 samples. Our data for Canada, Germany, and the U.K. begin in 1986, 1972, and 1985, respectively. We find similar patterns for Canada, Germany, and the U.K. Specifically, for all three countries, β_h is strongly decreasing in h in the post-2000 data, but not in the pre-2000 data.

2 Yield-curve dynamics and bond return predictability

We now pinpoint the term-structure dynamics that account for the stronger high-frequency sensitivity and weaker low-frequency sensitivity of long rates that we see in the post-2000 data. We show that this frequency-dependent sensitivity arises because, all else equal, past increases in short rates predict a subsequent flattening of the yield curve, as well as a subsequent decline in long-term yields and forwards. Statistically, this means that post-2000 yield curve dynamics are "path-dependent" or "non-Markovian": it is not enough to know the current shape of the yield curve; instead, to form the best forecast of future bond yields and returns, one also needs to know how the yield curve has shifted in recent months. These non-Markovian dynamics are themselves a reflection of a new short-lived form of bond return predictability. Since 2000, the expected excess returns on long-term bonds are temporarily elevated following past increases in short rates. Thus, relative to an expectations-hypothesis baseline, long-term yields exhibit excess sensitivity at high frequencies and *temporarily overreact* to changes in short rates.

2.1 Non-Markovian yield-curve dynamics

We first show that strong horizon-dependence of β_h in the post-2000 period arises because yield curve dynamics have become non-Markovian.

Predicting level and slope. When examining term structure dynamics, it is useful to study the dynamics of yield-curve factors, especially level and slope factors (Litterman and Scheinkman, 1991). We define the level factor as the 1-year yield ($L_t \equiv y_t^{(1)}$) and the slope factor as the 10-year yield less the 1-year yield ($S_t \equiv y_t^{(10)} - y_t^{(1)}$)—a.k.a., the "term spread." Most term structure models are Markovian with respect to current yield curve factors, meaning that the conditional mean of future yields depends only on today's yield-curve factors. However, our key finding—the post-2000 horizon-dependence of the relationship between long- and short-term yields—suggests that it may be useful to include lagged factors when forecasting yields. This idea has proven useful in other contexts, including in Cochrane and Piazzesi (2005) and Duffee (2013). Specifically, we consider the following system of predictive monthly regressions:

$$L_{t+1} = \delta_{0L} + \delta_{1L}L_t + \delta_{2L}S_t + \delta_{3L}(L_t - L_{t-6}) + \delta_{4L}(S_t - S_{t-6}) + \varepsilon_{L,t+1} \tag{2.1a}$$

$$S_{t+1} = \delta_{0S} + \delta_{1S}L_t + \delta_{2S}S_t + \delta_{3S}(L_t - L_{t-6}) + \delta_{4S}(S_t - S_{t-6}) + \varepsilon_{S,t+1}. \tag{2.1b}$$

These regressions include level and slope as well as their changes over the prior six months, which is a simple way of allowing for longer lags without estimating too many free parameters.

Table 2 reports estimates of equations (2.1a) and (2.1b) for both the pre-2000 and post-2000 samples. We include specifications omitting all lagged changes, omitting lagged changes in slope, and including all predictors. Based on the Akaike information criterion (AIC) or Bayesian information criterion (BIC), the model in column (1) with no lagged changes is chosen in the pre-2000 sample, while the model in column (5) with lagged changes in level is selected in the post-2000 sample. In the post-2000 sample, the lagged change in level is a highly significant negative predictor of the future slope—i.e., increases in the level of yields predict subsequent yield-curve flattening. For example, as shown in column (5), a 100 basis point increase in the level over the prior 6-months is associated with a 11 basis *per-month* decline in slope in the post-2000 sample (p-val < 0.001). By contrast, as shown in column (2), the coefficient on $L_t - L_{t-6}$ in the pre-2000 sample is zero. And, we can easily reject the hypothesis that the coefficients on $L_t - L_{t-6}$ in the pre- and post-2000 samples are equal (p-val < 0.001).

The model in equations (2.1a) and (2.1b) can match the puzzling post-2000 horizon-dependent behavior of β_h that we documented above. This model can be written as a restricted vector autoregression (VAR) in $\mathbf{y}_t = (L_t, S_t)'$ of the form: $\mathbf{y}_{t+1} = \boldsymbol{\mu} + \mathbf{A}_1\mathbf{y}_t + \mathbf{A}_2\mathbf{y}_{t-6} + \boldsymbol{\varepsilon}_{t+1}$. Let $\Gamma_{ij}(h)$ denote the ijth element of the autocovariance of $\mathbf{y}_t$ at a lag of h months—i.e., the ijth element of $\boldsymbol{\Gamma}(h) = E[(\mathbf{y}_t - E[\mathbf{y}_t])(\mathbf{y}_{t-h} - E[\mathbf{y}_{t-h}])']$. Given the estimated parameters from equations (2.1a) and (2.1b), we can work out $\Gamma_{ij}(h)$ to obtain the VAR-implied values of β_h in equation (1.1):

$$\beta_h = \frac{Var(L_t - L_{t-h}) + Cov(S_t - S_{t-h}, L_t - L_{t-h})}{Var(L_t - L_{t-h})} = 1 + \frac{2\Gamma_{12}(0) - \Gamma_{12}(h) - \Gamma_{12}(-h)}{2(\Gamma_{11}(0) - \Gamma_{11}(h))}. \tag{2.2}$$

In the pre-2000 sample, Table 1 reported estimates of $\beta_1 = 0.46$ and $\beta_{12} = 0.56$. In the post-2000 sample, the estimates are $\beta_1 = 0.66$ and $\beta_{12} = 0.23$. Table 2 reports the VAR-implied values of β_1 and β_{12} from equation (2.2). In the pre-2000 data, all of the VAR models can roughly match both β_1 and β_{12}. In the post-2000 sample, all models can match β_1, but only the models that include lagged changes in level—i.e., models that allow for non-Markovian dynamics—can match the sharp decline in β_{12}. Specifically, if the post-2000 VAR does not include lagged changes as in column (4), the VAR-implied values of β_{12} would be 0.59 and would be nowhere near what we observe in the data.

Predictable reversals in long-term rates. These post-2000 non-Markovian dynamics imply that there are predictable reversals in long rates following past increases in short rates. In Table 3 we estimate specifications that are reminiscent of the Jorda (2005) "local projection" approach to estimating impulse-response functions. Specifically, we predict the future changes in 10-year yields and forwards from month t to $t+h$ using the current level (L_t) and slope (S_t) of the yield curve as well as the prior month's change in level ($L_t - L_{t-1}$) and slope ($S_t - S_{t-1}$):

$$z_{t+h} - z_t = \delta_0^{(h)} + \delta_1^{(h)} L_t + \delta_2^{(h)} S_t + \delta_3^{(h)} (L_t - L_{t-1}) + \delta_4^{(h)} (S_t - S_{t-1}) + \varepsilon_{t\to t+h}. \tag{2.3}$$

Table 3 reports estimates of equation (2.3) for $z_t = y_t^{(10)}$ and $f_t^{(10)}$ in the pre- and post-2000 samples for $h =$ 3-, 6-, 9-, and 12- month changes. In Figure 4, we plot the coefficients $\delta_3^{(h)}$ on $L_t - L_{t-1}$ for $h = 1, 2, ..., 12$, tracing out the expected future change in z_t from month t to $t + h$ in response to an unexpected change in the level of rates between $t - 1$ and t.

In the post-2000 data, there are predictable reversals in both 10-year yields and forwards following an increase in short rates. However, there is no such reversal in the pre-2000 data. For 10-year yields, Table

3 reports that $\delta_3^{(6)} = -0.36$ (p-val $= 0.07$) after $h = 6$-months in the post-2000 data. (The difference between $\delta_3^{(6)}$ in the pre- and post-2000 data is significant with a p-value of 0.04.) Table 1 showed that, since 2000, a 100 bps increase in short-rates in month t is associated with a 66 bps contemporaneous rise of long-term yields. Thus, Table 3 suggests that 36 bps—or more than half—of this initial response is expected to reverse within 6 months. As in Table 1, the post-2000 reversion in 10-year forwards is even larger in magnitude and is statistically stronger. For 10-year forwards, we have $\delta_3^{(6)} = -0.52$ (p-val < 0.01) and the difference between $\delta_3^{(6)}$ in the pre- and post-2000 data is highly significant (p-val < 0.01). In summary, Table 3 and Figure 4 show that long rates appear to temporarily overreact to changes in short rates in the post-2000 data, but there was no such tendency before 2000.

To better understand these results, we decompose 10-year yields into the sum of a level component and a slope component as in Table 2—i.e., $y_t^{(10)} = L_t + S_t$—and plot the coefficients $\delta_3^{(h)}$ versus h for both level ($z_t = L_t$) and slope ($z_t = S_t$). Consistent with Table 2, Table 3 shows that the predictable reversals in long-term yields reflects the juxtaposition of two opposing forces in the post-2000. First, past increases in short-term rates predict subsequent increases in short-term rates in the post-2000 data, perhaps owing to the Fed's growing desire to gradually adjust short rates (Stein and Sunderam, 2018). However, past increase in short rates strongly predict a subsequent flattening of the yield curve since 2000. Since the latter effect outweighs the former, we see predictable reversals in long-term yields post-2000.

2.2 Predicting bond returns

We show that our main finding—the fact that, in recent years, β_h declines rapidly as a function of horizon h—reflects a new form of bond return predictability. Namely, this result arises because past increases in the level of rates lead to *temporary* rise in the expected excess returns on long-term bonds.

Results for 10-year bonds. The k-month log excess return on 10-year bonds over the riskless return on k-month bills, $(k/12)\, y_t^{(k/12)}$, is:

$$rx_{t\to t+k}^{(10)} \equiv (k/12)\,(y_t^{(10)} - y_t^{(k/12)}) - (10 - k/12)(y_{t+k}^{(10-k/12)} - y_t^{(10)}). \tag{2.4}$$

We forecast the k-month excess return on 10-year zero-coupon bonds using level, slope, and the 6-month past changes in these two yield-curve factors:

$$rx_{t\to t+k}^{(10)} = \delta_0 + \delta_1 L_t + \delta_2 S_t + \delta_3 (L_t - L_{t-6}) + \delta_4 (S_t - S_{t-6}) + \varepsilon_{t\to t+k}. \tag{2.5}$$

In Table 4, we report the results from estimating these predictive regressions for $k = 1$, 3, and 6-month returns. Panel A reports the results for the pre-2000 sample and Panel B shows the post-2000 results.

In the post-2000 data, Table 4 shows the past change in the level of rates is a robust predictor of the excess returns on long-term bonds. However, there is no such predictability in the pre-2000 data. For instance, in column (6) of Panel B, we see that, all else equal, a 100 bps increase in short-term rates over the prior 6 months is associated with a $\delta_3 = 166$ bps (p-val < 0.01) increase in expected 3-month bond returns and the difference between δ_3 in the pre- and post-2000 data is statistically significant (p-val < 0.01). In untabulated results, we find that the post-2000 return predictability associated with past increases in the level of rates is short-lived and generally dissipates after $k = 6$ months. In other words, past increases in the level of rates lead to a temporary increase in the risk premia on long-term bonds.[4]

Results for other bond maturities. In the Internet Appendix, we examine the predictability for bond maturities other than $n = 10$ years. If, as we argue, past increases in short rates temporarily raise the compensation that investors require for bearing interest-rate risk, this should have a larger impact on the expected returns of long-term bonds than intermediate bonds. However, such a short-lived increase in the compensation for bearing interest rate risk should have relatively constant or even a hump-shaped effect on the yield and forward curves. The intuition is that the impact on bond yields equals the effect on a bond's average expected returns over its lifetime. As a result, a *temporary* rise in the compensation for bearing interest rate risk can have a greater impact intermediate-term yields than on long-term yields. Indeed, this is precisely what we find in the post-2000 data.

In summary, since 2000, term premia on long-term bonds are *temporarily* elevated following increases in short rates. This implies that, relative to an expectations-hypothesis baseline, long rates *temporarily overreact* to movements in short rates and exhibit "excess sensitivity" at high frequencies.

2.3 Interpreting the evidence

Before developing our economic modelling framework, we pause to interpret our results. Our findings all point towards the view that, in recent years, the term premium on long-term bonds is increasing in the recent *change* in short rates, all else equal. This simple non-Markovian assumption can match the facts that, in the post-2000 data, (i) the sensitivity of long rates β_h declines with horizon h and (ii) that, controlling for current yield curve factors, past changes in short rates predict future yield-curve flattening, declines in long rates, and high excess returns on long-term bonds.

To develop these ideas, we shift notation slightly. Rather than identifying specific maturities, we now refer to the long-term yield as y_t and the short rate as i_t. We split the long-term yield into an expectations-hypothesis component, eh_t, that reflects expected future short-term rates and a term premium component, tp_t, that reflects expected future bond risk premia: $y_t = eh_t + tp_t$. Thus, by definition, β_h—the total sensitivity of long-term yields at horizon h—is the sum of the expectations-hypothesis, β_h^{eh} and the term premium, β_h^{tp}, components:

$$\overbrace{\frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]}}^{\beta_h} = \overbrace{\frac{Cov\left[eh_{t+h} - eh_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]}}^{\beta_h^{eh}} + \overbrace{\frac{Cov\left[tp_{t+h} - tp_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]}}^{\beta_h^{tp}}. \qquad (2.6)$$

First, consider the expectations-hypothesis piece. For now, assume the short-rate follows a univariate AR(1) process, implying $eh_t = \alpha^{eh} + \beta^{eh} \cdot i_t$ and $\beta_h^{eh} = \beta^{eh}$ for all h. Next, consider the term premium piece. In conventional asset-pricing theories, term premia only vary at business-cycle frequencies, so one would not expect β_h^{tp} to vary strongly with monthly horizon h. Thus, conventional theories suggest that $tp_t \approx \alpha^{tp} + \beta^{tp} \cdot i_t$, implying that $\beta_h^{tp} = \beta^{tp}$ and $\beta_h = \left(\beta^{eh} + \beta^{tp}\right)$ for all h. In other words, it is difficult for conventional theories to match the strong horizon-dependence of β_h seen in the post-2000 data.

To generate horizon-dependent sensitivity, consider, instead, the following non-Markovian assumption:

[4]Consistent with the predictable curve flattening discussed above, the Internet Appendix shows that the predictability we find for 10-year bond returns is related to predictability for portfolios that locally mimic changes in the slope factor.

$$tp_t = \alpha^{tp} + \beta^{tp} \cdot i_t + \delta^{tp} \cdot (i_t - i_{t-1}), \tag{2.7}$$

where $\delta^{tp} > 0$. This assumption implies that term premia depend on the current level of short rates and the recent *change* in short rates. Under this assumption, one can show that:

$$\beta_h = \beta^{eh} + \beta^{tp} + \delta^{tp} \cdot (1 - \gamma_h) \text{ where } \gamma_h \equiv \frac{Cov[i_{t+h-1} - i_{t-1}, i_{t+h} - i_t]}{Var[i_{t+h} - i_t]}. \tag{2.8}$$

The key is then to note that γ_h—the coefficient from a regression of $(i_{t+h-1} - i_{t-1})$ on $(i_{t+h} - i_t)$—is an increasing function of h. When $\delta^{tp} > 0$, this in turn explains why β_h^{tp} is decreasing in h.[5] Furthermore, when $\delta^{tp} > 0$, controlling for current level of short rates, the past change in short rates predicts future yield curve flattening, declines in long-term yields, and high excess returns on long-term bonds.

3 A model of temporary bond market overreaction

Because our key finding reflects a form of short-lived return predictability, the most natural explanations involve temporary supply-and-demand imbalances in financial markets (De Long et al. (1990), Shleifer and Vishny (1997), and Duffie (2010)). Our model emphasizes what we call "rate-amplifying" shocks to the supply and demand for long-term bonds. In Section 4, we build on this framework and explicitly microfound three rate-amplification mechanisms—mortgage refinancing waves, investor extrapolation, and investor reaching-for-yield—and assess empirically the extent to which each mechanism helps explain our key finding. The model here emphasizes the common underlying structure and shared asset-pricing implications of these rate-amplification mechanisms. By contrast, Section 4 emphasizes the idea that different amplification mechanisms have implications for different financial quantities.

Model setting. Time is discrete and infinite. Risk-averse bond arbitrageurs can hold either risky long-term nominal bonds or riskless short-term nominal bonds. The interest rate on short-term bonds follows an exogenous stochastic process. Long-term bonds are available in a given net supply that must be absorbed by bond arbitrageurs. Since the risk-bearing capacity of these specialized bond arbitrageurs is limited, shifts in the net supply of long-term bonds impact the term premium component of long-term yields as in Greenwood and Vayanos (2014) and Vayanos and Vila (2020).

We add two novel ingredients to this familiar setup. First, there are rate-amplifying supply-and-demand shocks: shocks to the net supply of long-term bonds are *positively* correlated with shocks to short rates. To induce arbitrageurs to absorb these supply shocks, the term premium component of long yields must increase when short rates rise, generating "excess sensitivity" of long-term yields relative to the expectations hypothesis baseline.

Second, arbitrage capital is slow-moving as in Duffie (2010): these net supply shocks walk down a short-run demand curve that is steeper than the long-run demand curve.[6] Thus, an increase in short rates leads to a *temporary* supply-and-demand imbalance in the market for long-term bonds and, thus, a short-lived increase in bond risk premia. As a result, the excess sensitivity of long rates is greatest at short horizons. Furthermore, this frequency-dependent excess sensitivity is most pronounced when the underlying rate-amplifying net supply shocks are themselves transitory.

[5] For instance, if i_t follows an AR(1) of the form $i_{t+1} - \bar{\imath} = \rho_i (i_t - \bar{\imath}) + \varepsilon_{i,t+1}$, then $\gamma_h = \left(2\rho_i - \rho_i^{h-1} - \rho_i^{h+1}\right) / \left(2 - 2\rho_i^h\right)$. We have $\gamma_1 = -(1-\rho_i)^2 / (2 - 2\rho_i) < 0$ and $\lim_{h\to\infty} \gamma_h = \rho_i > 0$. And, treating γ_h as continuous in h, we have $\partial\gamma_h/\partial h > 0$.

[6] Our model is related to Greenwood et al. (2018), who incorporate slow-moving capital into a model of the term structure.

The model can match our key finding in Section 1—that β_h has fallen for large h and risen for small h post-2000—if (i) shocks to short-term nominal rates have become less persistent and (ii) the kinds of rate-amplification mechanisms we emphasize have grown in importance. We argue that (i) is justified by the strong evidence that shocks to the persistent component of inflation have become less volatile since the mid-1990s (Stock and Watson, 2007). We argue that (ii) is justified since these rate amplification mechanisms appear to have become more powerful over time.

Short- and long-term nominal bonds. At time t, investors learn that short-term bonds will earn a riskless log return of i_t in nominal terms between time t and $t+1$. Short-term nominal bonds are available in perfectly elastic supply at this interest rate. One can think of the short-term nominal interest rate as being determined outside the model by monetary policy.

Long-term nominal bonds are available in a given net supply s_t that must be absorbed by the arbitrageurs in our model. The long-term nominal bond is a perpetuity. To generate a tractable linear model, we use the well-known Campbell and Shiller (1988) log-linear approximation to the return on this perpetuity. The log excess return on long-term bonds over short-term bonds from t to $t+1$ is approximately:

$$rx_{t+1} \equiv \ln(1+R_{t+1}) - i_t \approx \frac{1}{1-\phi} y_t - \frac{\phi}{1-\phi} y_{t+1} - i_t, \tag{3.1}$$

where y_t is the log yield-to-maturity on long-term bonds, $\phi \in (0,1)$, and $D = 1/(1-\phi)$ is the bond's duration—i.e., the sensitivity of the bond's price to its yield. Iterating equation (3.1) forward and taking expectations, the yield on long-term bonds is:

$$y_t = \overbrace{(1-\phi)\sum_{j=0}^{\infty} \phi^j E_t[i_{t+j}]}^{eh_t} + \overbrace{(1-\phi)\sum_{j=0}^{\infty} \phi^j E_t[rx_{t+j+1}]}^{tp_t}. \tag{3.2}$$

The long yield is the sum of an expectations hypothesis piece, eh_t, that reflects expected future short rates and a term premium, tp_t, reflecting expected future excess returns on long bonds over short bonds.

Arbitrageurs. There are two groups of specialized bond arbitrageurs, each with identical risk tolerance τ, who differ solely in the frequency with which they can rebalance their bond portfolios.

The first group of arbitrageurs are "fast-moving" and are free to adjust their holdings of long-term and short-term bonds each period. Fast-moving arbitrageurs are present in mass q and we denote their demand for long-term bonds at time t by b_t. Fast-moving arbitrageurs have mean-variance preferences over 1-period portfolio log returns. Their demand for long-term bonds at time t is:

$$b_t = \tau \frac{E_t[rx_{t+1}]}{Var_t[rx_{t+1}]}. \tag{3.3}$$

The second group of arbitrageurs are "slow-moving" and can only rebalance their holdings of long-term and short-term bonds every $k \geq 2$ periods. Slow-moving arbitrageurs are present in mass $1-q$. A fraction $1/k$ of slow-moving arbitrageurs are active each period and can rebalance their portfolios, but then cannot trade again for the next k periods. As in Duffie (2010), this is a reduced-form way to model the forces, whether due to institutional frictions or limited attention, that limit the speed of arbitrage capital flows. Since they only rebalance every k periods, slow-moving arbitrageurs have mean-variance preferences over their k-period *cumulative* portfolio excess return. Thus, the demand for long-term bonds from the subset of slow-moving arbitrageurs who are active at time t is:

$$d_t = \tau \frac{E_t[\sum_{j=1}^{k} rx_{t+j}]}{Var_t[\sum_{j=1}^{k} rx_{t+j}]}. \tag{3.4}$$

Holders of long-term bonds face two different types of risk. First, they are exposed to *short rate risk*. they will suffer a capital loss on their long-term bond holdings if short-term rates unexpectedly rise. Second, they are exposed to *supply risk*: there are shocks to the net supply of long-term bonds that impact the term premium component of long-term bond yields. We make the following assumptions about the evolution of these two risk factors.

Short-term nominal interest rates. The short-term nominal interest rate is the sum of a highly persistent component $i_{P,t}$ and a more transient component $i_{T,t}$:

$$i_t = i_{P,t} + i_{T,t}. \tag{3.5}$$

A natural interpretation is that the persistent component reflects long-run inflation expectations and the transient component reflects cyclical variation in short-term real rates and expected inflation. The persistent component $i_{P,t}$ follows an exogenous AR(1) process:

$$i_{P,t+1} = \bar{\imath} + \rho_P \left(i_{P,t} - \bar{\imath}\right) + \varepsilon_{P,t+1}, \tag{3.6}$$

where $0 < \rho_P < 1$ and $Var_t\left[\varepsilon_{P,t+1}\right] = \sigma_P^2$. The transient component $i_{T,t}$ also follows an exogenous AR(1):

$$i_{T,t+1} = \rho_T i_{T,t} + \varepsilon_{T,t+1}, \tag{3.7}$$

where $0 < \rho_T \leq \rho_P < 1$ and $Var_t\left[\varepsilon_{T,t+1}\right] = \sigma_T^2$.

If $\rho_T < \rho_P$ and σ_P is large relative to σ_T, then short-term nominal rates will be highly persistent. As a result, long-term nominal rates will be highly sensitive to movements in short-term nominal rates due to standard expectations-hypothesis logic. Indeed, a large value of σ_P is a good explanation for the high sensitivity of long-term rates observed in the 1970s, 1980s, and the 1990s when long-run inflation expectations were not well-anchored (Gürkaynak et al., 2005). However, long-run inflation expectations have become firmly anchored in recent decades and there is strong evidence that shocks to the persistent component of nominal inflation have become far less volatile since the mid-1990s.

Rate-amplifying shocks to the net supply of long-term bonds. Long-term nominal bonds are available in an exogenous, time-varying net supply s_t that must be held in equilibrium by arbitrageurs. This net supply equals the gross supply of long-term bonds minus the demand from other, non-arbitrageur investors outside the model who have inelastic demands. We assume that s_t follows an AR(1) process:

$$s_{t+1} = \bar{s} + \rho_s \left(s_t - \bar{s}\right) + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1} + \varepsilon_{s,t+1}, \tag{3.8}$$

where $0 < \rho_s \leq \rho_T < 1$, $C \geq 0$, and $Var_t\left[\varepsilon_{s,t+1}\right] = \sigma_s^2$.

When $C > 0$, there are *rate-amplifying* net supply shocks—shocks to short rates are positively associated with shocks to net bond supply—and C parameterizes the strength of these amplification mechanisms. Equation (3.8) is a reduced-form way of capturing three different rate-amplification mechanisms that we detail in Section 4: (i) mortgage refinancing waves, (ii) investors who extrapolate recent changes in short rates, and (iii) investors who "reach for yield" by buying more long-term bonds when short rates are low. Rate-amplifying net supply shocks can arise either because increases in short rates are associated with increases in the gross supply of long-term bonds (as in the mortgage refinancing channel) or because they are associated with reductions in the demands of other, non-arbitrageur investors (as in the investor extrapolation and reaching-for-yield channels). The $\varepsilon_{s,t+1}$ shocks in (3.8) capture forces that are unrelated to short rates which also impact the net supply of long-term bonds. While the model can be solved for any arbitrary correlation structure between the $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$ shocks, we assume, for simplicity, that these three shocks are mutually orthogonal.

The difference between the persistence of these rate-amplifying net supply shocks and that of the underlying shocks to short-term rates plays an important role in our model's ability to generate excess sensitivity that is most pronounced at high frequencies. To see why, note that equation (3.8) implies that the net supply of long-term bond is given by

$$\begin{aligned} s_t &= \bar{s} + C[(i_{P,t} - \bar{\imath}) - (\rho_P - \rho_s) \textstyle\sum_{j=0}^{\infty} \rho_s^j \, (i_{P,t-j-1} - \bar{\imath})] \\ &\quad + C[i_{T,t} - (\rho_T - \rho_s) \textstyle\sum_{j=0}^{\infty} \rho_s^j i_{T,t-j-1}] + [\textstyle\sum_{j=0}^{\infty} \rho_s^j \varepsilon_{s,t-j}]. \end{aligned} \tag{3.9}$$

When $\rho_s < \rho_T$, the rate-amplifying net supply shocks are less persistent than the underlying short rate shocks. As a result, net bond supply is increasing in the differences between the current level of each component of the short rate and a geometric moving-average of its past values. Thus, when $\rho_s < \rho_T$, s_t will be high when short rates have recently risen. By contrast, if $\rho_s = \rho_T = \rho_P$, s_t will be just as persistent as short rates. In this case, $s_t = \bar{s} + C(i_t - \bar{\imath}) + [\sum_{j=0}^{\infty} \rho_s^j \varepsilon_{s,t-j}]$ and only the current *level* of short rates—as opposed to recent changes in short rates—impacts net bond supply.

Equilibrium yields. At time t, there is a mass q of fast-moving arbitrageurs, each with demand b_t, and a mass $(1-q)\,k^{-1}$ of active slow-moving arbitrageurs who rebalance their portfolios, each with demand d_t. These arbitrageurs must accommodate the *active net supply* of long-term bonds, which is the total net supply s_t less any supply held by inactive slow-moving arbitrageurs who do not rebalance at time t, $(1-q)\,k^{-1}\sum_{j=1}^{k-1} d_{t-j}$. Thus, the market-clearing condition for long-term bonds at time t is:

$$\overbrace{qb_t}^{\text{Fast demand}} + \overbrace{(1-q)k^{-1}d_t}^{\text{Active slow demand}} = \overbrace{s_t}^{\text{Total net supply}} - \overbrace{(1-q)(k^{-1}\textstyle\sum_{j=1}^{k-1} d_{t-j})}^{\text{Inactive slow holdings}}. \tag{3.10}$$

We conjecture that equilibrium yields y_t and the demands of active slow-moving arbitrageurs d_t are linear functions of a state vector, $\mathbf{x}_t$, that includes the steady-state deviations of both components of short-term nominal interest rates, the net supply of long-term bonds, and holdings of bonds by inactive slow-moving arbitrageurs. Formally, we conjecture that the yield on long-term bonds is $y_t = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t$ and that slow-moving arbitrageurs' demand for long-term bonds is $d_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$, where the $(k+2)\times 1$ dimensional state vector, $\mathbf{x}_t$, is given by $\mathbf{x}_t = [i_{P,t} - \bar{\imath},\ i_{T,t},\ s_t - \bar{s}, d_{t-1} - \delta_0, \cdots, d_{t-(k-1)} - \delta_0]'$. These assumptions imply that the state vector follows a VAR(1) process $\mathbf{x}_{t+1} = \boldsymbol{\Gamma}\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1}$, where $\boldsymbol{\Gamma}$ depends on the parameters $\boldsymbol{\delta}_1$ governing slow-moving arbitrageurs' demand.

In the Internet Appendix, we show how to solve for equilibrium yields in this setting. A rational expectations equilibrium of our model is a fixed point of a specific operator involving the "price-impact" coefficients, $(\boldsymbol{\alpha}_1')$, which show how the state variables impact bond yields, and the "demand-impact" coefficients, $(\boldsymbol{\delta}_1')$, which show how these variables impact the demand of active slow-moving investors. Specifically, let $\boldsymbol{\omega} = (\boldsymbol{\alpha}_1', \boldsymbol{\delta}_1')'$ and consider the operator $\mathbf{f}(\boldsymbol{\omega}_0)$ which gives (i) the price-impact coefficients that will clear the market for long-term bonds and (ii) the demand-impact coefficients consistent with optimization on the part of active slow-moving investors when agents conjecture that $\boldsymbol{\omega} = \boldsymbol{\omega}_0$ at all future dates. A rational expectations equilibrium of our model is a fixed point $\boldsymbol{\omega}^* = \mathbf{f}(\boldsymbol{\omega}^*)$. Solving the model involves numerically finding a solution to a system of $2k$ non-linear equations in $2k$ unknowns.

An equilibrium solution only exists if arbitrageurs are sufficiently risk tolerant (i.e., for τ sufficiently large). When an equilibrium exists, there can be multiple equilibria. Equilibrium non-existence and multiplicity of this sort are common in overlapping-generations, rational-expectations models such as ours where risk-averse arbitrageurs with finite investment horizons trade an infinitely-lived asset that is subject to supply shocks. Different equilibria correspond to different self-fulfilling beliefs that arbitrageurs can hold about the price-impact of supply shocks and, hence, the risks of holding long-term bonds. However, we always find a unique equilibrium that is stable in the sense that equilibrium is robust to a small perturbation in arbitrageurs' beliefs regarding the equilibrium that will prevail in the future. Consistent with the "correspondence principle" of Samuelson (1947), this unique stable equilibrium has comparative statics that accord with standard economic intuition. We focus on this unique stable equilibrium in our analysis. See Greenwood et al. (2018) for an extensive discussion of these issues.

The sensitivity of long-term yields. We now explain the factors that shape the sensitivity of long-term rates in our model and how this sensitivity depends on horizon. Consider the model-implied counterpart of the empirical regression coefficient in equation (1.1). In the model, the coefficient β_h from a regression of $y_{t+h} - y_t$ on $i_{t+h} - i_t$ is:

$$\beta_h = \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} = \frac{\boldsymbol{\alpha}_1'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}, \tag{3.11}$$

where $\mathbf{V} = Var\left[\mathbf{x}_t\right]$ denotes the variance of the state vector $\mathbf{x}_t$ and $\mathbf{e}$ denotes the $(k+2) \times 1$ vector with ones in the first and second positions and zeros elsewhere.[7] We can then establish the following result:

Proposition 1. *The dependence of the coefficient β_h on time horizon h is governed by (i) the persistence ρ_x of the three shocks $x \in \{s, T, P\}$, (ii) the volatilities of the two short-rate shocks, σ_T and σ_P, (iii) the strength of the rate-amplification mechanisms C, and (iv) the degree to which capital is slow moving q.*

1. *When there are no rate-amplifying net supply shocks ($C = 0$), changes in term premia are unrelated to shifts in short rates and long-term yields do not exhibit excess sensitivity. Furthermore,*
   - *(a) if $\rho_T = \rho_P$, β_h is independent of h, σ_T, and σ_P.*
   - *(b) if $\rho_T < \rho_P$, β_h is* increasing *in h; the level of β_h falls with σ_T and rises with σ_P for all h.*
2. *When there are rate-amplifying net supply shocks ($C > 0$), changes in term premia are positively correlated with changes in short rates and long-term yields exhibit excess sensitivity. Furthermore,*
   - *(a) if $\rho_s = \rho_T = \rho_P$, and all capital is fast-moving ($q = 1$), then β_h is independent of h;*
   - *(b) if $\rho_s \leq \rho_T = \rho_P$ and either (i) supply shocks are transient ($\rho_s < \rho_T$) or (ii) capital is slow-moving ($q < 1$), then β_h is decreasing in h;*

[7] To derive this expression, note that $y_{t+h} - y_t = \boldsymbol{\alpha}_1'(\mathbf{x}_{t+h} - \mathbf{x}_t)$ and $i_{t+h} - i_t = \mathbf{e}'(\mathbf{x}_{t+h} - \mathbf{x}_t)$. Since the state-vector $\mathbf{x}_t$ follows a VAR(1) process $\mathbf{x}_{t+1} = \boldsymbol{\Gamma}\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1}$, we have $Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] = 2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h$ and the result follows.

(c) *if $\rho_s \leq \rho_T < \rho_P$, β_h can be non-monotonic in h.*

Proof. See the Internet Appendix for all proofs. □

When there are no rate-amplifying net supply shocks ($C = 0$), long rates are not excessively sensitive to short rates when judged relative to the expectations hypothesis. If short rates contain both a transient and a more persistent component ($\rho_T < \rho_P$), one should actually expect the β_h coefficients to increase in horizon h when $C = 0$. This effect arises since movements in the persistent short rate component are associated with larger movements in long rates by standard expectations-hypothesis logic and because the persistent component dominates changes in short rates at longer horizons. Furthermore, when $\rho_T < \rho_P$, the level of β_h for any horizon h depends on σ_T and σ_P. For instance, an increase in σ_P raises the fraction of total short-rate variation that is due to the persistent component. Since shocks to the persistent component of short rates have a larger impact on long rates, an increase in σ_P raises β_h for all h.

Rate-amplifying net supply shocks ($C > 0$) generate excess sensitivity. However, Part 2.(a) of Proposition 1 shows that rate-amplification ($C > 0$) need not generate *horizon-dependent* excess sensitivity—i.e., *temporary* overreaction of long rates when judged relative to the expectations hypothesis. To generate β_h coefficients that decline with h, Part 2.(b) clarifies that either (i) the rate-amplifying net supply shocks must be less persistent than the underlying short-rate shocks ($\rho_s < \rho_T$) or (ii) these rate-amplifying shocks must be met by a slow-moving arbitrage response ($q < 1$). Under either of these conditions, shifts in short rates trigger a short-lived supply-and-demand imbalance in the market for long-term bonds, leading long rates to *temporarily* overreact to short rates. In practice, we suspect that *both* transitory rate-amplifying net supply shocks and slow-moving capital play a role in explaining why β_h declines steeply with h in the recent data. Furthermore, these two mechanisms reinforce one another: it is easier to quantitatively match the steep decline in β_h as a function of h in calibrations that feature both elements.[8]

Model calibration. Our main findings are that β_h has risen at high frequencies (low h) but has fallen at low frequencies (high h) in recent decades, leading β_h to decline steeply with horizon h in the post-2000 data. Guided by Proposition 1, we discuss how to understand the changing sensitivity of long rates. We focus on the role of changes in the volatility of persistent short rate shocks (σ_P) and the strength of any rate-amplifying mechanisms (C). Our model can match the data if (1) shocks to the persistent component of short-term nominal rates have become less volatile in the post-2000 period (σ_P has fallen) and (2) the rate-amplifying supply-and-demand mechanisms we emphasize have become more important in recent decades (C has risen).

We consider an illustrative calibration of the model in which each period is a month. We assume the following parameters were the same in the 1971-1999 and post-2000 periods:

- **Persistence:** $\rho_P = 0.995$, $\rho_T = 0.96$, and $\rho_s = 0.80$. These parameters imply that shocks to the persistent short rate component have a half-life of 11.5 years, shocks to the transient component

[8]Technically, when capital is slow-moving ($q < 1$) and $\rho_s \leq \rho_T = \rho_P$, β_h is only guaranteed to be *locally* decreasing in h for $h \leq k$—i.e., for horizons shorter than over which all slow-moving arbitrageurs will have rebalanced their portfolios. While we always have $\beta_h < \beta_{h-1}$ for $h \leq k$, we can have $\beta_h > \beta_{h-1}$ for $h > k$. However, even when there are local non-monoticities, β_h is globally decreasing in the sense that $\lim_{h\to\infty} \beta_h < \beta_1$. What explains the potential for these local non-monoticities? As in Duffie (2010), the gradual adjustment of slow-moving arbitrageurs can gives rise to modest echo effects for $h > k$, generating a series of damping oscillations that converge to $\lim_{h\to\infty} \beta_h$. These oscillations arise because the slow-moving arbitrageurs who reallocate soon after a supply shock lands take large opportunistic positions. These positions temporarily reduce the active supply of long-term bonds and need to be re-absorbed in later periods.

have a half-life of 1.4 years, and shocks to the net supply of long bonds have a half-life of 3 months.

- **Slow-moving capital:** $q = 30\%$ and $k = 12$. Thus, $1 - q = 70\%$ of the arbitrageurs are slow-moving and only rebalance their bond portfolios every 12 months. These assumptions capture the idea that many large institutional investors only rebalance their portfolios annually.
- **Volatility of the transient component of short rates:** $\sigma_T^2 = 0.15\%$.
- **No independent net supply shocks:** $\sigma_s^2 = 0$. This assumption is without loss of generality.
- **Other parameters:** $\tau = 0.5$ and $\phi = 119/120$, so the duration of the perpetuity is 10 years—i.e., $D = 1/(1-\phi) = 120$ months.

For the pre-2000 period, we assume:

- **A large persistent component of short rates:** $\sigma_P^2 = 0.15\%$. The implied volality of the short rate is 4.12% which compares with a volatility of 1-year yields of 2.63% in the 1971-1999 sample.
- **No rate-amplifying net supply shocks:** $C = 0$.

By contrast, for the post-2000 period, we assume:

- **A small persistent component of short rates:** $\sigma_P^2 = 0.012\%$. The implied standard deviation of the short rate is 1.77% which is similar to the post-2000 volatility of 1-year yields of 1.85%.
- **Net supply shocks induced by short rate shocks:** $C = 0.55$. Thus, we assume a meaningful increase in the strength of rate-amplifying supply-and-demand mechanisms.

The first graph in Figure 5 plots the model-implied regression coefficients β_h from equation (3.11) against monthly horizon h for the pre- and post-2000 calibrations. In the pre-2000 calibration where σ_P is large and $C = 0$, β_h is high and largely independent of h. In fact, β_h rises gradually with h—as it does in the pre-2000 data—because the more persistent component of short rates dominates when changes are computed at longer horizons. By contrast, in the post-2000 calibration where σ_P is smaller and C is large, β_h declines steeply with h. And, since σ_P is lower, β_h reaches a lower level for large h.

β_h declines with h in the post-2000 calibration because short rate shocks give rise to transient rate-amplifying shocks to the net supply of long-term bonds ($C > 0$ and $\rho_s < \rho_T$) that encounter a short-run demand curve that is steeper than the long-run demand curve due to slow-moving capital ($q < 1$), triggering short-lived market imbalances. The second graph in Figure 5 shows that β_h only declines moderately with h in our post-2000 calibration if we drop the assumption that arbitrage capital is slow-moving. Thus, transient rate-amplifying net supply shocks and slow-moving capital are both helpful for quantitatively matching the fact that β_h declines so steeply with h in the post-2000 data.

Another way of understanding the mechanism is to study the model-implied impulse response functions following a surprise increase to short rates. We carry out this exercise in the Internet Appendix. To summarize, an initial positive shock to short rates leads to a rise in term premia. Thus, relative to the expectations-hypothesis, long rates are excessively sensitive to short rates. However, the rise in term premia wears off quickly, explaining our key finding that β_h declines sharply with horizon h. Furthermore, the initial rise in short rates predicts future yield curve flattening and future reversals in long rates.

In addition to matching the fact that β_h declines steeply with h in the post-2000 period, the model can also match the related empirical findings documented above. First, the model is consistent with our return forecasting evidence: in the post-2000 calibration, bond risk premia $E_t [rx_{t+1}] = \tau^{-1} V^{(1)} b_t$ are elevated when short-term rates have recently risen. Intuitively, if rate-amplifying net supply shocks ($C > 0$) are either transient ($\rho_s < \rho_T$) or are met by a slow-moving arbitrage response ($q < 1$), then fast-moving arbitrageurs must bear greater interest-rate risk when short rates have recently risen—b_t will be higher—and they will require additional compensation for bearing this extra risk

Second, let $L_t = i_t$ and $S_t = y_t - i_t$ denote the model-implied level and slope factors. If we estimate equation (2.1b) in data simulated from the model, we find that past increases in the level of rates predict a flattening of the yield curve in the post-2000 calibration but not in the pre-2000 calibration. In the post-2000 calibration, past increases in the level of rates are associated with a higher current risk premium on long-term bonds. Since the risk premium is $E_t [rx_{t+1}] = S_t - \phi (1 - \phi)^{-1} (E_t [\Delta S_{t+1}] + E_t [\Delta L_{t+1}])$, all else equal, $E_t [\Delta S_{t+1}]$ is lower when short rates have recently risen.

4 Rate-amplification mechanisms

We now explore three rate-amplification mechanisms—mortgage refinancing, extrapolation, and reaching for yield—that may help explain why increases in short rates trigger temporary supply-and-demand imbalances in the market for long-term bonds. For each mechanism, we show how it microfounds rate-amplifying net supply shocks like those we introduced in reduced-form in Section 3 and then embed it the modelling framework developed above.[9] Next, we discuss why the strength of each channel may have increased in recent decades. Finally, by examining the relationship between bond yields and different financial quantities, we assess empirically the extent to which each channel contributes to the frequency-dependent sensitivity of long-term rates we observe after 2000.

4.1 Mortgage refinancing

Negative shocks to short-term rates trigger mortgage refinancing waves in the U.S. that lead to temporary reductions in the effective gross supply of long-term bonds and, thus, temporary declines in bond term premia (Hanson, 2014; Malkhozov et al., 2016). The mortgage refinancing channel is only relevant in countries such as the U.S. where fixed-rate mortgages with an embedded prepayment option are an important source of mortgage financing. However, Domanski et al. (2017) point to a similar rate-amplification mechanism—one that may be more important in the Eurozone—stemming from the desire of insurers and pensions to dynamically match the duration of their assets and liabilities.

Modeling the mechanism. Most fixed-rate, residential mortgages in the U.S. give the borrower the option to prepay at any time without a penalty (Boyarchenko et al., 2019). When rates fall, the option to prepay and refinance older, higher-coupon mortgages becomes more attractive to borrowers. Households exercise their prepayment options only gradually after a decline in rates, leading the effective maturity or "duration" of outstanding mortgages—i.e., the sensitivity of mortgage prices to changes in interest rates—to decline when long-term rates fall. And, the amount of expected mortgage refinancing activity varies significantly over time: depending on the past path of rates, there are times when many households move from being far from refinancing to being close and vice versa. The resulting shifts in expected refinancing activity trigger large changes to the total quantity of interest rate risk that must be borne by bond market investors, leading to transient, but sizable fluctuations in bond term premia .

[9] The Internet Appendix provides additional details and illustrative calibrations of these three microfounded models.

This mortgage refinancing channel can be used to micro-found a specification for the net supply of long-term bonds that is similar to equation (3.9). Following (Malkhozov et al., 2016), we assume that (i) there is a constant quantity M of outstanding fixed-rate mortgages with an embedded prepayment option; (ii) the primary mortgage rate, y_t^M, equals the long-term yield, y_t, plus a constant spread; (iii) the average coupon on outstanding mortgages evolves according to $c_{t+1}^M - c_t^M = -\eta \cdot (c_t^M - y_t^M)$, where $(c_t^M - y_t^M)$ is the "refinancing incentive" at time t and $\eta \in [0, 1]$ is the sensitivity of c_{t+1}^M to the refinancing incentive at t; (iv) the average "duration" or effective maturity of outstanding mortgages is $DUR_t^M = \overline{DUR}^M - N \cdot \left(c_t^M - y_t^M\right)$, where $\overline{DUR}^M > 0$ and $N > 0$ is the "negative convexity" of the average mortgage; and (v) the effective gross supply of long-bonds at time t is $s_t = M \cdot DUR_t^M$.

Each of these assumptions captures a well-known and reliable empirical regularity about the U.S. mortgage market. In particular, assumption (iii) captures the fact that, when the refinancing incentive $(c_t^M - y_t^M)$ is higher, more households refinance their existing high-coupon mortgages at time t, leading the average coupon to fall from t to $t + 1$. Assumption (iv) captures the fact that, when the refinancing incentive is higher, more households are expected to refinance their existing mortgages in the near future, implying that the average outstanding mortgage behaves more like a short-term bond. These assumptions imply that the effective gross supply of long-term bonds at time t is:

$$s_t = M \cdot \overline{DUR}^M + MN \cdot (y_t - \eta \textstyle\sum_{j=0}^{\infty} (1-\eta)^j \, y_{t-1-j}). \tag{4.1}$$

Thus, the mortgage refinancing channel implies that bond investors must bear greater interest rate risk when long-term rates have recently risen. And, the strength of this channel is given by the product MN.

There have been two structural shifts that are relevant for the strength of the refinancing channel. First, mortgage-backed securities (MBS) have become a larger share of the U.S. bond market over time. In the language of the model, this means that M has risen. From 1976 to 1999, MBS on average accounted for 21% of the value of the Bloomberg-Barclays Aggregate Index, a proxy for the broad U.S. bond market. From 2000 to 2019, the corresponding figure was 33%. As a result, movements in the duration of the outstanding mortgages (DUR_t^M) now generate far larger shifts in the effective supply of long-term bonds when judged relative to the overall U.S. bond market. Second, due a secular decline in refinancing costs, mortgage refinancing has become more interest-rate elastic over time (Bennett and Peristiani, 2001; Fuster et al., 2019). More elastic refinancing corresponds to a rise in both N and η. As a result, the association between DUR_t^M and recent changes in long rates has grown stronger. Together these changes suggest that the strength of the mortgage refinancing channel has grown in recent decades.

To solve our model of mortgage refinancing, we substitute the expression for supply in (4.1) into the market-clearing condition in (3.10) from Section 3. As above, fraction q of investors are fast-moving with demands given by equation (3.3) and fraction $(1 - q)$ are slow-moving and only rebalance their portfolios every $k \geq 2$ periods with demands given by (3.4). We can then establish the following proposition:

Proposition 2. ***Mortgage refinancing model.*** *For simplicity suppose $\rho_T = \rho_P$. When $MN > 0$, long-term yields are excessively sensitive to short rates. When $MN > 0$ and $\eta = 0$, this excess sensitivity is only horizon-dependent—i.e., the model-implied regression coefficients β_h in equation (3.11) only decline with horizon h—when arbitrage capital is slow moving ($q < 1$). By contrast, when $MN > 0$ and $\eta > 0$, β_h declines with horizon h even if all arbitrage capital is fast-moving ($q = 1$).*

When $\eta > 0$, shocks to short rates trigger shifts in effective bond supply that are less persistent than the underlying short rate shocks, giving rise to horizon-dependent excess sensitivity even without slow-moving capital. However, we are best able to quantitatively match the post-2000 behavior of the β_h coefficients using calibrations of this model in which (i) MN has risen substantially from the pre-2000 level and (ii) the resulting rate-amplifying supply shocks are met by a gradual arbitrage response. In addition, our model of mortgage refinancing predicts that (1) mortgage duration DUR_t^M is high when interest rates have recently risen and (2) the level of mortgage duration positively predicts future excess returns on long-term bonds (i.e., $E_t[rx_{t+1}]$ is high when DUR_t^M is high).

Evidence from mortgage-related quantities. To assess the contribution of the refinancing channel to our findings, we use two proxies for the impact of mortgage refinancing on the effective supply long-term bonds. The first is $y_t^M - c_t^M$, the mortgage refinancing *disincentive*. Here y_t^M is the average primary rate for 30-year, fixed-rate mortgages from Freddie Mac's Primary Mortgage Market Survey and c_t^M is the average outstanding coupon of MBS in the Bloomberg-Barclays U.S. MBS index. The index covers pass-through MBS backed by conventional fixed-rate mortgages that are guaranteed by Fannie Mae, Freddie Mac, and Ginnie Mae. This refinancing disincentive measure, which is associated with a higher duration on outstanding mortgages, is available beginning in Jan-1976. The second is the duration-to-worst of the Bloomberg-Barclays U.S. MBS index, DUR_t^M , a measure of the sensitivity of MBS prices to changes in long-term yields. This MBS duration measure is available on a monthly basis beginning in Jan-1976.[10] The correlation between $y_t^M - c_t^M$ and DUR_t^M is 0.55 from 1976 to 1999 and 0.66 in the post-2000 sample.

Using each of these proxies (X_t) for mortgage duration, we first estimate

$$X_t = \gamma_0 + \gamma_1 L_t + \gamma_2 S_t + \gamma_3 (L_t - L_{t-6}) + \gamma_4 (S_t - S_{t-6}) + \varepsilon_t^{MBS}, \tag{4.2}$$

for the pre- and post-2000 samples. We are interested in the coefficient on $L_t - L_{t-6}$, which tells us how MBS duration responds to recent changes in the level of short rates. Second, we estimate

$$rx_{t\to t+3}^{(10)} = \delta_0 + \delta_1 L_t + \delta_2 S_t + \delta_3 (L_t - L_{t-6}) + \delta_4 (S_t - S_{t-6}) + \delta_5 X_t + \varepsilon_{t\to t+3}^{(10)}, \tag{4.3}$$

for the pre-2000 and post-2000 samples. That is, we run horse race regressions to assess whether mortgage waves help explain why past changes in short rates forecast excess bond returns in the post-2000 data. We are interested in the coefficients on X_t and $L_t - L_{t-6}$ and how these coefficients change when these two variables are included jointly as opposed to one at a time.

Panel A of Table 5 shows the results from estimating (4.2) using the refinancing disincentive ($X_t = y_t^M - c_t^M$) and shows that it is responsive to past changes in level. Comparing columns (3) and (6), we see that it has become more responsive to past changes in level since 2000 (p-value = 0.06). Panel B reports the results for estimating this same equation using the duration of the Barclays MBS index ($X_t = DUR_t^M$) and delivers a similar message. Panel C reports the results from estimating (4.3) using the refinancing disincentive and suggest that the refinancing channel helps explain why past changes in the level of rates predict high excess returns on long-term bonds in the post-2000 data. In Panel C column (5), we see that

[10] Barclays uses its proprietary prepayment model to estimate the expected cashflows for each MBS. Yield-to-worst is the internal rate of return that equates MBS price and the present value of expected cash flows. Barclays computes the Macaulay duration of MBS treating these expected cashflows as given and index duration is the valued-weighted average of security-level durations. Beginning in 1989, Barclays reports the option-adjusted duration measure used in Hanson (2014). In the post-2000 sample, this slightly more sophisticated measure has a correlation of 0.84 with the measure we use.

$y_t^M - c_t^M$ attracts a positive and significant coefficient when forecasting 3-month excess bond returns after 2000. By contrast, the corresponding coefficient in column (2) for the pre-2000 sample is near zero and insignificant. And, the difference between the coefficients in columns (2) and (5) is significant (p-value < 0.01). However, when we use both $(y_t^M - c_t^M)$ and $(L_t - L_{t-6})$ to forecast $rx_{t\to t+3}^{(10)}$ in column (6), the coefficients on both variables decline noticeably relative to those in columns (4) and (5) where they are considered in isolation. This is precisely what we should expect if the refinancing channel plays an important role in explaining the short-lived excess sensitivity that we see in the post-2000 data.[11] Panels D shows that DUR_t^M also strongly forecasts $rx_{t\to t+3}^{(10)}$ in the post-2000 sample with the expected signs.

4.2 Investor extrapolation

Several recent papers, including Cieslak (2018), Giglio and Kelly (2018), Brooks et al. (2019) and D'Arienzo (2020) argue that some bond investors make biased forecasts of future interest rates. Positive shocks to short rates lead extrapolative investors to overestimate the future path of short rates and demand fewer long-term bonds. As a result, the quantity of bonds that must be held by unbiased investors rises when short rates rise, leading to an rise in term premium and generating excess sensitivity. If these expectational errors are transitory or if the arbitrage response is slow, extrapolation will create a short-lived market imbalance, leading long rates to temporarily overreact to changes in short rates.

Modeling the mechanism. We assume that some investors have "diagnostic expectations" about short rates in the sense that they "overweight future outcomes that have become more likely in light of incoming data" (Bordalo et al., 2017). In contrast to most recent work on diagnostic expectations—which takes a representative agent approach—we adopt a heterogenous agent approach, enabling us to study the dynamic arbitrage response of unbiased bond investors to these rate-amplifying demand shocks.

We assume that fraction f of investors have diagnostic expectations about short rates. Diagnostic investors' demand for long-term bonds is $h_t = \tau(E_t^D[rx_{t+1}]/Var_t^D[rx_{t+1}])$ where $E_t^D[\cdot]$ denotes diagnostic investors' biased expectations. Fraction $(1-f)$ of a bond investors have fully rational expectations about short-term interest rates. Of these rational investors, fraction q are fast-moving with demands given by equation (3.3) and fraction $(1-q)$ are slow-moving with demands given by (3.4). We assume the gross supply of long-term bonds is constant over time at $s_t = \bar{s}$. Following Maxted (2020), we assume that diagnostic investors' expectation of the transient component of short-term rates ($i_{T,t}$) is

$$E_t^D[i_{T,t+1}] = \rho_T i_{T,t} + \theta \cdot [i_{T,t} - (\rho_T - \kappa_T)\sum\nolimits_{j=0}^{\infty} \kappa_T^j i_{T,t-j-1}], \tag{4.4}$$

where $\theta \geq 0$ and $\kappa_T \in [0, \rho_T]$. The parameter θ governs the extent to which diagnostic expectations depart from rationality ($\theta = 0$) and κ_T governs the persistence of investors' mistaken beliefs about short rates.[12] When $\theta > 0$ and $\kappa_T < \rho_T$, equation (4.4) says that diagnostic investors overestimate $i_{T,t+1}$ when $i_{T,t}$ has recently risen. Thus, extrapolation leads to a model that is similar to the reduced-form specification for net bond supply in equation (3.9). We adopt an analogous specification for diagnostic

[11] If mortgage refinancing was the only source of rate amplification in the U.S. bond market and there was no slow-moving arbitrage capital, then $(y_t^M - c_t^M)$ should be a sufficient statistic for bond risk premia and should completely drive out $(L_t - L_{t-6})$ in a horse race specification. However, if mortgage refinancing was one of several amplification mechanisms, or if arbitrage capital was slow moving, then one expected both $(y_t^M - c_t^M)$ and $(L_t - L_{t-6})$ to attract meaningful coefficients.

[12] In the limit where $\kappa_T = 0$, $E_t^D[i_{T,t+1}] = \rho_T i_{T,t} + \theta\varepsilon_{T,t}$, so investors' mistakes ($\theta\varepsilon_{T,t}$) are serially uncorrelated over time. In the opposite limit where $\kappa_T = \rho_T$, $E_t^D[i_{T,t+1}] = \rho_T i_{T,t} + \theta i_{T,t}$, so investors' mistakes ($\theta i_{T,t}$) are just as persistent as $i_{T,t}$.

investors' expectations of the persistent component of short rates ($i_{P,t}$), but assume for simplicity that diagnostic investors form rational forecasts of all other state variables.

The strength of this extrapolation channel is given by $f\theta$—i.e., the mass of diagnostic investors (f) times the extent to which their expectations depart from rationality (θ). Why might $f\theta$ have risen in recent decades? While many bond investors may have a tendency to extrapolate past changes in interest rates, it is natural to think that this tendency is most pronounced amongst investors in bond mutual funds. Indeed, there is a long literature arguing that mutual fund investors—who are predominantly households and smaller institutions—tend to be more prone to common psychological biases than larger institutional investors (Barberis et al., 1998; Dichev, 2007; Frazzini and Lamont, 2008). Furthermore, mutual funds have become more important players in the U.S. bond market in recent decades. Based on data from Federal Reserve's Financial Accounts, mutual funds' share of Treasury and MBS holdings has gradually risen from 5% in the early 1990s to nearly 10% today. And, mutual funds have rapidly gained share in the corporate bond market, rising from a 7% share in early 2009 to over 20% today. Thus, even if individual mutual fund investors have not become more extrapolative since 2000 (i.e., if θ has not changed), this group of extrapolation-prone investors has become more important in the bond market (corresponding to a rise in f). In this setting, we can demonstrate the following result:

Proposition 3. ***Investor extrapolation model.*** *For simplicity suppose $\rho_T = \rho_P$ and $\kappa_T = \kappa_P$. When $f\theta > 0$, long rates are excessively sensitive to short rates. When $f\theta > 0$ and $\kappa_T = \rho_T$, this excess sensitivity is only horizon-dependent—i.e., the regression coefficients β_h only decline with horizon h—when unbiased arbitrage capital is slow moving ($q < 1$). By contrast, when $f\theta > 0$ and $\kappa_P < \rho_T$, β_h declines with horizon h even if all arbitrage capital is fast-moving ($q = 1$).*

When $f\theta > 0$ and $\kappa_P < \rho_P$, extrapolation generates transitory rate-amplifying demand shocks, giving rise to frequency-dependent excess sensitivity even without slow-moving capital. However, this frequency-dependent excess sensitivity becomes more pronounced when these demand shocks are met by a gradual arbitrage response from unbiased investors. Thus, we are best able to quantitatively match the post-2000 behavior of the β_h coefficients using calibrations of our extrapolation model in which (i) $f\theta$ has risen from its pre-2000 level and (ii) arbitrage is gradual. Our extrapolation model also predicts that: (1) the bond holdings of extrapolative investors, h_t, are low when interest rates have recently risen and (2) the level of extrapolative investors' bond holdings negatively predicts excess returns on long bonds.

Evidence from bond mutual fund flows. To assess whether investor extrapolation contributes to high-frequency excess sensitivity, we obtain monthly data from 1984 to 2019 on the total net assets of taxable bond mutual funds and the net dollar flows into these funds from the Investment Company Institute. We then compute the 3-month cumulative percentage flow into bond funds, $\%FLOW_{t-3\to t}$. Using bond fund flows as a proxy for the rate-amplifying demand of extrapolative investors in our model (h_t), we first estimate equation (4.2) with $X_t = \%FLOW_{t-3\to t}$ for the pre-2000 and post-2000 samples. The results are presented in Panel A of Table 6 and show that bond mutual funds tend to suffer investor outflows when short-term interest rates decline. This result is consistent with the vast literature on return-chasing behavior by mutual fund investors (Warther, 1995; Sirri and Tufano, 1998). Interesting, this relationship is actually stronger in the pre-2000 sample than in the post-2000 sample, consistent with other evidence that mutual fund flows have become less performance sensitive in recent years. However, the importance of mutual funds within the bond market has increased meaningfully since 2000.

In Panel B, we estimate equation (4.3) with $X_t = \%FLOW_{t-3\to t}$ for the pre- and post-2000 samples. As shown in column (5), past mutual fund flows predict low future excess returns on 10-year bonds in the post-2000 data. By contrast, as shown in column (2), there are no such relationships in the pre-2000 sample. However, when we use both $(L_t - L_{t-6})$ and $\%FLOW_{t-3\to t}$ to forecast returns in column (6), the coefficients on both variables decline meaningfully relative to those shown in columns (4) and (5) where they are considered in isolation. As above, this is what one would expect if extrapolation plays a role in explaining why long-term yields temporarily overreact to short rates in the post-2000 data.

4.3 Investors who reach for yield

Investors who "reach for yield" when short rates decline are a final potential source of rate-amplifying demand. According to the reaching-for-yield channel, negative shocks to short rates boost the demand for long-term bonds from "yield-seeking investors." Holding fixed the gross supply, the net supply of long-term bonds that must be held by fast- and slow-moving arbitrageurs declines when short rates fall, leading term premia to decline when short rates fall.

Modeling the mechanism. We assume that fraction f of bond investors are "yield-seeking" and have non-standard preferences as in Hanson and Stein (2015). The idea is that, for either behavioral or institutional reasons, these investors only care about the current yield on their portfolios instead of their expected portfolio returns. Yield-seeking investors' demand for long-term bonds is:

$$h_t = \tau \frac{y_t - i_i}{Var_t\left[rx_{t+1}\right]}. \tag{4.5}$$

Since $E_t\left[rx_{t+1}\right] = (y_t - i_i) - (\phi/\left(1-\phi\right)) \cdot E_t\left[y_{t+1} - y_t\right]$, equation (4.5) means that yield-seeking investors neglect any expected capital gains or losses from holding long-term bonds. And, because expectations-hypothesis logic implies that long-term yields are expected to rise when short rates are low, this implies that these investors have an elevated demand for long bonds when short rates are low. As above, the gross supply of long-term bonds is constant. A mass $(1-f)$ of a bond investors are expected-return-oriented and have standard mean-variance preferences. Of these, fraction q are fast-moving with demands given by (3.3) and fraction $(1-q)$ are slow-moving with demands given by (3.4). And, prior research suggests the reaching-for-yield channel may have grown stronger—corresponding to a rise in f—in recent years as interest rates have reached historically low levels.[13] Using this model, we can then show:

Proposition 4. *Investor reaching for yield model.* *Suppose $\rho_T = \rho_P$. When $f > 0$, long rates are excessively sensitive to short rates. However, this excess sensitivity is only horizon-dependent when arbitrage capital is slow moving ($q < 1$).*

Our model of reaching-for-yield also predicts that: (1) the bond holdings of yield-seeking investors, h_t, are low when interest rates are high and (2) the level of yield-seeking investors' bond holdings negatively predicts future excess returns on long-term bonds.

Since these rate-amplifying shifts in demand are tied to the level of short rates as opposed to recent changes in short rates, reaching for yield generates persistent shifts in demand. Thus, while reaching-

[13] Lian et al. (2017) provide experimental evidence that the tendency to take on greater risk when short rates decline becomes more pronounced when the level of short rates is already low. Building on Prospect Theory (Kahneman and Tversky, 1979), they argue that this yield-seeking behavior becomes more pronounced as rates fall further below some reference level that investors are accustomed to based on past experience.

for-yield can generate excess sensitivity, in the absence of gradual arbitrage, this excess sensitivity is not horizon-dependent. And, while the combination of reaching-for-yield and gradual arbitrage generates horizon-dependent sensitivity, our calibrations of this model struggle to quantitatively match the profile of β_h seen in the post-2000 data. Thus, it is not clear that reaching-for-yield can explain why excess sensitivity has become so pronounced at high frequencies. Going further, reaching-for-yield itself may be a slow-moving phenomenon: investors may only gradually take on greater risk following a decline in short rates. If true, this would further weaken the ability of reaching-for-yield to explain why the excess sensitivity of long rates has become so horizon-dependent.

Evidence from sectoral bond market flows. We use quarterly data from the Federal Reserve's Financial Accounts on the aggregate net bond acquisitions by insurers, pension funds, and banks to construct proxies for the bond demand of yield-seeking investors, h_t. We focus on these intermediaries since prior research argues that they are likely to be concerned about the current yield on their portfolios and, thus, to reach for yield when interest rates fall.[14] For intermediaries in sector i, we compute bond flows in quarter t as $\%FLOW_{i,t} = FLOW_{i,t}/HOLD_{i,t-1}$, where $FLOW_{i,t}$ denotes net bond acquisitions in quarter t and $HOLD_{i,t-1}$ is prior bond holdings. Bonds include the sum of U.S. Treasuries, agency debt and GSE-guaranteed mortgage-backed securities, and corporate bonds.

In the Internet Appendix, we estimate quarterly regressions that are analogous to equations (4.2) and (4.3) using these bond flows $\%FLOW_{i,t}$ as X_t. In the post-2000 data, we find little evidence that increases in short rates lead to reductions in bond purchases by insurers, pensions, and banks. Furthermore, bond purchases by these intermediaries do not predict low excess returns on long-term bonds.

In summary, we find evidence that mortgage refinancing and investor extrapolation both help explain why long yields rates have temporarily overreacted to short rates since 2000. However, we find little evidence that reaching-for-yield plays a major role in driving the temporary overreaction of long rates.[15]

5 Implications

5.1 High-frequency identification

Our findings have clear implications for identification approaches based on the high-frequency responses of long-term yields to macroeconomic news and policy announcements. Papers in the vast event-study literature often implicitly assume that one can directly infer the expected long-run effects of news shocks on future fundamentals by looking at the high-frequency reactions of long-term asset prices (MacKinlay, 1997). And, several recent papers—e.g., Hördahl et al. (2015) and Nakamura and Steinsson (2018)—have made this assumption more explicitly. Intuitively, if changes in long rates in a tight window around a macro announcement are governed by the expectations hypothesis—e.g., because bond risk premia only vary at lower frequencies, then the high-frequency reaction of long rates directly reveals the expected long-run effect of the news shock on future short rates. For instance, if the 10-year forward rate fell by 20 basis points in a short window around an FOMC announcement, one would infer that this led expected short rates in 10 years to drop by 20 basis points.

[14] Insurers and banks are generally not required to include any changes in mark-to-market value in their reported earnings, potentially leading to yield-seeking behavior. For prior work on reaching-for-yield by insurers, see Becker and Ivashina (2015). For banks, see Maddaloni and Peydró (2011) and Hanson and Stein (2015). For pensions, see Lu et al. (2019).

[15] This need not imply that reaching-for-yield plays an unimportant role in determining financial risk premia more generally.

Our evidence casts serious doubt on this assumption. If, as we argue, a large portion of the impact of news shock on long rates reflects rapidly-reverting shifts in term premia, then the short-run impact of news shocks on long rates will differ meaningfully from their expected long-run impact on future short rates. As a result, the high-frequency responses of long rates are likely to provide a highly biased estimate of the longer-run impact of announcements. Fortunately, it is relatively straightforward to eliminate this bias: one needs to use an methodology that does not assume that we can directly infer the expected long-run effects of news shocks simply by looking the high-frequency reactions of long-term asset prices.[16] Of course, these unbiased approaches lead to far less precise estimates, so economists face a steep bias-variance trade-off. The short-run market impact of news on long rates can be estimated very precisely, but these are likely to be biased estimates of the longer-run impact that is typically of greatest interest.

Still, it is possible that changes in 1-year yields that coincide with macro announcements are different, and do not trigger movements in term premia, as argued by Hördahl et al. (2015) and Nakamura and Steinsson (2018). To provide some direct evidence, we form an macro news index for month t, $News_t$, by cumulating daily changes in 1-year yields within month t on days with important announcements. Our data on announcement timing is from Money Market Services/Action Economics and begins in 1980. The announcements we consider are: FOMC announcements, the employment situation report, retail sales, durable goods orders, new and existing home sales, housing starts, CPI, and PPI. We then estimate the following predictive regression for the subsequent change in 10-year forward rates:

$$f_{t+h}^{(10)} - f_t^{(10)} = \delta_0 + \delta_1 L_t + \delta_2 S_t + \delta_3 (L_t - L_{t-1}) + \delta_4 (S_t - S_{t-1}) + \lambda \cdot News_t + \varepsilon_{t+h}, \qquad (5.1)$$

where L_t and S_t denote the level and slope of the yield curve at the end of month t. Thus, equation (5.1) adds $News_t$ to the Jorda (2005) projections we previously estimated in equation (2.3). Table 7 shows the results for both pre- and post-2000 samples and for $h =$ 3-, 6-, 9-, and 12- month future changes.

In Panel A, we omit $News_t$, so the estimates are the same as those in Table 3. As previously shown, past increases in short-term rates are associated with predictable future declines in long-term forward rates in the post-2000 data, but there is no such tendency in the pre-2000 data. In Panel B, we add $News_t$, but omit the prior changes in level and slope. We see that positive values of the news index predict subsequent declines in long-term forwards in the post-2000 data. Indeed, the coefficients on $News_t$ in Panel B are similar to those on $L_t - L_{t-1}$ in Panel A.

In Panel C, we include $News_t$ as well as the prior changes in level and slope. The goal is to see if shifts in short rates on announcement and non-announcement days have different implications. Once we control for the total change in short rates in month t, $L_t - L_{t-1}$, we find that the coefficient on $News_t$ is small and insignificant, indicating that the response of long-term forwards rates on announcement days is just as likely to reverse as the response on non-announcement days.

In Panel D, we break $News_t$ into two pieces—one reflecting changes in short rates on FOMC announcement days and one for all other announcements—to see if FOMC announcements differ from other macro announcements. We exclude the 1980-81 monetary targeting regime and thus FOMC announce-

[16] For instance, one could estimate the long-run effects of a shock using a Structural VAR in which high-frequency asset prices movements are used as external instruments for monetary policy or other shocks. And, then IV-estimates of the SVAR would be used to trade out the long-run dynamic effects of the shock—see, e.g., Gertler and Karadi (2015) and Eberly et al. (2020). Similarly, one could estimate the long-run effects of news shocks using Jorda (2005) style "local projections" in which one regresses outcomes at future horizons on high-frequency market reactions to news.

ment dates begin in 1982. As in Panel C, we include $L_t - L_{t-1}$ as an independent variable. If anything, Panel D suggests that, since 2000, changes in short rates on FOMC announcement days are *more* likely to be followed by reversals in long-term forward rates than changes on non-announcement days.

5.2 Monetary policy transmission

Our results also have important implications for the transmission of monetary policy. Central banks conduct conventional monetary policy by adjusting short-term nominal rates. According to the standard New Keynesian view (Gali, 2008), changes in nominal short rates affect real short rates because of nominal rigidities. And, the resulting shifts in real short rates affects long-term real rates via the expectations hypothesis, which then influence household consumption and firm investment. Stein (2013) points out that the excess sensitivity of long-term yields—whereby shocks to short rates move term premia in the same direction—should strengthen the effects of monetary policy relative to the canonical view. Stein (2013) refers to this as the "recruitment" channel of monetary transmission.

In our framework from Section 3, the strength of this recruitment channel at business-cycle frequencies (e.g., over a 1 to 3-year horizon) depends on (i) the strength of the demand-based amplification mechanisms (i.e., the size of C) and (ii) the persistence of the associated demand shocks. When ρ_s is well below ρ_T as under the mortgage refinancing interpretation of C, the associated shifts in term premia would be quite transient and would likely have only modest effects on investment and consumption at medium-run frequencies. (A caveat here is that reductions in short rates that trigger mortgage refinancing waves may only temporarily lower term premia, but the effect of refinancing waves on distribution of household disposable income, and hence consumption, may persist long after term premia have reverted in heterogeneous agent settings.[17]) By contrast, when $\rho_s \approx \rho_T$ as under the reaching-for-yield interpretation, the shifts in term premia would be more persistent and have larger effects on aggregate demand.

Our results indicate that a significant part of the influence of short rates on term premia is quite transitory, suggesting that recruitment channel may be smaller than one would conclude based on a simple extrapolation of the high-frequency response of term premia to policy shocks documented by Hanson and Stein (2015), Gertler and Karadi (2015), and Gilchrist et al. (2015). More generally, our findings suggest that central banks should heed the way that monetary policy impacts financial conditions at business-cycle frequencies, but should focus less on the immediate market response to their announcements since much of the latter may be quite transitory. In this way, our findings lend support to the argument in Stein and Sunderam (2018) that the Fed has become too focused on high-frequency asset price movements.

5.3 Bond market "conundrums"

Our findings help explain the rising prevalence of episodes like the one Greenspan (2005) famously called the "conundrum"—the period after June 2004 when short rates rose and long rates fell. Consistent with the weaker low-frequency sensitivity of long rates, "conundrum" episodes—defined as 6-month periods where short and long rates move in *opposite* directions—have grown increasingly common. Since 2000, 1- and 10-year yields have moved in the *opposite* direction in 37% of all 6-month periods. By contrast, from 1971 to 1999, this figure was 18%, and the difference is significant (p-val < 0.001). In the Internet

[17] To the extent that mortgage refinancing plays an important role in U.S. monetary policy transmission as in recent heterogeneous agent models (Beraja et al., 2018; Berger et al., 2018; Wong, 2019), then even short-lived excess sensitivity may make monetary policy more potent than in a world where long rates are not excessively sensitive.

Appendix, we show that the non-Markovian dynamics documented in Section 2 help explain several noteworthy "conundrums" episodes, including Greenspan's 2004 "conundrum," 2008, and 2017. In each case, 1-year and 10-year yields moved in opposite directions, but, if the slope of the yield curve had not responded to past changes short rates, 10- and 1-year yields would have moved in the same direction.

5.4 Affine term structure models

Finally, we explore the implications for affine term structure models which are a widely-used, reduced-form tools for understanding the term structure of bond yields (Duffie and Kan, 1996; Duffee, 2002). In these models, the n-year zero coupon yield is $y_t^{(n)} = \alpha_{0(n)} + \boldsymbol{\alpha}'_{1(n)}\mathbf{x}_t$, where $\mathbf{x}_t$ is a vector of state variables and the $\alpha_{0(n)}$ and $\boldsymbol{\alpha}_{1(n)}$ coefficients satisfy a set of recursive equations. In the Internet Appendix, we fit affine term structure models using the first K principal components of 1- to 10-year yields as the state variables $\mathbf{x}_t$. We show that standard affine models—models that are Markovian with respect to these current yield-curve factors—cannot fit the fact that β_h declines so strongly with horizon h in the post-2000 data. This remains so even if we estimate models that include many (e.g., $K = 5$) yield-curve factors as state variables. However, we show that our key finding is consistent with non-Markovian term structure models in which past lags of the yield-curve factors are treated as "unspanned state variables."[18]

6 Conclusion

The strong sensitivity of long-term interest rates to changes in short rates is a long-standing puzzle. We have shown that since 2000 this sensitivity has become even stronger at high frequencies, but has weakened significantly at low-frequencies. As a result, in the post-2000 data, the sensitivity of long rates to changes in short rates declines steeply with the horizon over which these changes are computed.

Before 2000, long rates were quite sensitive to short rates because inflation expectations were relatively unanchored, making short rates highly persistent. Since 2000, the sensitivity of long rates has become horizon-dependent and arises because past increases in short rates temporarily raise the term premium, leading long rates to temporarily overreact to changes in short rates. Consistent with this view, we show that, controlling for current yields, past changes in short rates predict future yield-curve flattening, declines in long-term yields and forwards, and high excess returns on long-term bonds after 2000.

We proposed a model that can explain this puzzling post-2000 pattern. The tendency of long rates to temporarily overreact to changes in short rates is due to the combination of (i) rate-amplifying shifts in the demand for long-term bonds and (ii) a limited and slow arbitrage response to these demand shifts. We presented evidence that two specific rate-amplifying demand mechanisms—mortgage refinancing waves and extrapolation of past changes in short rates—each help explain this post-2000 pattern.

Our findings have important implications for the recruitment channel of monetary policy transmission (Stein, 2013). In recent years this channel appears far more short-lived than one might conclude from high-frequency evidence alone: Part of the high-frequency response of long rates to shocks to short rates represents transitory term premium movements. Lastly, it is important to remember that event-study approaches only measure high-frequency responses of long-term rates to news; the impact may be more muted at the lower frequencies that are typically of greatest interest to economists and policymakers.

[18] An unspanned state variable is a variable that is useful for forecasting future bond yields and returns but that has no impact on the current yield curve (Duffee, 2002). To be clear, we do not argue that the past increase in the level of rates is *literally* unspanned. Instead, as discussed in the Internet Appendix, we think this variable is *close* to being unspanned.

References

ANDREWS, D. W. (1993): "Tests for parameter instability and structural change with unknown change point," *Econometrica*, 61, 821–856.

BARBERIS, N., A. SHLEIFER, AND R. VISHNY (1998): "A model of investor sentiment," *Journal of Financial Economics*, 49, 307–343.

BECKER, B. AND V. IVASHINA (2015): "Reaching for yield in the bond market," *Journal of Finance*, 70, 1863–1902.

BEECHEY, M. J. AND J. H. WRIGHT (2009): "The high-frequency impact of news on long-term yields and forward rates: Is it real?" *Journal of Monetary Economics*, 56, 535–544.

BENNETT, P. R. P. AND S. PERISTIANI (2001): "Structural change in the mortgage market and the propensity to refinance," *Journal of Money, Credit, and Banking*, 33, 955975.

BERAJA, M., A. FUSTER, E. HURST, AND J. VAVRA (2018): "Regional heterogeneity and the refinancing channel of monetary policy," *Quarterly Journal of Economics*, 134, 109–183.

BERGER, D. W., K. MILBRADT, F. TOURRE, AND J. VAVRA (2018): "Mortgage prepayment and path dependent effects of monetary policy," NBER Working Paper 25157.

BORDALO, P., N. GENNAIOLI, AND A. SHLEIFER (2017): "Diagnostic expectations and credit cycles," *Journal of Finance*, 73, 199–227.

BOYARCHENKO, N., A. FUSTER, AND D. O. LUCCA (2019): "Understanding mortgage spreads," *The Review of Financial Studies*, 32, 3799–3850.

BROOKS, J., M. KATZ, AND H. LUSTIG (2019): "Post-FOMC announcement drift in U.S. bond markets," Working Paper.

CAMPBELL, J. Y. AND R. J. SHILLER (1988): "Stock prices, earnings, and expected dividends," *Journal of Finance*, 43, 661–676.

——— (1991): "Yield spreads and interest rate movements: A bird's eye view," *The Review of Economic Studies*, 58, 495–514.

CHO, C.-K. AND T. J. VOGELSANG (2017): "Fixed-b inference for testing structural change in a time series regression," *Econometrics*, 5, 2.

CHOW, G. C. (1960): "Tests of equality between sets of coefficients in two linear regressions," *Econometrica*, 28, 591–605.

CIESLAK, A. (2018): "Short-rate expectations and unexpected returns in Treasury bonds," *Review of Financial Studies*, 31, 3265–3306.

COCHRANE, J. H. AND M. PIAZZESI (2002): "The Fed and interest ratesa high-frequency identification," *American Economic Review: Papers and Proceedings*, 92, 90–95.

——— (2005): "Bond risk premia," *American Economic Review*, 95, 138–160.

D'ARIENZO, D. (2020): "Maturity increasing over-reaction and bond market puzzles," Working paper, Bocconi University.

DE LONG, J. B., A. SHLEIFER, L. H. SUMMERS, AND R. J. WALDMAN (1990): "Noise trader risk in financial markets," *Journal of Political Economy*, 98, 703–738.

Dichev, I. D. (2007): "What are stock investors actual historical returns? Evidence from dollar-weighted returns," *American Economic Review*, 97, 386–401.

Domanski, D., H. S. Shin, and V. Shushko (2017): "The hunt for duration: Not waving but drowning?" *IMF Economic Review*, 65, 113–153.

Duffee, G. (2002): "Term premia and interest rate forecasts in affine models," *Journal of Finance*, 57, 405–443.

Duffee, G. R. (2013): "Forecasting interest rates," in *Handbook of Economic Forecasting, Volume 2*, ed. by G. Elliott and A. Timmermann, Elsevier.

Duffie, D. (2010): "Asset price dynamics with slow-moving capital," *Journal of Finance*, 65, 1238–1268.

Duffie, D. and R. Kan (1996): "Yield factor models of interest rates," *Mathematical Finance*, 64, 379–406.

Eberly, J. C., J. H. Stock, and J. H. Wright (2020): "The Federal Reserve's Current Framework for Monetary Policy: A Review and Assessment," *International Journal of Central Banking*, 16, 5–71.

Fama, E. and R. R. Bliss (1987): "The information on long-maturity forward rates," *American Economic Review*, 77, 680–692.

Frazzini, A. and O. A. Lamont (2008): "Dumb money: Mutual fund flows and the cross-section of stock returns," *Journal of Financial Economics*, 88, 299–322.

Fuster, A., M. Plosser, P. Schnabl, and J. Vickery (2019): "The role of technology in mortgage lending," *Review of Financial Studies*, 32, 1854–1899.

Gali, J. (2008): *Monetary Policy, Inflation, and the Business Cycle*, Princeton, New Jersey: Princeton University Press.

Gertler, M. and P. Karadi (2015): "Monetary policy surprises, credit costs, and economic activity," *American Economic Journal: Macroeconomics*, 7, 44–76.

Giglio, S. and B. Kelly (2018): "Excess volatility: Beyond discount rates," *Quarterly Journal of Economics*, 133, 71–127.

Gilchrist, S., D. Lpez-Salido, and E. Zakrajek (2015): "Monetary policy and real borrowing costs at the zero lower bound," *American Economic Journal: Macroeconomics*, 7, 77–109.

Greenspan, A. (2005): "Federal Reserve Board's semiannual Monetary Policy Report to the Congress," Testimony on February 16, 2005 before the Senate Committee on Banking, Housing, and Urban Affairs.

Greenwood, R., S. G. Hanson, and G. Y. Liao (2018): "Asset price dynamics in partially segmented markets," *Review of Financial Studies*, 31, 3307–3343.

Greenwood, R. and D. Vayanos (2014): "Bond supply and excess bond returns," *Review of Financial Studies*, 27, 663–713.

Gürkaynak, R. S., B. Sack, and E. T. Swanson (2005): "The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models," *American Economic Review*, 95, 425–436.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007): "The U.S. Treasury yield curve: 1961 to the present," *Journal of Monetary Economics*, 54, 2291–2304.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2010): "The TIPS yield curve and inflation compensation," *American Economic Journal: Macroeconomics*, 2, 70–92.

Gurkaynak, R. S., B. P. Sack, and E. T. Swanson (2005): "Do actions speak louder than words? The response of asset prices to monetary policy actions and statements," *International Journal of Central Banking*, 1, 55–93.

Hansen, L. P. and R. J. Hodrick (1980): "Forward exchange rates as optimal predictors of future spot rates: An econometric analysis," *Journal of Political Economy*, 88, 829–853.

Hanson, S. G. (2014): "Mortgage convexity," *Journal of Financial Economics*, 113, 270–299.

Hanson, S. G. and J. C. Stein (2015): "Monetary policy and long-term real rates," *Journal of Financial Economics*, 115, 429–448.

Hördahl, P., E. M. Remolona, and G. Valente (2015): "Expectations and risk premia at 8:30AM: Macroeconomic announcements and the yield curve," BIS working paper 527.

Jorda, O. (2005): "Estimation and inference of impulse responses by local projections," *American Economic Review*, 95, 161–182.

Kahneman, D. and A. Tversky (1979): "Prospect Theory: An analysis of decision under risk," *Econometrica*, 47, 263–292.

Kiefer, N. M. and T. J. Vogelsang (2005): "A new asymptotic theory for heteroskedasticity-autocorrelation robust tests," *Econometric Theory*, 21, 1130–1164.

Lian, C., Y. Ma, and C. Wang (2017): "Low interest rates and risk taking: Evidence from individual investment decisions," .

Litterman, R. and J. Scheinkman (1991): "Common factors affecting bond returns," *Journal of Fixed Income*, 1, 54–61.

Lu, L., M. Pritsker, A. Zlate, K. Anadu, and J. Bohn (2019): "Reach for yield by U.S. public pension funds," .

MacKinlay, A. C. (1997): "Event studies in economics and finance," *Journal of Economic Literature*, 35, 13–39.

Maddaloni, A. and J.-L. Peydró (2011): "Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the euro-area and the US lending standards," *Review of Financial Studies*, 24, 2121–2165.

Malkhozov, A., P. Mueller, A. Vedolin, and G. Venter (2016): "Mortgage risk and the yield curve," *Review of Financial Studies*, 29, 1220–1253.

Mankiw, N. G. and L. H. Summers (1984): "Do long term interest rates overreact to short-term interest rates?" *Brookings Papers on Economic Activity*, 1, 223–242.

Maxted, P. (2020): "A macro-finance model with sentiment," .

Nakamura, E. and J. Steinsson (2018): "High-frequency identification of monetary non-neutrality: The information effect," *Quarterly Journal of Economics*, 133, 1283–1330.

Newey, W. K. and K. D. West (1987): "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix," *Econometrica*, 55, 703–708.

Samuelson, P. A. (1947): *Foundations of Economic Analysis*, Cambridge, MA: Harvard University Press.

Shiller, R. J. (1979): "The volatility of long-term interest rates and expectations models of the term structure," *Journal of Political Economy*, 87, 1190–1219.

Shiller, R. J., J. Y. Campbell, and K. L. Schoenholtz (1983): "Forward rates and future policy: Interpreting the term structure of interest rates," *Brookings Papers on Economic Activity*, 1, 173–217.

Shleifer, A. and R. W. Vishny (1997): "The limits of arbitrage," *Journal of Finance*, 52, 33–55.

Sirri, E. R. and P. Tufano (1998): "Costly search and mutual fund flows," *Journal of Finance*, 53, 1589–1622.

Stein, J. C. (2013): "Yield-oriented investors and the monetary transmission mechanism," .

Stein, J. C. and A. Sunderam (2018): "The Fed, the bond market, and gradualism in monetary policy," *Journal of Finance*, 1015–1060.

Stock, J. H. and M. W. Watson (2007): "Why has U.S. inflation become harder to forecast?" *Journal of Money, Credit and Banking*, 39, 3–33.

Vayanos, D. and J.-L. Vila (2020): "A preferred-habitat model of the term structure of interest rates," *Econometrica*, Forthcoming.

Warther, V. A. (1995): "Aggregate mutual fund flows and security returns," *Journal of Financial Economics*, 39, 209–235.

Wong, A. (2019): "Refinancing and the transmission of monetary policy to consumption," Working paper, Princeton University.

Table 1: Regressions of changes in long-term rates on short-term rates. This table reports the estimated regression coefficients from equations (1.1) and (1.2) for each reported sample. The dependent variable is the change in the 10-year U.S. Treasury zero-coupon yield or forward rate, either nominal, real or their difference (IC, or inflation compensation). The independent variable is the change in the 1-year nominal U.S. Treasury zero-coupon yield in all cases. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. In the 1971-1999 monthly sample, time t runs from Aug-1971 to Dec-1999 and the number of monthly observations is 341 irrespective of h. In the 2000-2019 monthly sample, t runs from Jan-2000 to Dec-2019, so the number of monthly observations runs 239 from for $h = 1$ to 228 for $h = 12$. For $h > 1$, we report Newey-West (1987) standard errors are in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, ** $p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

Panel A. 10-year zero coupon yields and IC

	(1) Nominal	(2) Nominal	(3) Real	(4) IC
Daily	0.56***	0.87***	0.54***	0.33***
	[0.02]	[0.03]	[0.03]	[0.02]
Monthly	0.46***	0.66***	0.38***	0.26***
	[0.04]	[0.11]	[0.09]	[0.09]
Quarterly	0.48***	0.44***	0.22**	0.22*
	[0.04]	[0.07]	[0.10]	[0.12]
Semi-annual	0.50***	0.34***	0.21**	0.13
	[0.04]	[0.07]	[0.08]	[0.09]
Yearly	0.56***	0.23***	0.15**	0.08
	[0.05]	[0.05]	[0.06]	[0.05]
Sample	1971-1999	2000-2019	2000-2019	2000-2019

Panel B. 10-year instantaneous forward yields and IC

	(1) Nominal	(2) Nominal	(3) Real	(4) IC
Daily	0.39***	0.49***	0.31***	0.17***
	[0.03]	[0.04]	[0.03]	[0.03]
Monthly	0.29***	0.26*	0.18**	0.06
	[0.04]	[0.14]	[0.08]	[0.09]
Quarterly	0.31***	0.06	0.09*	-0.03
	[0.05]	[0.09]	[0.05]	[0.05]
Semi-annual	0.33***	-0.02	0.04	-0.06
	[0.06]	[0.08]	[0.04]	[0.05]
Yearly	0.39***	-0.13**	-0.02	-0.11**
	[0.07]	[0.06]	[0.04]	[0.04]
Sample	1971-1999	2000-2019	2000-2018	2000-2019

Table 2: Estimates of predictive equations for level and slope. This table reports the estimated regression coefficients from monthly predictive equations (2.1a) and (2.1b) for the Aug-1971 to Dec-1999 and Jan-2000 to Dec-2019 samples. Dependent variables are the level ($L_t \equiv y_t^{(1)}$) and slope ($S_t \equiv y_t^{(10)} - y_t^{(1)}$) of the U.S. Treasury zero-coupon yield curve. Heteroskedasticity robust standard errors are in brackets. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. The table also shows AIC and BIC values (to be minimized) for each possible specification of the system of two equations. Lastly, the implied β_1 and β_{12} coefficients from equation (2.2) for each possible specification of the system are reported.

	Pre-2000			Post-2000		
	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable: Level						
L_t	0.98***	0.97***	0.96***	0.97***	0.98***	0.98***
	[0.02]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]
S_t	0.00	-0.01	-0.02	-0.02*	-0.01	-0.00
	[0.04]	[0.04]	[0.04]	[0.01]	[0.01]	[0.01]
$L_t - L_{t-6}$		-0.01	0.05		0.08***	0.06**
		[0.04]	[0.05]		[0.03]	[0.03]
$S_t - S_{t-6}$			0.13*			-0.03*
			[0.07]			[0.02]
Dependent Variable: Slope						
L_t	0.01	0.01	0.01	0.00	-0.01	-0.01
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
S_t	0.96***	0.96***	0.97***	0.98***	0.96***	0.96***
	[0.03]	[0.02]	[0.03]	[0.02]	[0.02]	[0.02]
$L_t - L_{t-6}$		0.00	-0.03		-0.11***	-0.12***
		[0.02]	[0.03]		[0.02]	[0.03]
$S_t - S_{t-6}$			-0.08			-0.02
			[0.05]			[0.03]
N	341	335	335	239	239	239
Implied β_1	0.46	0.46	0.46	0.66	0.71	0.71
Implied β_{12}	0.52	0.51	0.58	0.59	0.38	0.30
AIC	-5720.3	-5607.9	-5608.7	-4529.0	-4567.0	-4565.6
BIC	-5697.3	-5577.4	-5570.5	-4508.1	-4539.2	-4530.8
Sample	1971-1999	1972-1999	1972-1999	2000-2019	2000-2019	2000-2019

Table 3: Predictable yield-curve dynamics following an impulse to short-term interest rates. This table reports the estimated regression coefficients in equation (2.3) for the Aug-1971 to Dec-1999 and Jan-2000 to Dec-2019 samples. For $h = 3, 6, 9,$ and, 12-months changes, we show results for 10-year yields ($z_t = y_t^{(10)}$), 10-year forward rates ($z_t = f_t^{(10)}$), level ($z_t = L_t$), and slope ($z_t = S_t$). We report Newey-West standard errors in brackets using a lag truncation parameter of $\lceil 1.5 \times h \rceil$. Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Pre-2000				Post-2000			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep. var with h	3	6	9	12	3	6	9	12
Dependent variable: $y_{t+h}^{(10)} - y_t^{(10)}$								
$L_t - L_{t-1}$	-0.03	0.10	0.04	0.23*	-0.11	-0.36*	-0.41*	-0.20
	[0.12]	[0.13]	[0.11]	[0.12]	[0.13]	[0.19]	[0.20]	[0.20]
$S_t - S_{t-1}$	0.03	0.30	0.34	0.55**	-0.09	-0.16	-0.11	-0.16
	[0.18]	[0.19]	[0.20]	[0.26]	[0.15]	[0.17]	[0.15]	[0.20]
L_t	-0.06*	-0.11*	-0.17*	-0.23**	-0.07**	-0.14***	-0.19***	-0.23***
	[0.03]	[0.06]	[0.08]	[0.10]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.13*	-0.27**	-0.45***	-0.62***	-0.10**	-0.20**	-0.28**	-0.35**
	[0.07]	[0.11]	[0.16]	[0.19]	[0.05]	[0.09]	[0.11]	[0.12]
Adj.R^2	0.03	0.08	0.14	0.20	0.05	0.11	0.16	0.20
Dependent variable: $f_{t+h}^{(10)} - f_t^{(10)}$								
$L_t - L_{t-1}$	-0.08	0.09	-0.05	0.12	-0.30***	-0.52***	-0.71***	-0.74***
	[0.12]	[0.11]	[0.10]	[0.12]	[0.11]	[0.17]	[0.14]	[0.15]
$S_t - S_{t-1}$	0.01	0.18	0.16	0.34	-0.14	-0.11	-0.02	-0.17
	[0.18]	[0.19]	[0.20]	[0.22]	[0.16]	[0.20]	[0.19]	[0.21]
L_t	-0.04	-0.09	-0.14*	-0.19**	-0.03	-0.05	-0.06	-0.06
	[0.03]	[0.05]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.16**	-0.31***	-0.48***	-0.66***	-0.09*	-0.17*	-0.27**	-0.35**
	[0.06]	[0.11]	[0.15]	[0.19]	[0.05]	[0.09]	[0.11]	[0.14]
Adj.R^2	0.04	0.10	0.17	0.24	0.04	0.09	0.17	0.23
Dependent variable: $L_{t+h} - L_t$								
$L_t - L_{t-1}$	0.07	0.26	0.18	0.60***	0.66***	0.86**	1.33**	1.70***
	[0.24]	[0.21]	[0.20]	[0.19]	[0.17]	[0.38]	[0.53]	[0.59]
$S_t - S_{t-1}$	0.13	0.64**	0.57*	1.07**	-0.23**	-0.38*	-0.59*	-0.58
	[0.30]	[0.30]	[0.31]	[0.49]	[0.11]	[0.21]	[0.29]	[0.39]
L_t	-0.09*	-0.18**	-0.26**	-0.36***	-0.08**	-0.19**	-0.28***	-0.38**
	[0.05]	[0.08]	[0.11]	[0.12]	[0.03]	[0.07]	[0.11]	[0.15]
S_t	-0.00	-0.09	-0.24	-0.33	-0.03	-0.07	-0.04	-0.01
	[0.11]	[0.15]	[0.20]	[0.25]	[0.04]	[0.09]	[0.15]	[0.21]
Adj.R^2	0.03	0.08	0.12	0.17	0.20	0.26	0.36	0.42
Dependent variable: $S_{t+h} - S_t$								
$L_t - L_{t-1}$	-0.10	-0.16	-0.14	-0.37***	-0.77***	-1.23***	-1.74***	-1.90***
	[0.16]	[0.12]	[0.12]	[0.11]	[0.16]	[0.33]	[0.39]	[0.51]
$S_t - S_{t-1}$	-0.10	-0.34*	-0.23	-0.51	0.14	0.22	0.48*	0.43
	[0.17]	[0.18]	[0.16]	[0.30]	[0.12]	[0.22]	[0.27]	[0.32]
L_t	0.03	0.07*	0.10**	0.13***	0.01	0.05	0.09	0.15
	[0.03]	[0.03]	[0.04]	[0.04]	[0.03]	[0.07]	[0.10]	[0.14]
S_t	-0.12*	-0.18*	-0.21*	-0.29**	-0.07*	-0.13	-0.24	-0.34
	[0.07]	[0.10]	[0.11]	[0.13]	[0.04]	[0.09]	[0.13]	[0.20]
Adj.R^2	0.08	0.16	0.20	0.28	0.16	0.24	0.37	0.43
N	340	340	340	340	237	234	231	228
Sample	1971-1999	1971-1999	1971-1999	1971-1999	2000-2019	2000-2019	2000-2019	2000-2019

Table 4: Estimates of predictive equations for bond excess returns. This table reports the estimated regression coefficients in equation (2.5) using monthly data from the Aug-1971 to Dec-1999 and Jan-2000 to Dec-2019 samples. We report results various return forecast horizon (k). The yield on k-month Treasury bills, $y_t^{(k/12)}$, is from the yield curve estimates in Gürkaynak et al. (2007). However, this curve is based on coupon securities with at least three months to maturity and does not fit the very short end of the curve well in the pre-2000 data. Therefore, we take the 1-month bill yield from Ken French's website for the pre-2000 sample. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. For $k = 1$-month returns, we report heteroskedasticity robust standard errors are in brackets. For $k = 3$ and 6-month returns, we report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 5 and 9 months, respectively. In this case, p-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$rx_{t\to t+k}^{(10)}$ for $k =$	1	1	1	3	3	3	6	6	6
Panel A: Pre-2000									
L_t	0.17	0.18	0.22*	0.53*	0.54	0.65*	0.99*	1.01	1.25**
	[0.13]	[0.13]	[0.12]	[0.31]	[0.33]	[0.34]	[0.57]	[0.62]	[0.60]
S_t	0.55**	0.61**	0.66***	1.57**	1.91***	2.07***	3.17***	3.62***	3.98***
	[0.24]	[0.24]	[0.24]	[0.64]	[0.65]	[0.64]	[1.04]	[1.13]	[1.09]
$L_t - L_{t-6}$		0.10	-0.18		0.59	-0.22		0.79	-1.00
		[0.19]	[0.29]		[0.49]	[0.54]		[0.69]	[0.90]
$S_t - S_{t-6}$			-0.57			-1.67**			-3.67***
			[0.44]			[0.81]			[1.30]
Adj.R^2	0.02	0.01	0.02	0.05	0.06	0.07	0.11	0.12	0.17
N	341	335	335	341	335	335	341	335	335
Sample	1971-1999	1972-1999	1972-1999	1971-1999	1972-1999	1972-1999	1971-1999	1972-1999	1972-1999
Panel B: Post-2000									
L_t	0.30***	0.32***	0.28**	0.73***	0.81***	0.71**	1.37***	1.50***	1.34**
	[0.11]	[0.11]	[0.11]	[0.27]	[0.28]	[0.30]	[0.44]	[0.47]	[0.51]
S_t	0.56***	0.63***	0.53**	1.41***	1.65***	1.42**	2.71***	3.07***	2.73***
	[0.21]	[0.20]	[0.21]	[0.48]	[0.52]	[0.58]	[0.83]	[0.93]	[0.95]
$L_t - L_{t-6}$		0.33	0.62*		0.98**	1.66***		1.33*	2.39**
		[0.24]	[0.33]		[0.46]	[0.61]		[0.73]	[1.07]
$S_t - S_{t-6}$			0.48			1.12			1.76
			[0.35]			[0.72]			[1.22]
Adj.R^2	0.03	0.03	0.03	0.08	0.10	0.12	0.16	0.18	0.20
N	239	239	239	237	237	237	234	234	234
Sample	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019

Table 5: The role of mortgage refinancing: Evidence from mortgage-related quantities. Panels A and B report the estimated coefficients for equation (4.2) where the dependent variable is the mortgage refinancing disincentive: $X_t = y_t^M - c_t^M$ or the duration of the Barclay's MBS index: $X_t = DUR_t^M$. Panels C and D report the estimated coefficients using equation (4.3) to forecast 3-month returns using either the mortgage refinancing disincentive or the duration. All regressions are estimated using monthly data for the Jan-1976 to Dec-1999 and Jan-2000 to Dec-2019 samples. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. We report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 9 months in Panels A and B and 5 months in Panels C and D. p-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	(1)	(2)	(3)	(4)	(5)	(6)		(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Dependent Variable: $y_t^M - c_t^M$							Panel B. Dependent Variable: DUR_t^M						
L_t	0.76***	0.76***	0.72***	0.23***	0.24***	0.20***		0.16***	0.16***	0.15***	0.05	0.08	0.01
	[0.07]	[0.06]	[0.06]	[0.05]	[0.05]	[0.05]		[0.04]	[0.04]	[0.04]	[0.09]	[0.09]	[0.08]
S_t	0.17	0.13	0.05	-0.03	0.00	-0.08		0.15**	0.08	0.06	-0.06	0.03	-0.11
	[0.10]	[0.11]	[0.10]	[0.07]	[0.06]	[0.06]		[0.08]	[0.08]	[0.08]	[0.14]	[0.12]	[0.12]
$L_t - L_{t-6}$		-0.07	0.18*		0.17**	0.42***			-0.13**	-0.06		0.44***	0.90***
		[0.08]	[0.10]		[0.08]	[0.07]			[0.05]	[0.06]		[0.13]	[0.12]
$S_t - S_{t-6}$			0.56***			0.42***				0.17**			0.76***
			[0.16]			[0.06]				[0.08]			[0.12]
Adj.R^2	0.86	0.86	0.88	0.52	0.55	0.67		0.23	0.29	0.30	0.02	0.16	0.36
Panel C. Dependent Variable: $rx_{t\rightarrow t+3}^{(10)}$							Panel D. Dependent Variable: $rx_{t\rightarrow t+3}^{(10)}$						
L_t	0.67*	1.10*	0.86	0.71**	0.21	0.34		0.67*	0.82**	0.88**	0.71**	0.68**	0.70**
	[0.36]	[0.59]	[0.59]	[0.30]	[0.30]	[0.34]		[0.36]	[0.36]	[0.36]	[0.30]	[0.28]	[0.30]
S_t	2.20**	1.83**	2.21***	1.42**	1.50***	1.57***		2.20***	1.98***	2.28***	1.42**	1.51***	1.55***
	[0.75]	[0.77]	[0.76]	[0.58]	[0.49]	[0.57]		[0.75]	[0.71]	[0.73]	[0.58]	[0.50]	[0.58]
$L_t - L_{t-6}$	-0.30		-0.25	1.66***		0.88		-0.30		-0.38	1.66***		0.71
	[0.56]		[0.60]	[0.61]		[0.79]		[0.56]		[0.55]	[0.61]		[0.83]
$S_t - S_{t-6}$	-1.70*		-1.55	1.12		0.35		-1.70*		-1.45	1.12		0.31
	[0.92]		[1.08]	[0.72]		[0.80]		[0.92]		[0.90]	[0.72]		[0.85]
$y_t^M - c_t^M$		-0.74	-0.27		2.34***	1.82*	DUR_t^M		-1.77*	-1.45*		1.31***	1.05**
		[0.71]	[0.77]		[0.80]	[0.98]			[0.94]	[0.81]		[0.35]	[0.51]
Adj.R^2	0.07	0.05	0.06	0.12	0.13	0.14		0.07	0.07	0.08	0.12	0.14	0.14
Sample	1976-1999	1976-1999	1976-1999	2000-2019	2000-2019	2000-2019		1976-1999	1976-1999	1976-1999	2000-2019	2000-2019	2000-2019

Table 6: The role of investor over-extrapolation: Evidence from bond mutual fund flows. Data on flows into taxable bond mutual funds is from the Investment Company Institute. Letting $\%FLOW_t = FLOW_t/TNA_{t-1}$ denote the percentage flow in month t, the 3-month cumulative percentage flow is $\%FLOW_{t-3\to t} = (1+\%FLOW_t)(1+\%FLOW_{t-1})(1+\%FLOW_{t-2})-1$. Panels A reports the estimated regression coefficients for equation (4.2) using $X_t = \%\Delta FLOW_{t-3\to t}$. Panels B and C report the estimated regression coefficients when we use $X_t = \%\Delta FLOW_{t-3\to t}$ in equation (4.3) to forecast 3-month returns. We estimate these regressions using monthly data for the Apr1984 to Dec-1999 and Jan-2000 to Dec-2019 subsamples. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. We report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 9 months in Panel A and 5 months in Panel B. p-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Pre-2000			Post-2000		
	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable: $FLOW_{t-3\to t}$						
L_t	1.96**	2.15***	2.24***	0.06	0.01	0.04
	[0.89]	[0.67]	[0.69]	[0.23]	[0.20]	[0.23]
S_t	4.08**	3.24**	3.92**	0.60	0.44	0.51
	[1.64]	[1.27]	[1.45]	[0.37]	[0.36]	[0.39]
$L_t - L_{t-6}$		-4.00***	-5.29***		-0.77**	-1.00**
		[1.04]	[1.29]		[0.32]	[0.46]
$S_t - S_{t-6}$			-4.31**			-0.38
			[2.15]			[0.45]
Adj.R^2	0.24	0.43	0.50	0.09	0.16	0.16
N	189	189	189	240	240	240
Dependent Variable: $rx^{(10)}_{t\to t+3}$						
L_t	1.27***	1.13***	1.04***	0.71**	0.76***	0.72**
	[0.30]	[0.29]	[0.35]	[0.30]	[0.27]	[0.31]
S_t	1.88***	1.37**	1.47*	1.42**	1.63***	1.55**
	[0.66]	[0.64]	[0.74]	[0.58]	[0.50]	[0.60]
$L_t - L_{t-6}$	-0.01		0.54	1.66***		1.42**
	[0.61]		[0.89]	[0.61]		[0.64]
$S_t - S_{t-6}$	-1.01		-0.56	1.12		1.02
	[0.88]		[0.98]	[0.72]		[0.73]
$FLOW_{t-3\to t}$		0.07	0.10		-0.34**	-0.24
		[0.08]	[0.11]		[0.14]	[0.15]
Adj.R^2	0.16	0.17	0.17	0.12	0.10	0.12
N	189	189	189	237	237	237
Sample	1984-1999	1984-1999	1984-1999	2000-2019	2000-2019	2000-2019

Table 7: Economic news and subsequent changes in forward rates. This table reports the regression coefficents in equation (5.1) using monthly data from the Aug1971 to Dec-1999 and Jan-2000 to Dec2019 samples. Newey-West (1987) standard errors are in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Pre-2000				Post-2000			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$f^{(10)}_{t+h} - f^{(10)}_{t}$ with h	3	6	9	12	3	6	9	12
Panel A								
$L_t - L_{t-1}$	-0.09	0.11	-0.04	0.12	-0.30***	-0.52***	-0.71***	-0.74***
	[0.12]	[0.12]	[0.12]	[0.13]	[0.11]	[0.17]	[0.16]	[0.15]
$S_t - S_{t-1}$	0.03	0.29	0.19	0.35	-0.14	-0.11	-0.02	-0.17
	[0.19]	[0.20]	[0.22]	[0.24]	[0.16]	[0.20]	[0.20]	[0.21]
L_t	-0.04	-0.08	-0.13	-0.18*	-0.03	-0.05	-0.06	-0.06
	[0.03]	[0.06]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.11	-0.24*	-0.37**	-0.52**	-0.09*	-0.17*	-0.27**	-0.35**
	[0.07]	[0.13]	[0.17]	[0.21]	[0.05]	[0.09]	[0.11]	[0.14]
Adj.R^2	0.02	0.05	0.09	0.14	0.04	0.09	0.17	0.23
Panel B								
$News_t$	-0.29	-0.12	-0.80***	-0.46	-0.45**	-0.63*	-0.82***	-0.67**
	[0.21]	[0.30]	[0.24]	[0.26]	[0.18]	[0.34]	[0.27]	[0.30]
L_t	-0.04	-0.08	-0.12	-0.17*	-0.04	-0.05	-0.06	-0.06
	[0.03]	[0.06]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.10	-0.23*	-0.37*	-0.52**	-0.10**	-0.18**	-0.26**	-0.35**
	[0.07]	[0.12]	[0.17]	[0.20]	[0.04]	[0.08]	[0.11]	[0.13]
Adj.R^2	0.02	0.05	0.11	0.14	0.04	0.08	0.15	0.21
Panel C								
$News_t$	-0.11	-0.11	-0.99**	-0.72**	-0.28	-0.05	0.15	0.38
	[0.34]	[0.35]	[0.42]	[0.29]	[0.29]	[0.47]	[0.36]	[0.29]
$L_t - L_{t-1}$	-0.06	0.14	0.19	0.29	-0.19	-0.50*	-0.77***	-0.89***
	[0.16]	[0.15]	[0.16]	[0.17]	[0.17]	[0.24]	[0.22]	[0.18]
$S_t - S_{t-1}$	0.03	0.29	0.23	0.38	-0.15	-0.11	-0.02	-0.15
	[0.19]	[0.20]	[0.20]	[0.23]	[0.16]	[0.20]	[0.18]	[0.21]
L_t	-0.04	-0.08	-0.12	-0.17*	-0.03	-0.05	-0.06	-0.06
	[0.03]	[0.06]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.11	-0.24*	-0.37*	-0.52**	-0.09*	-0.17*	-0.27**	-0.35**
	[0.07]	[0.13]	[0.17]	[0.21]	[0.05]	[0.09]	[0.11]	[0.14]
Adj.R^2	0.01	0.05	0.10	0.14	0.04	0.08	0.16	0.23
N	240	240	240	240	237	234	231	228
Sample	1980-1999	1980-1999	1980-1999	1980-1999	2000-2019	2000-2019	2000-2019	2000-2019
Panel D								
$News_{t,\text{FOMC}}$	0.11	-0.01	-1.89	-0.44	-0.99**	-0.74*	-1.18	0.09
	[0.77]	[1.04]	[1.56]	[1.56]	[0.46]	[0.39]	[0.75]	[0.56]
$News_{t,\text{Other}}$	-0.25	-0.17	-0.45	-0.32	-0.02	0.21	0.65	0.49
	[0.33]	[0.45]	[0.48]	[0.43]	[0.29]	[0.57]	[0.41]	[0.41]
$L_t - L_{t-1}$	0.09	0.03	0.03	-0.00	-0.20	-0.51**	-0.79***	-0.89***
	[0.18]	[0.21]	[0.24]	[0.27]	[0.16]	[0.24]	[0.20]	[0.18]
$S_t - S_{t-1}$	0.20	0.30	0.33	0.55**	-0.17	-0.13	-0.05	-0.16
	[0.24]	[0.27]	[0.23]	[0.25]	[0.16]	[0.20]	[0.18]	[0.21]
L_t	-0.07*	-0.13*	-0.18*	-0.20*	-0.03	-0.05	-0.06	-0.06
	[0.03]	[0.06]	[0.09]	[0.10]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.07	-0.17	-0.28	-0.42*	-0.10**	-0.18*	-0.28**	-0.36**
	[0.07]	[0.14]	[0.19]	[0.21]	[0.05]	[0.09]	[0.11]	[0.14]
Adj.R^2	0.03	0.08	0.14	0.16	0.05	0.09	0.18	0.23
N	216	216	216	216	237	234	231	228
Sample	1982-1999	1982-1999	1982-1999	1982-1999	2000-2019	2000-2019	2000-2019	2000-2019

Figure 1: Regressions of changes in long-term yields on short-term rates. This figure plots the estimated regression coefficients β_h from equation (1.1) versus horizon (h) for the pre-2000 and post-2000 sample: $y_{t+h}^{(10)} - y_t^{(10)} = \alpha_h + \beta_h (y_{t+h}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+h}$. The dependent variable is the h-month change in the 10-year nominal zero-coupon U.S. Treasury yield and the independent variable is the h-month change in the 1-year nominal zero-coupon U.S. Treasury yield. Changes are considered with daily data (plotted as $h = 0$ in the figure) and with monthly data using $h = 1$, ..., 12-month changes.

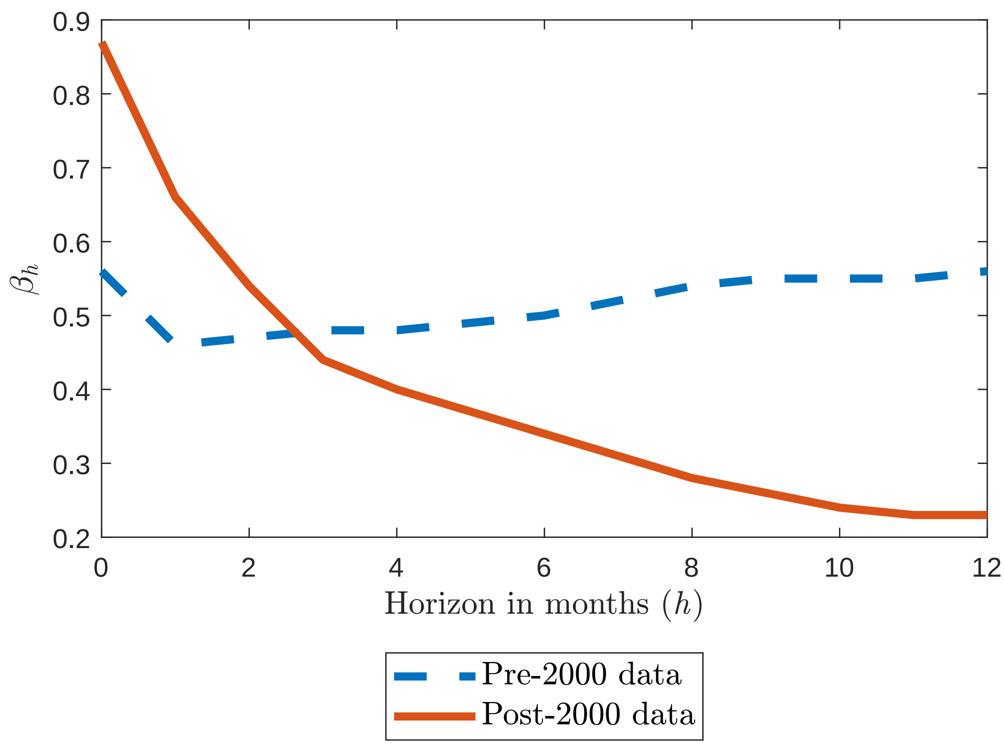


Figure 2: Rolling regression estimates of equations (1.1) and (1.2) This figure plots rolling estimates of the slope coefficients in equations (1.1) and (1.2) with $h = 12$-month changes using 10-year rolling windows for estimation. Results are plotted against the midpoint of the 10-year rolling window. 95% confidence intervals are included (shaded areas), formed using Newey-West standard errors with a lag truncation parameter of 18 and 95% critical values from the asymptotic theory of Kiefer and Vogelsang (2005). Specifically, the 95% confidence interval is ± 2.41 times the estimated standard errors as opposed to ± 1.96 under traditional asymptotic theory.

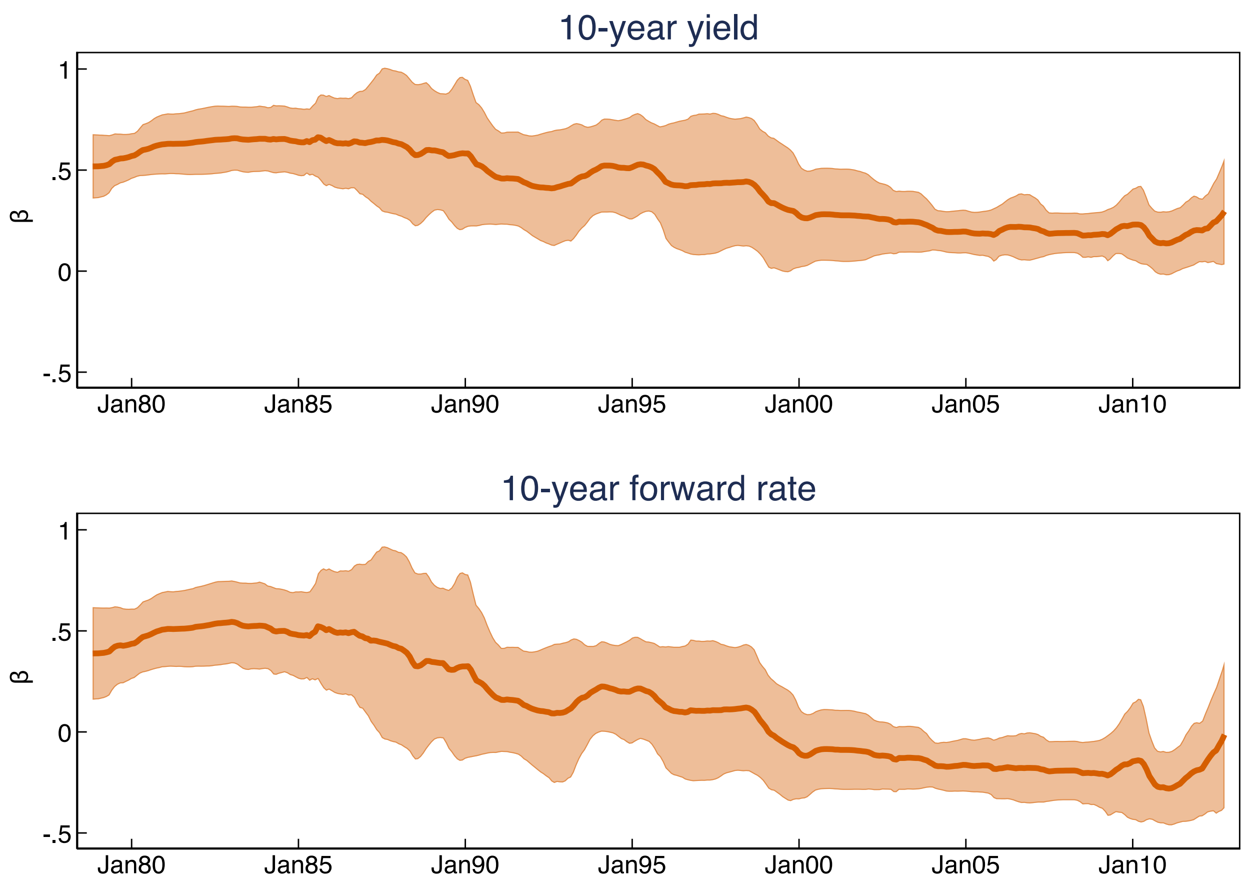


Figure 3: Break tests for equations (1.1) and (1.2) This figure plots the Wald test statistic for each possible break date in equations (1.1) and (1.2) with h = 12-month changes from a fraction 15% of the way through the sample to 85% of the way through the sample. The horizontal red dashed lines denote 10%, 5%, and 1% critical values for the maximum of these Wald statistics as in Andrews (1993). Our Wald tests use a Newey and West (1987) variance matrix with a lag truncation parameter of 18. To address the tendency for tests based on the Newey-West variance estimator to over-reject in finite samples, we use the Cho and Vogelsang (2017) critical values for a null of no structural break. The Cho and Vogelsang (2017) critical values are based on the asymptotic theory of Kiefer and Vogelsang (2005) and are slightly larger than the traditional critical values from Andrews (1993).

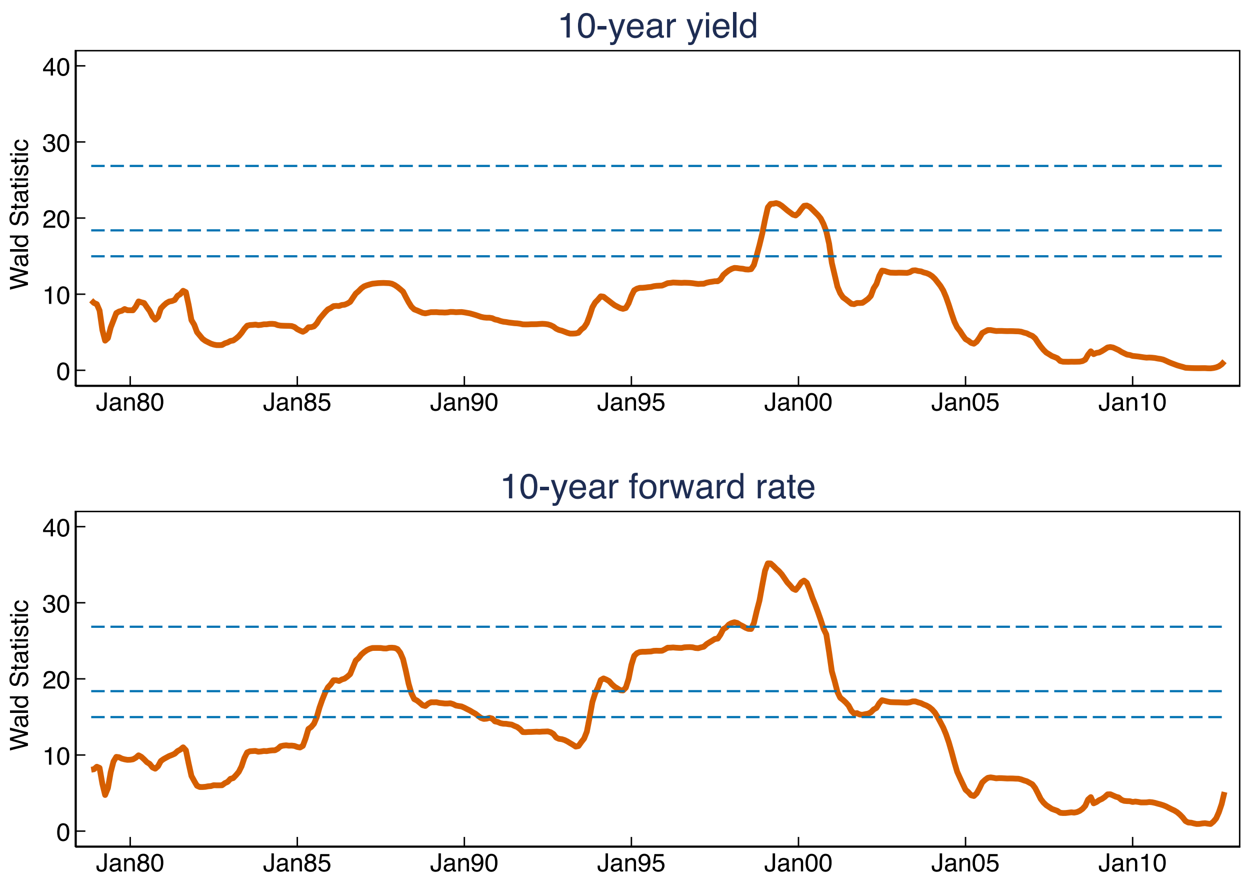


Figure 4: Predictable yield-curve dynamics following an impulse to short-term interest rates. The figures plot the coefficients $\delta_3^{(h)}$ versus horizon h from estimating equations (2.3) for various horizons $h = 1..., 12$-months in the pre-2000 and post-2000 samples. We show results for 10-year yields ($z_t = y_t^{(10)}$), 10-year forward rates ($z_t = f_t^{(10)}$), level ($z_t = L_t$) and slope ($z_t = S_t$). 95% confidence intervals are shown as dashed lines, formed using Newey-West standard errors and 95% critical values from the asymptotic theory of Kiefer and Vogelsang (2005). We use a Newey-West lag truncation parameter of 0 for $h = 1$ and $\lceil 1.5 \times h \rceil$ for $h > 1$.

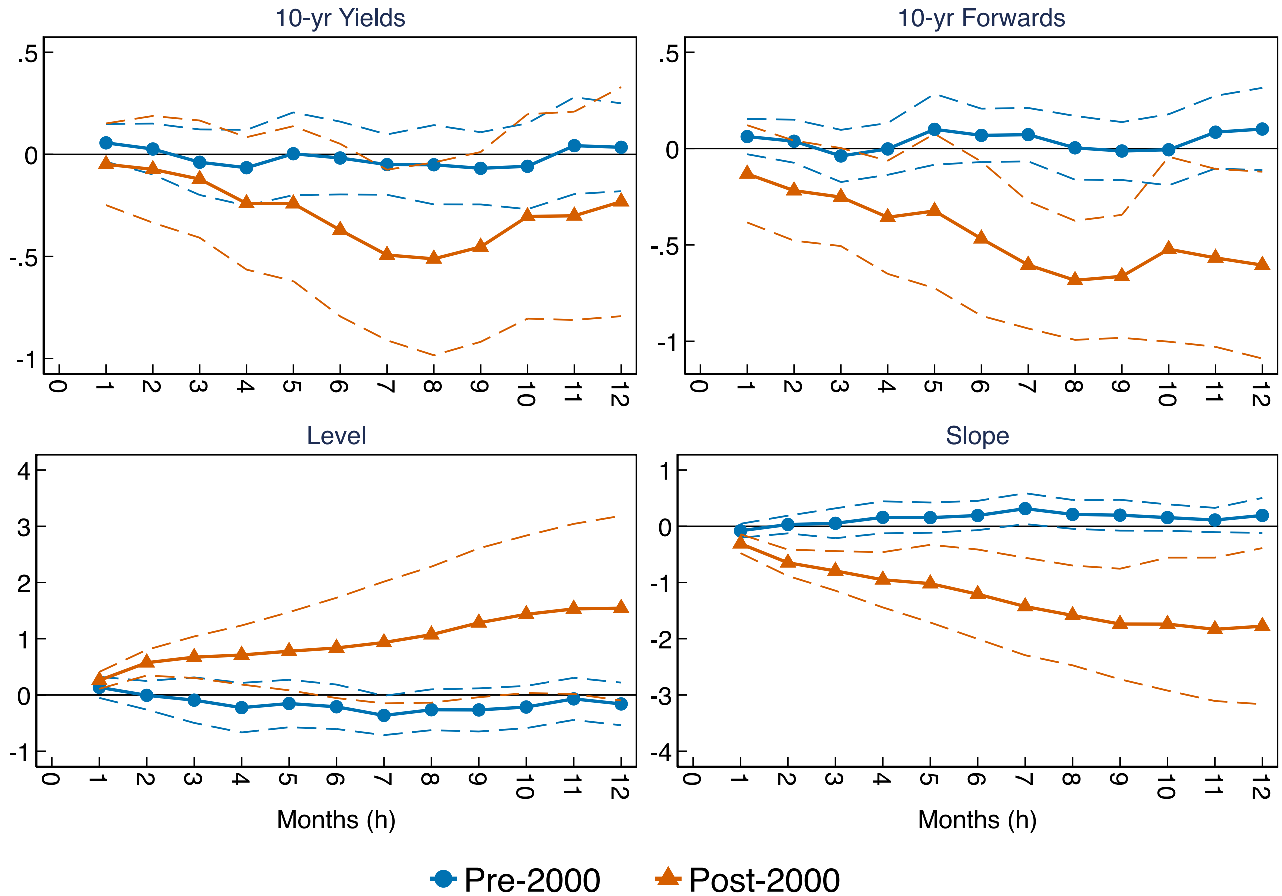
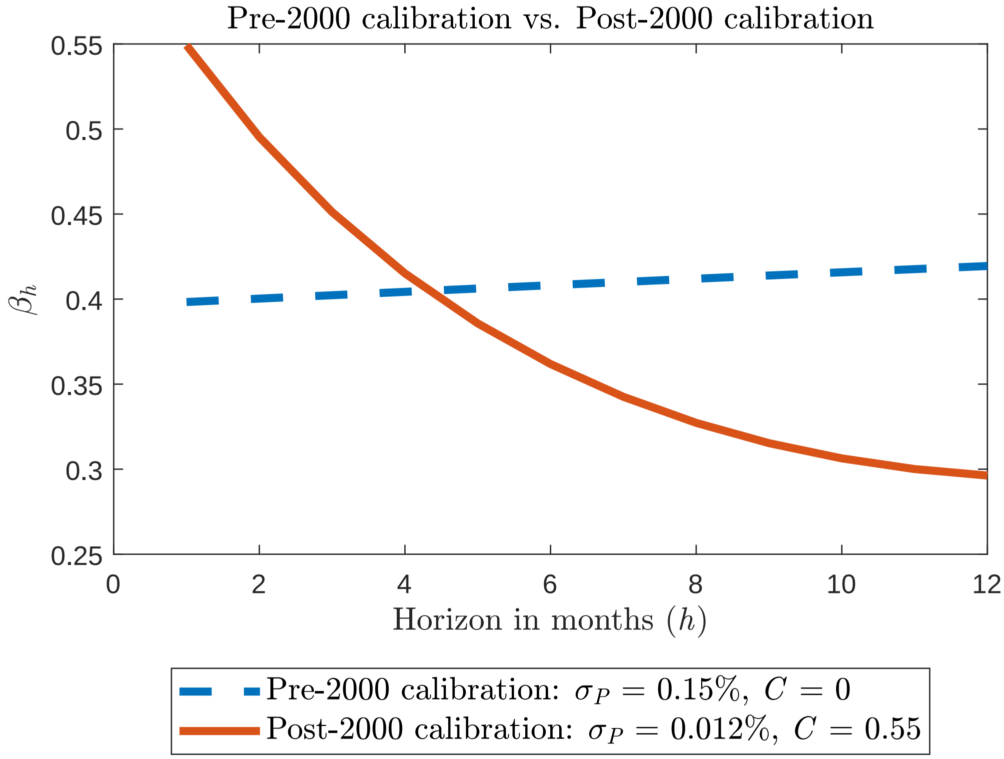
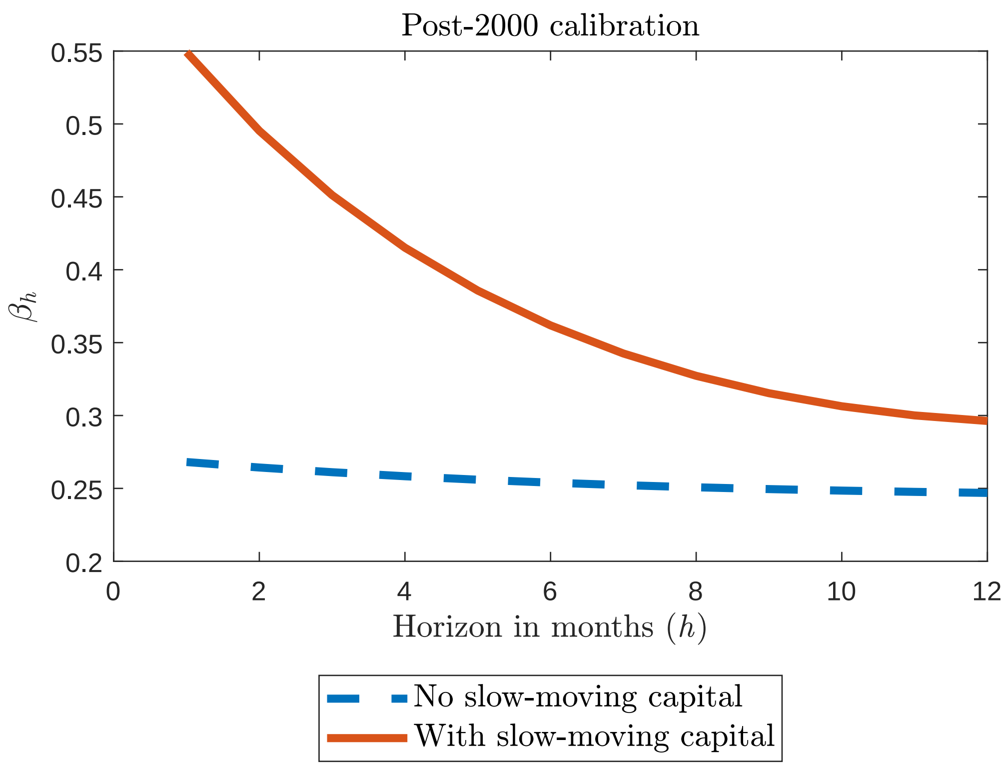


Figure 5: Model-implied coefficients β_h versus horizon (h) in months. The first figure shows the model-implied β_h coefficients from equation (3.11) for the pre-2000 and post-2000 calibrations discussed in the text. The second figure isolates the role of slow-moving capital in the post-2000 calibration, alternately setting $q = 100\%$ ("No slow-moving capital") and $q = 30\%$ ("With slow-moving capital").





Appendix of supplementary materials for:

RATE-AMPLIFYING DEMAND AND THE EXCESS SENSITIVITY OF LONG-TERM RATES

Samuel G. Hanson, David O. Lucca and Jonathan H. Wright

Contents

A Additional empirical results

This Appendix collects several supplementary empirical results that are mentioned in the main text.

A.1 Additional empirical results for Section 1

A.1.1 Long-term private yields

Table A.1 shows that we obtain very similar results for the U.S. using a host of long-term private yields as the dependent variable in equation (1.1). We report results for both Aaa and Baa seasoned corporate bond yields from Moodys, the 10-year swap yield, and the yield on current coupon Fannie Mae MBS (FNCL). For of all these long-term private yields, the sensitivity to changes in 1-year Treasury rate is similar at high- and low- frequencies in the pre-2000 sample. Post-2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines significantly.

A.1.2 Different short-term rates

Table A.2 shows that similar results for the U.S. hold using different proxies for the short rate in equation (1.1) of the main text—i.e., using changes in 3-month, 6-month, or 2-year Treasury yields as the independent variable in equation (1.1). (The results in Table 1 of main the text correspond to those reported in Table A.2 for the 1-year Treasury rate.) For all of these short rate proxies, the sensitivity of 10-year yields to changes in short rates was similar irrespective of frequency prior to 2000. After 2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines.

Table A.1: Regression of changes in corporate bond, swap and secondary mortgage market rates on short-term rates. This table reports the estimated slope coefficients from equation (1.1) in the main text for each reported sample. Specifically, the dependent variables are long-term corporate bond yields with Moody's ratings of Baa and Aaa (labled BAA and AAA), the 10-year swap yield (SWAP10), and the yield on current-coupon Fannie Mae mortgage-backed-securities (FNCL). The independent variable in all regressions is the change in the 1-year nominal Treasury yield. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. We report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $1.5\times(h-1)$ (rounded to the nearest integer). Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	(1) BAA	(2) AAA	(3) SWAP10	(4) FNCL	(5) BAA	(6) AAA	(7) SWAP10	(8) FNCL
Daily	0.48***	0.51***	0.73***	0.95***	0.56***	0.58***	0.95***	0.90***
	[0.02]	[0.02]	[0.04]	[0.04]	[0.04]	[0.04]	[0.03]	[0.04]
Monthly	0.52***	0.59***	0.82***	0.92***	0.24*	0.37***	0.83***	0.75***
	[0.05]	[0.05]	[0.07]	[0.06]	[0.13]	[0.10]	[0.10]	[0.10]
Quarterly	0.49***	0.57***	0.80***	0.84***	0.08	0.24***	0.63***	0.54***
	[0.06]	[0.06]	[0.07]	[0.07]	[0.12]	[0.07]	[0.09]	[0.09]
Yearly	0.41***	0.55***	0.67***	0.72***	-0.01	0.11	0.39***	0.34***
	[0.08]	[0.09]	[0.07]	[0.10]	[0.12]	[0.06]	[0.05]	[0.07]
Sample	1986-1999	1983-1999	1988-1999	1984-1999	2000-2019	2000-2019	2000-2019	2000-2019

Table A.2: Regressions of changes in long-term rates on short-term rates. This table reports the estimated regression coefficients from equation (1.1) in the main text for each reported sample. The dependent variable is the change in the 10-year nominal U.S. Treasury yield. The independent variable is alternately the change in the 3-month, 6-month, 1-year, and 2-year nominal U.S. Treasury yield. Changes are considered with daily data, and with monthly data using monthly ($h = 1$) and annual ($h = 12$) horizons. In the 1971-1999 monthly sample, time t runs from 1971m8 to 1999m12. In the 2000-2019 monthly sample, t runs from 2000m1 to 2019m12. For $h > 1$, we report Newey-West (1987) standard errors are in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Pre-2000			Post-2000		
	(1) Daily	(2) Monthly	(3) Annual	(4) Daily	(5) Monthly	(6) Annual
Short rate = 3-month	0.16***	0.26***	0.39***	0.27***	0.24**	0.17***
	[0.02]	[0.03]	[0.05]	[0.02]	[0.10]	[0.04]
Short rate = 6-month	0.41***	0.37***	0.46***	0.65***	0.45***	0.18***
	[0.02]	[0.03]	[0.06]	[0.03]	[0.11]	[0.05]
Short rate = 1-year	0.56***	0.46***	0.56***	0.87***	0.66***	0.23***
	[0.02]	[0.04]	[0.05]	[0.03]	[0.11]	[0.05]
Short rate = 2-year	0.65***	0.57***	0.68***	0.87***	0.78***	0.34***
	[0.01]	[0.04]	[0.05]	[0.02]	[0.09]	[0.07]
Observations	7062	341	341	5002	239	228

A.1.3 Hansen-Hodrick standard errors

One might be concerned about our use of overlapping changes in equations (1.1) and (1.2) when $h > 1$. In the main text, we computed Newey and West (1987) standard errors with a lag truncation parameter of $\lceil 1.5 \times h \rceil$. And, to address the tendency for statistical tests based on Newey and West (1987) standard errors to over-reject in finite samples, we compute p-values using the asymptotic theory of Kiefer and Vogelsang (2005) which gives more conservative p-values and has better finite-sample properties than traditional Gaussian asymptotic theory.

Table A.3 report the results from estimating equations (1.1) and (1.2) when $h > 1$, but instead using Hansen and Hodrick (1980) standard errors with a lag truncation parameter equal to h. Hansen and Hodrick (1980) standard errors are sometimes preferred when working with overlapping data, although they have the disadvantage that, unlike the Newey and West (1987) counterpart, the estimated variance-covariance matrix is not guaranteed to be positive-definite. The point estimates are identical by construction to those in Table 1 of the paper and the standard errors are nearly identical to those reported in the main text. Thus, we would draw almost identical inferences using Hansen and Hodrick (1980) standard errors instead of Newey and West (1987) standard errors.

Table A.3: Regressions of changes in long-term rates on short-term rates with Hansen-Hodrick (1980) standard errors. This table reports the estimated regression coefficients from equations (1.1) and (1.2) for $h > 1$. We report Hansen and Hodrick (1980) with lag truncation parameter of h. By construction the point estimates are the same as in Table 1. Results are not included for daily or monthly data, because there are no overlapping changes at those frequencies. Significance: $^{*}p < 0.1$, ** $p < 0.05$, $^{***}p < 0.01$.

Panel A. Ten-year zero coupon yields and IC				
	Nominal	Nominal	Real	IC
Quarterly	0.48***	0.44***	0.22**	0.22
	[0.04]	[0.07]	[0.11]	[0.14]
Semi-Annual	0.50***	0.34***	0.21**	0.13
	[0.05]	[0.08]	[0.09]	[0.10]
Yearly	0.56***	0.23***	0.15**	0.08*
	[0.06]	[0.05]	[0.06]	[0.04]
Panel B. Ten-year instantaneous forward yields and IC				
	Nominal	Nominal	Real	IC
Quarterly	0.31***	0.06	0.09*	-0.03
	[0.05]	[0.09]	[0.05]	[0.05]
Semi-Annual	0.33***	-0.02	0.04	-0.06
	[0.06]	[0.08]	[0.04]	[0.05]
Yearly	0.39***	-0.13	-0.02	-0.11**
	[0.07]	[0.06]	[0.05]	[0.05]
Sample	1971-1999	2000-2019	2000-2019	2000-2019

A.1.4 Estimates using non-overlapping data

Here we show the estimates and our inferences are quite similar if we simply use non-overlapping h-month changes. Consider our estimates of β_{12}—the coefficient from a regression involving 12-month changes—where concerns about the use of overlapping data are greatest. Table A.4 reports the results for estimating equations (1.1) and (1.2) with non-overlapping data for each possible month in the year—i.e., a regression in changes from month m of one year to month m of the next year—along with the averages across all 12 months. Naturally, the results differ slightly across the individual months. However, averaging across all 12 months, in the pre-2000 data, the regression has an average $\beta_{12} = 0.561$ with an average robust standard error of 0.076**;** in the post-2000 data, the regression has an average point estimate of $\beta_{12} = 0.222$ with an average standard error of 0.088. Thus, as one would expect, the average point estimate across these 12 separate estimators that each use non-overlapping changes are almost identical to our baseline estimator that use overlapping changes reported in Table 1 of the main text. (In Table 1, we have $\beta_{12} = 0.557$ in the pre-2000 period and $\beta_{12} = 0.230$ in the post-2000 period.) However, the average of the standard errors across these 12 estimators is larger than the Newey-West standard error from our baseline estimates which are 0.052 in the pre-2000 period and 0.053 post-2000. We attribute the larger average standard error to the loss of efficiency from discarding some of the data. Specifically, the standard error of the average of the 12 estimates will be less than average of the 12 standard errors to the extent the 12 estimates are imperfectly correlated.

Nonetheless, we draw similar conclusions even using this highly conservative approach that deliberately discards data. The difference between the resulting pre-2000 and post-2000 estimates of β_{12} in yields are significant at the 5% level for 10 of the 12 monthly estimators using non-overlapping changes. And the difference between the two estimates of β_{12} in forward rates are significant at the 5% level for all 12 of the monthly estimators using non-overlapping changes.

A.1.5 U.S. evidence prior to the Great Inflation

Our baseline findings on the sensitivity of long-term U.S. Treasury yields to short-term Treasury yields use data beginning in 1971. Data on the Treasury term structure prior to the 1970s is far more limited because the U.S. Treasury did not regularly issue large quantities of debt at fixed, long-term maturities (Gürkaynak et al., 2007). Nevertheless, it is useful to examine the sensitivity of long-term yields to short-term yields prior to the Great Inflation, which ran from the late-1960s to the mid-1980s. Specifically, one plausible explanation for the strong sensitivity of long-term nominal yields during the 1971-1999 sample is that this was a period when long-run inflation expectations became unanchored and were continuously being revised in response to news (Gürkaynak et al., 2005). Since inflation-expectations have become firmly anchored in the past two decades, it is useful to compare the patterns we see in the post-2000 data to the those witnessed prior to the Great Inflation—another period when inflationary expectations were also more firmly anchored.

To examine the sensitivity of long-term yields prior to the Great Inflation, we use data on 10-year and 1-year Constant Maturity Treasury (CMT) yields from the Federal Reserve's H.15 statistical release, which are available on a monthly basis dating back to Apr 1953. As in equation (1.1), we use these data to estimate the coefficient β_h from a regression of h-month changes in the 10-year CMT yield on h-month changes in the 1-year CMT yield. Our pre-Great Inflation sample ends in Aug-1968, since this is when standard measures of long-term inflation expectations began to drift up in the U.S.[1] For comparison, we also show the analogous β_h coefficients obtained from regressions using 1- and 10-year CMT yields for the Aug-1971 to Dec-1999 and Jan-2000 to Dec-2019 samples we considered in the main text.

The results are shown in Figure A.1. The β_h coefficients for the 1971-1999 and 2000-2019 subsamples using CMT yields are very similar to those shown in Figure 1 for zero-coupon yields. The β_h coefficients for the 1953-1968 sample are around 0.44 and largely independent of the horizon h. The pre-1968 coefficients are lower than those in the 1971-2000 subsample, which hover around

[1] The results are similar if we end the analysis in July 1971, the month before our baseline analysis in Table 1 begins.

Table A.4: Regressions of 12-month changes in long rates on 12-month changes in short-rate using non-overlapping data This Table reports the estimate of β_{12} from estimating equations (1.1) and (1.2) at the annual frequency ($h = 12$) but using non-overlapping 12-month changes from each possible month of the year to the same month in the following year. Heteroskedasticity robust standard errors are show in brackets. We also report the p-value from a test of the hypothesis that the pre-2000 and post-2000 estimates of β_{12} are equal. Significance: $^{*}p < 0.1$, ** $p < 0.05$, $^{***}p < 0.01$.

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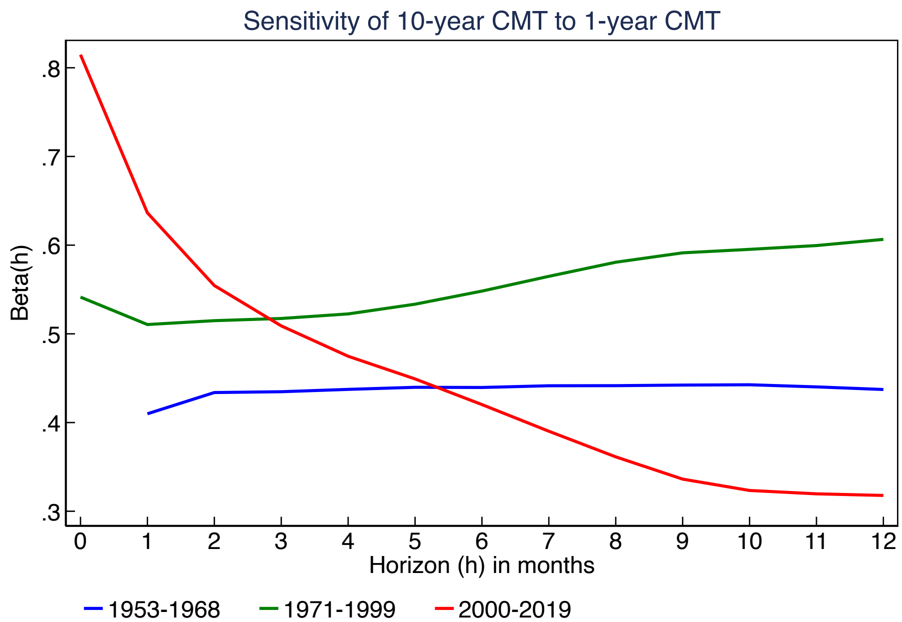
	Yields			Forwards		
Month	Pre-2000	Post-2000	$p-$val	Pre-2000	Post-2000	$p-$val
Jan	0.57***	0.22**	0.00	0.38***	-0.14*	0.00
	[0.07]	[0.09]		[0.10]	[0.07]	
Feb	0.56***	0.21**	0.00	0.43***	-0.19**	0.00
	[0.08]	[0.09]		[0.12]	[0.07]	
Mar	0.48***	0.29***	0.21	0.38**	-0.14*	0.00
	[0.10]	[0.08]		[0.14]	[0.07]	
Apr	0.57***	0.23**	0.01	0.39***	-0.15	0.00
	[0.09]	[0.09]		[0.12]	[0.10]	
May	0.48***	0.27**	0.16	0.29**	-0.11	0.02
	[0.08]	[0.12]		[0.11]	[0.14]	
Jun	0.59***	0.30**	0.03	0.40***	-0.04	0.01
	[0.06]	[0.12]		[0.08]	[0.15]	
Jul	0.47***	0.21**	0.01	0.24***	-0.17	0.00
	[0.06]	[0.08]		[0.07]	[0.10]	
Aug	0.56***	0.22**	0.00	0.41***	-0.13	0.00
	[0.07]	[0.08]		[0.07]	[0.11]	
Sep	0.65***	0.26***	0.00	0.49***	-0.08	0.00
	[0.06]	[0.06]		[0.09]	[0.08]	
Oct	0.65***	0.22**	0.00	0.49***	-0.12	0.00
	[0.07]	[0.09]		[0.09]	[0.14]	
Nov	0.53***	0.14**	0.00	0.32**	-0.16**	0.00
	[0.11]	[0.06]		[0.13]	[0.07]	
Dec	0.62***	0.19	0.00	0.45***	-0.08	0.00
	[0.07]	[0.13]		[0.10]	[0.13]	
Average	0.56	0.22	0.03	0.39	-0.14	0.00
	[0.08]	[0.09]		[0.10]	[0.10]	
Sample	1971-1999	2000-2019		1971-1999	2000-2019	

Figure A.1: Pre-Great Inflation regressions of changes in long-term yields on short-term rates. This figure plots the estimated regression coefficients β_h from equation (1.1) versus horizon (h) for the Apr1953/Aug1968, Aug1971/Dec-1999, and Jan-2000/Dec201 9 subsamples: $y^{(10)}_{t+h} - y^{(10)}_t = \alpha_h + \beta_h(y^{(1)}_{t+h} - y^{(1)}_t) + \varepsilon_{t,t+h}$. The dependent variable is the h-month change in the 10-year nominal Constant Maturity Treasury yield and the independent variable is the h-month change in the 1-year nominal Constant Maturity Treasury yield. Changes are considered with daily data (plotted as $h = 0$ in the figure) and with monthly data using $h = 1$, ..., 12-month changes. Daily data on Constant Maturity Treasury yields is not available until 1962, so β_{day} is missing for the Apr1953/Aug1968 subsample.



0.55. This lower level is consistent with the view that inflation expectations were better-anchored in the pre-1968 sample, although perhaps not as stable as in the post-2000 sample. However, while the level of the β_h coefficients is lower in the pre-1968 sample, we also do not observe the strong dependence on horizon that is so evident in the post-2000 data. In summary, while the unanchoring and then reanchoring of long-run inflation expectations may help explain shifts in the *level* of β_h over time, the strongly *frequency-dependent* sensitivity of long-term rates that we see in the post-2000 subsample appears to be something new under the sun.

A.1.6 International evidence

Our focus is on the U.S., but it is useful to consider whether these same patterns are also observed in other large, highly-developed economies. In Table A.5, we briefly explore evidence for the U.K., Germany, and Canada. Panel A of Table A.5 shows estimates of equation (1.1) for the U.K., where data is available beginning in 1985. For the U.K., the estimates are broken out into real yields and inflation compensation. The evidence for the U.K. is remarkably similar to the U.S. evidence in Table 1. Before 2000, the daily coefficient ($\beta_{day} = 0.44$) and the yearly coefficient ($\beta_{12} = 0.38$) are similar in the U.K. After 2000, the daily sensitivity increases ($\beta_{day} = 0.89$), and the yearly sensitivity declines ($\beta_{12} = 0.29$). Because we have data on real yields prior to 2000 in the U.K., we can decompose the change in β_h into its real and inflation compensation components. As shown in Table A.5, the inflation compensation component of β_h is stable across sample periods and frequency h. Thus, most of the changes in β_h are accounted for by changes in the real component of nominal yields.

Panel B of Table A.5 shows estimates of equation (1.1) for Germany and Canada. For Germany, monthly data is available beginning in 1972 and daily data is available starting in 2000. For Canada, monthly and daily data are available beginning in 1986. Again, we observe similar patterns to those in the U.S. In the pre-2000 sample, β_h is stable across frequencies in Germany and Canada. After 2000, we observe greater sensitivity at high frequencies and less sensitivity at lower frequencies.

We also consider estimates of equation (1.1) for emerging market countries. Specifically, we look at changes in long-term and short-term yields for Mexico, South Africa, and the Philippines. The results are shown in Table A.6. The sample period is 2000-2019 because there was very little local-currency long-term debt in emerging markets before 2000. The results show a fairly flat coefficient β_h and one that, if anything, rise with h. At long horizons, the coefficient is bigger than in advanced economies. This suggests that these emerging countries have higher long-run inflation uncertainty but that their bond yields are not highly affected by temporary rate-amplifying demand shocks.

A.1.7 Rolling estimation of $\beta_{12} - \beta_1$

Here we use an alternate procedure to date the break in the sensitivity of long-term rates to movements in short-term rates. As opposed to focusing simply on the break the low-frequency sensitivity, β_{12}, here we seek to date the emergence of the *frequency-dependent* sensitivity of long-term rates by examining $\beta_{12} - \beta_1$. To do so, we estimate

$$y_{t+1}^{(10)} - y_t^{(10)} = \alpha_1 + \beta_1 (y_{t+12}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+1} \tag{A.1}$$

$$y_{t+12}^{(10)} - y_t^{(10)} = \alpha_{12} + \beta_{12} (y_{t+12}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+12} \tag{A.2}$$

as a joint system using 10-year rolling windows and report $\beta_{12} - \beta_1$. These rolling estimates of $\beta_{12} - \beta_1$ are shown in Figure (A.2) below. As shown, $\beta_{12} - \beta_1$ is positive is earlier windows, hovers just below zero from the late 1980s and to the late 1990s, before turning significantly negative around 2000.

Table A.5: Regressions of changes in long-term international rates on short-term rates. This table reports the estimated regression coefficients from equation (1.1) for the United Kingdom (UK), Germany (DE), and Canada (CAN) on each reported sample. We obtain data on each country's zero-coupon government bond yield curve from each country's central bank website. The dependent variable is the change in the 10-year zero-coupon yield, either nominal, real, or their difference—i.e., inflation compensation (IC). The independent variable is the change in the 1-year nominal yield in all cases. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. For $h > 1$, we report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

Panel A: UK 10-year zero-coupon yields

	(1) Nominal	(2) Nominal	(3) Real	(4) Real	(5) IC	(6) IC
Daily	0.44***	0.89***	0.14***	0.67***	0.29***	0.22***
	[0.04]	[0.03]	[0.01]	[0.03]	[0.04]	[0.02]
Monthly	0.47***	0.56***	0.19***	0.15	0.28***	0.41***
	[0.06]	[0.13]	[0.04]	[0.23]	[0.08]	[0.14]
Quarterly	0.49***	0.43***	0.23***	0.06	0.26***	0.38***
	[0.08]	[0.11]	[0.04]	[0.17]	[0.10]	[0.09]
Semi-annual	0.45***	0.39***	0.22***	0.07	0.23**	0.32***
	[0.09]	[0.08]	[0.05]	[0.11]	[0.11]	[0.06]
Yearly	0.38***	0.29***	0.16**	0.06	0.22***	0.23***
	[0.06]	[0.07]	[0.06]	[0.08]	[0.08]	[0.03]
Sample	1985-1999	2000-2019	1985-1999	2000-2019	1985-1999	2000-2019

Panel B: German and Canadian 10-year nominal zero-coupon yields

	(1) DE	(2) DE	(3) CAN	(4) CAN
Daily		0.66***	0.42***	0.71***
		[0.03]	[0.03]	[0.03]
Monthly	0.34***	0.50***	0.46***	0.53***
	[0.05]	[0.10]	[0.05]	[0.08]
Quarterly	0.41***	0.45***	0.51***	0.39***
	[0.04]	[0.07]	[0.05]	[0.05]
Semi-annual	0.41***	0.41***	0.50***	0.29***
	[0.04]	[0.08]	[0.07]	[0.06]
Yearly	0.43***	0.33***	0.43***	0.16**
	[0.04]	[0.10]	[0.08]	[0.07]
Sample	1972-1999	2000-2019	1986-1999	2000-2019

Figure A.2: Rolling regression estimates of $\beta_{12} - \beta_1$ This figure plots rolling estimates of $\beta_{12} - \beta_1$ using 10-year rolling windows for estimation. Results are plotted against the midpoint of the 10-year rolling window. 95% confidence intervals are included (shaded areas). To conduct inference on $\beta_{12} - \beta_1$, we estimate these two regression equations jointly as a system using the Generalized Method of Moments (GMM). Standard errors for $\beta_{12} - \beta_1$ are formed using Newey-West standard errors with a lag truncation parameter of 18 and 95% critical values from the asymptotic theory of Kiefer and Vogelsang (2005). Specifically, the 95% confidence interval is ±2.41 times the estimated standard errors as opposed to ±1.96 under traditional asymptotic theory.

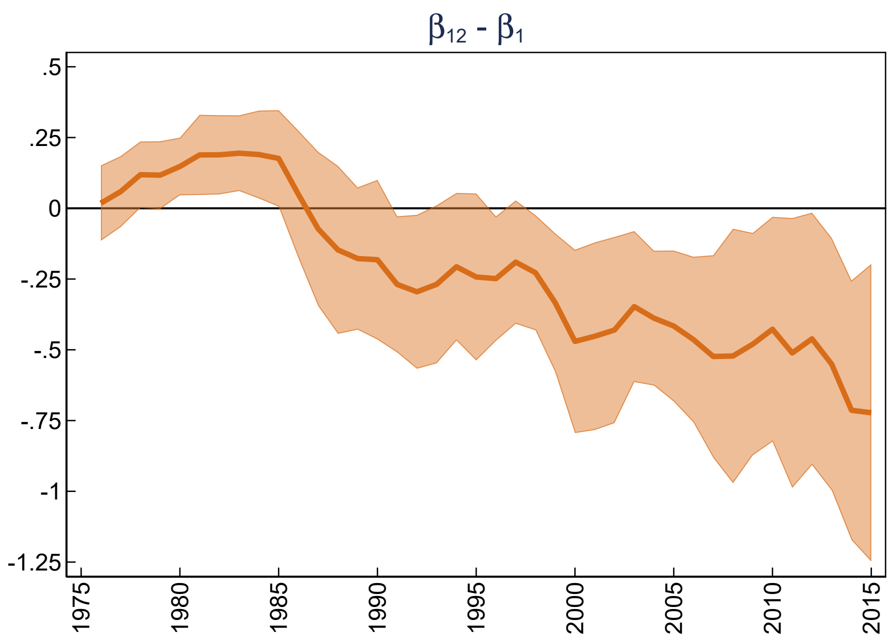


Table A.6: Regressions of changes in long-term emerging market rates on short-term rates. This table reports the estimated regression coefficients from equation (1.1) for Mexico, South Africa and the Philippines. These data come from the IMF International Financial Statistics and are not zero coupon yield curve data but rather use the long-term bond and short-term bill rates as reported in International Financial Statistics which are monthly. The sample period is 2000-2019. Changes are considered at monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. For $h > 1$, we report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Mexico	South Africa	Philippines
Monthly	0.28***	0.20***	0.44*
	[0.06]	[0.07]	[0.23]
Quarterly	0.33***	0.21**	0.59***
	[0.05]	[0.09]	[0.20]
Semi-Annual	0.34***	0.24**	0.64***
	[0.04]	[0.12]	[0.19]
Annual	0.44***	0.40**	0.62***
	[0.04]	[0.17]	[0.20]

A.2 Additional empirical results for Section 2

A.2.1 Predicting the returns on "level-mimicking" and "slope-mimicking" portfolios

As in Table 3 in main text, we first forecast the k-month excess return on $n = 10$-year zero-coupon bonds using level, slope, and the 6-month past changes in these two yield-curve factors:

$$rx^{(10)}_{t \to t+k} = \delta_0 + \delta_1 L_t + \delta_2 S_t + \delta_3 (L_t - L_{t-6}) + \delta_4 (S_t - S_{t-6}) + \varepsilon_{t \to t+k}. \tag{A.3}$$

In Table A.7, we report the results from estimating these predictive regressions for $k = 1$, 3, and 6-month returns. Panel A reports the results for the pre-2000 sample and Panel B shows the post-2000 results.[2,3]

In the post-2000 data, Table A.7 shows the past change in the level of rates is a robust predictor of the excess returns on long-term bonds. However, there is no such predictability in the pre-2000 data. For instance, in column (6) of Panel B, we see that, all else equal, a 100 bps increase in short-term rates over the prior 6 months is associated with a $\delta_3 = 166$ bps (p-val < 0.01) increase in expected 3-month bond returns and the difference between δ_3 in the pre- and post-2000

[2]The yield on k-month Treasury bills, $y_t^{(k/12)}$, is from the yield curve estimates in Gürkaynak et al. (2007). However, this curve is based on coupon securities with at least three months to maturity and does not fit the very short end of the curve well in the pre-2000 data. Therefore, we take the 1-month bill yield from Ken French's website for the pre-2000 sample.

[3]We obtain broadly similar results in Table A.7 if we forecast returns using 3- or 12-month past changes in level and slope. And, the return predictability associated with past changes in level remains similar if, instead of controlling for level and slope, we control for the first five forward rates as in Cochrane and Piazzesi (2005).

data is statistically significant (p-val < 0.01). In untabulated results, we find that the post-2000 return predictability associated with past increases in the level of rates is short-lived and generally dissipates after $k = 6$ months. In other words, past increases in the level of rates lead to a temporary increase in the risk premia on long-term bonds.[4]

To draw out the connection to the predictable curve flattening shown in Table 2 of the main text, we show that these results for 10-year returns are related to predictability of the returns on what we refer to as "level-mimicking" and "slope-mimicking" portfolios. Specifically, we follow Joslin et al. (2014) and construct bond portfolios that locally mimic changes in the level and slope factors. Consider a factor-mimicking portfolio that places weight w_n on zero-coupon bonds with n years to maturity. The k-month excess return on this portfolio from t to $t+k$ is $rx^P_{t\to t+k} = (\sum_n w_n \cdot rx^{(n)}_{t\to t+k})/|\sum_n w_n|$. The level-mimicking portfolio has a weight of -1 on 1-year bonds and no weight on any other bonds. For small k, we have $rx^{(10)}_{t\to t+k} \approx -10 \cdot (\Delta_k L_{t+k} + \Delta_k S_{t+k})$ and $rx^{(1)}_{t\to t+k} \approx -1 \cdot \Delta_k L_{t+k}$. Thus, the level- mimicking portfolio has a k-month excess return of $rx^{LEVEL}_{t\to t+k} = -1 \cdot rx^{(1)}_{t\to t+k} \approx \Delta_k L_{t+k}$. The slope-mimicking portfolio has a weight of 1 on 1-year bonds and of -0.1 on 10-year bonds, so $rx^{SLOPE}_{t\to t+k} = (1 \cdot rx^{(1)}_{t\to t+k} - 0.1 \cdot rx^{(10)}_{t\to t+k})/0.9 \approx \Delta_k S_{t+k}/0.9$. Finally, we note that the excess returns on 10-year bonds are just a linear combination of the excess returns on the level- and slope-mimicking portfolios: $rx^{(10)}_{t\to t+k} = -9 \cdot rx^{SLOPE}_{t\to t+k} - 10 \cdot rx^{LEVEL}_{t\to t+k}$.

In the two bottom blocks of Table A.7, we estimate equation (A.3) using $rx^{LEVEL}_{t\to t+k}$ and $rx^{SLOPE}_{t\to t+k}$ as the dependent variable. In the post-2000 sample, the excess returns on the slope-mimicking portfolio depend negatively on $L_t - L_{t-6}$, but the excess returns on the level-mimicking portfolio depends positively on $L_t - L_{t-6}$.[5] While the two effects partially offset when predicting 10-year excess returns, the net effect is positive and statistically significant in the post-2000 data. Furthermore, the results in Table A.7 where we forecast $rx^{LEVEL}_{t\to t+k}$ and $rx^{SLOPE}_{t\to t+k}$ are entirely consistent with those in Table 2 in the main text.

A.2.2 Unspanned macroeconomic factors

In Table 4 of the paper (and Table A.7 above), we showed that lagged changes in levels can help predict excess returns in the post-2000 period. As detailed below, in standard affine models, if the true model is known, one can obtain the full set of state variables by inverting an appropriate set of yields—i.e., the state variables are spanned by current yields. An unspanned state variable is a variable that is useful for forecasting future bond yields and returns but that has no impact on the current yield curve—i.e., it is not " spanned" by current yields—and cannot be recovered in this

[4]Consistent with the vast literature on lower-frequency movements in bond risk premia initiated by Fama and Bliss (1987) and Campbell and Shiller (1991), we find $\delta_2 > 0$—i.e., expected bond returns are high when the yield curve is steep.

[5]The latter fact is consistent with Piazzesi et al. (2015) and Cieslak (2018), who account for it either with expectational errors or time-varying risk premia. Brooks et al. (2019) also show that the federal funds rate displays short-term momentum.

Table A.7: Estimates of predictive equations for bond excess returns. This table reports the estimated regression coefficients in equation (A.3) using monthly data from the Aug-1971 to Dec-1999 and Jan-2000 to Dec-2019 samples. We report results various return forecast horizon (k). Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. For $k = 1$-month returns, we report heteroskedasticity robust standard errors are in brackets. For $k = 3$ and 6-month returns, we report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 5 and 9 months, respectively. In this case, p-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

Panel A: Pre-2000 sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dep. Var for k	1	1	1	3	3	3	6	6	6
Dependent Variable: $rx^{(10)}_{t\to t+k}$									
L_t	0.17	0.18	0.22*	0.53*	0.54	0.65*	0.99*	1.01	1.25**
	[0.13]	[0.13]	[0.12]	[0.31]	[0.33]	[0.34]	[0.57]	[0.62]	[0.60]
S_t	0.55**	0.61**	0.66***	1.57**	1.91***	2.07***	3.17***	3.62***	3.98***
	[0.24]	[0.24]	[0.24]	[0.64]	[0.65]	[0.64]	[1.04]	[1.13]	[1.09]
$L_t - L_{t-6}$		0.10	-0.18		0.59	-0.22		0.79	-1.00
		[0.19]	[0.29]		[0.49]	[0.54]		[0.69]	[0.90]
$S_t - S_{t-6}$			-0.57			-1.67**			-3.67***
			[0.44]			[0.81]			[1.30]
Adj.R^2	0.02	0.01	0.02	0.05	0.06	0.07	0.11	0.12	0.17
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent Variable: $rx^{LEVEL}_{t\to t+k}$									
L_t	-0.04*	-0.04*	-0.04**	-0.08*	-0.09*	-0.10**	-0.11**	-0.11**	-0.13**
	[0.02]	[0.02]	[0.02]	[0.04]	[0.05]	[0.05]	[0.05]	[0.05]	[0.05]
S_t	-0.06	-0.07*	-0.08**	-0.15*	-0.21**	-0.24***	-0.23***	-0.26***	-0.29***
	[0.04]	[0.03]	[0.04]	[0.09]	[0.08]	[0.08]	[0.09]	[0.09]	[0.09]
$L_t - L_{t-6}$		-0.01	0.04		-0.10	0.03		-0.04	0.09
		[0.03]	[0.04]		[0.08]	[0.08]		[0.09]	[0.09]
$S_t - S_{t-6}$			0.11*			0.26**			0.27*
			[0.06]			[0.11]			[0.14]
Adj.R^2	0.01	0.01	0.02	0.04	0.06	0.08	0.09	0.10	0.12
Dependent Variable: $rx^{SLOPE}_{t\to t+k}$									
L_t	0.02	0.02	0.03**	0.03	0.04	0.04	0.01	0.01	0.01
	[0.01]	[0.01]	[0.01]	[0.02]	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
S_t	0.00	0.01	0.01	-0.01	0.02	0.03	-0.09	-0.11	-0.12
	[0.03]	[0.02]	[0.03]	[0.06]	[0.05]	[0.06]	[0.08]	[0.08]	[0.08]
$L_t - L_{t-6}$		0.00	-0.03		0.04	-0.00		-0.04	0.01
		[0.02]	[0.03]		[0.05]	[0.06]		[0.04]	[0.05]
$S_t - S_{t-6}$			-0.06			-0.10			0.11
			[0.05]			[0.09]			[0.10]
Adj.R^2	0.01	0.01	0.01	0.02	0.03	0.03	0.04	0.05	0.06
N	341	335	335	341	335	335	341	335	335
Sample	1971-1999	1972-1999	1972-1999	1971-1999	1972-1999	1972-1999	1971-1999	1972-1999	1972-1999

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dep. Var for k	1	1	1	3	3	3	6	6	6
Dependent Variable: $rx_{t\to t+k}^{(10)}$									
L_t	0.30***	0.32***	0.28**	0.73***	0.81***	0.71**	1.37***	1.50***	1.34**
	[0.11]	[0.11]	[0.11]	[0.27]	[0.28]	[0.30]	[0.44]	[0.47]	[0.51]
S_t	0.56***	0.63***	0.53**	1.41***	1.65***	1.42**	2.71***	3.07***	2.73***
	[0.21]	[0.20]	[0.21]	[0.48]	[0.52]	[0.58]	[0.83]	[0.93]	[0.95]
$L_t - L_{t-6}$		0.33	0.62*		0.98**	1.66***		1.33*	2.39**
		[0.24]	[0.33]		[0.46]	[0.61]		[0.73]	[1.07]
$S_t - S_{t-6}$			0.48			1.12			1.76
			[0.35]			[0.72]			[1.22]
Adj.R^2	0.03	0.03	0.03	0.08	0.10	0.12	0.16	0.18	0.20
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent Variable: $rx_{t\to t+k}^{LEVEL}$									
L_t	-0.03***	-0.03***	-0.03***	-0.09***	-0.08***	-0.07***	-0.12***	-0.11***	-0.10***
	[0.01]	[0.01]	[0.01]	[0.03]	[0.02]	[0.02]	[0.04]	[0.03]	[0.03]
S_t	-0.04***	-0.03**	-0.03**	-0.10***	-0.07**	-0.05*	-0.13***	-0.08*	-0.05
	[0.01]	[0.01]	[0.01]	[0.03]	[0.03]	[0.03]	[0.04]	[0.04]	[0.04]
$L_t - L_{t-6}$		0.05**	0.03		0.12**	0.06		0.19***	0.12
		[0.02]	[0.02]		[0.06]	[0.06]		[0.07]	[0.07]
$S_t - S_{t-6}$			-0.04**			-0.09**			-0.11**
			[0.02]			[0.04]			[0.05]
Adj.R^2	0.06	0.10	0.11	0.13	0.21	0.23	0.21	0.37	0.39
Dependent Variable: $rx_{t\to t+k}^{SLOPE}$									
L_t	0.01	-0.00	0.00	0.02	-0.00	0.00	-0.01	-0.05	-0.04
	[0.01]	[0.01]	[0.01]	[0.03]	[0.03]	[0.03]	[0.06]	[0.04]	[0.05]
S_t	-0.01	-0.03*	-0.03	-0.05	-0.10**	-0.10*	-0.16*	-0.26***	-0.24***
	[0.02]	[0.02]	[0.02]	[0.05]	[0.05]	[0.05]	[0.08]	[0.08]	[0.08]
$L_t - L_{t-6}$		-0.09***	-0.10***		-0.24***	-0.25***		-0.36***	-0.40***
		[0.02]	[0.03]		[0.05]	[0.06]		[0.07]	[0.11]
$S_t - S_{t-6}$			-0.01			-0.02			-0.07
			[0.03]			[0.07]			[0.13]
Adj.R^2	0.00	0.06	0.06	0.03	0.19	0.18	0.08	0.28	0.28
N	239	239	239	237	237	237	234	234	234
Sample	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019	2000-2019

way. Lagged changes in level and slope are unspanned factors, as in Joslin et al. (2013).

Other authors have considered macroeconomic variables as unspanned factors, and this is the more typical choice of unspanned factors. Thus, following Joslin et al. (2014) , we augment equation (A.3) with growth (measured by the 3 month moving average of the Chicago Fed National Activity Index) and inflation (measured by the Blue Chip forecast of CPI inflation over the next four quarters). Because of the availability of the inflation forecast data, the sample only starts in March 1980. The results are shown in Table A.8. In the post-2000 sample, we find that lagged changes in level are significant predictors of excess returns on 10-year bonds and are also significant predictors of returns on the slope-mimicking portfolio. Inflation is also a significant predictor of 10-year excess bond returns in this post-2000 sample, though it is not a significant predictor of returns on the slope-mimicking portfolio. Overall, our finding that lagged changes in level predict future bond returns in the post-2000 data is robust to the inclusion of the standard unspanned macroeconomic variables.

A.2.3 Predicting the returns on bonds with different maturities

In this section, we examine the predictability for bond maturities other than $n = 10$ years. If, as we argue, past increases in short rates temporarily raise the net supply of long-term bonds that investors must hold, thereby raising the compensation investors require for bearing interest-rate risk, this should have a larger impact on the expected returns of long-term bonds than intermediate bonds. This is because the returns on long-term bonds are more sensitive to shifts in yields than those on intermediate bonds (Vayanos and Vila, 2020; Greenwood and Vayanos, 2014). We explore this prediction in Figure A.3. We separately forecast the 3-month returns on bonds with different maturities n, estimating:

$$rx_{t\to t+3}^{(n)} = \delta_0^{(n)} + \delta_1^{(n)} L_t + \delta_2^{(n)} S_t + \delta_3^{(n)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t\to t+3}^{(n)} \tag{A.4}$$

separately for $n = 1, ...20$-year bonds. We then plot the coefficients $\delta_3^{(n)}$ on the past change in level from estimating equation (A.4) versus bond maturity n for the pre-2000 and post-2000 samples. (For the pre-2000 sample, the longest available maturity is $n = 15$ years). Consistent with the idea that past increases in short rates temporarily raise the compensation for bearing interest-rate risk, the coefficients $\delta_3^{(n)}$ are monotonically increasing in bond maturity n in the post-2000 sample. By contrast, there is no such predictability in the pre-2000 sample.

A.2.4 Predictable changes in the shape of the yield and forward rate curves

This temporary rise in the compensation for bearing interest rate risk impacts the yield and forward rate curves. As explained in Greenwood and Vayanos (2014), a short-lived rise in the compensation for bearing interest rate risk may have relatively constant or even a hump-shaped effect on the yield

Table A.8: Estimates of predictive equations for bond excess returns including macro factors This table reports the estimation of equation (2.6) augmented with growth (measured by the 3 month moving average of the Chicago Fed National Activity Index) and inflation (measured by the Blue Chip forecast of CPI inflation over the next four quarters) using monthly data from the subsample from March 1980 to December 1999 and the subample from January 2000 to December 2019. We report results various return forecast horizon (k). Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. For k = 1-month returns, we report heteroskedasticity robust standard errors are in brackets. For k = 3 and 6-month returns, we report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 5 and 9 months, respectively. In this case, p-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	(1)	(2)	(3)	(4)	(5)	(6)
Dep. Var for k	1	3	6	1	3	6
Dependent Variable: $rx^{(10)}_{t\to t+k}$						
L_t	0.46**	1.77***	3.47***	0.28**	0.51	1.01*
	(0.22)	(0.38)	(0.48)	(0.14)	(0.32)	(0.53)
S_t	0.76**	2.37***	4.55***	0.61***	1.65***	2.93***
	(0.30)	(0.73)	(1.09)	(0.23)	(0.57)	(0.95)
$L_t - L_{t-6}$	0.31	1.01*	0.98	0.73*	1.81***	2.25**
	(0.34)	(0.55)	(0.87)	(0.42)	(0.63)	(1.00)
$S_t - S_{t-6}$	-0.64	-1.32	-2.78**	0.43	0.72	1.21
	(0.51)	(0.93)	(1.30)	(0.35)	(0.69)	(1.17)
Growth	-2.34***	-6.17***	-9.68***	-0.32	-1.37	-1.13
	(0.77)	(1.85)	(2.72)	(0.88)	(1.00)	(0.82)
Inflation	-0.56*	-2.45***	-4.69***	0.26	2.34**	3.24***
	(0.31)	(0.52)	(0.58)	(0.79)	(0.96)	(1.11)
Dependent Variable: $rx^{LEVEL}_{t\to t+k}$						
L_t	-0.07*	-0.26***	-0.36***	-0.02**	-0.05***	-0.08***
	(0.04)	(0.06)	(0.03)	(0.01)	(0.02)	(0.03)
S_t	-0.07	-0.25***	-0.29***	-0.03**	-0.06*	-0.06
	(0.05)	(0.08)	(0.08)	(0.01)	(0.03)	(0.04)
$L_t - L_{t-6}$	-0.06	-0.15**	-0.06	0.01	0.02	0.08
	(0.05)	(0.06)	(0.07)	(0.03)	(0.07)	(0.08)
$S_t - S_{t-6}$	0.12**	0.20**	0.15	-0.02	-0.06	-0.08
	(0.06)	(0.09)	(0.10)	(0.02)	(0.04)	(0.05)
Growth	0.43***	0.86***	0.69***	0.09**	0.22***	0.20***
	(0.13)	(0.17)	(0.19)	(0.04)	(0.08)	(0.07)
Inflation	0.07	0.36***	0.48***	-0.06*	-0.18**	-0.14
	(0.07)	(0.10)	(0.05)	(0.03)	(0.09)	(0.09)
Dependent Variable: $rx^{SLOPE}_{t\to t+k}$						
L_t	0.03	0.09**	0.01	-0.01	0.00	-0.02
	(0.03)	(0.04)	(0.05)	(0.01)	(0.03)	(0.05)
S_t	-0.00	0.02	-0.18*	-0.04*	-0.11**	-0.26***
	(0.03)	(0.07)	(0.10)	(0.02)	(0.05)	(0.09)
$L_t - L_{t-6}$	0.03	0.06	-0.04	-0.09**	-0.22***	-0.34***
	(0.03)	(0.06)	(0.05)	(0.04)	(0.07)	(0.10)
$S_t - S_{t-6}$	-0.06	-0.08	0.15	-0.02	-0.01	-0.04
	(0.04)	(0.09)	(0.11)	(0.03)	(0.07)	(0.13)
Growth	-0.21**	-0.27**	0.30*	-0.06	-0.09	-0.10
	(0.09)	(0.13)	(0.17)	(0.10)	(0.10)	(0.08)
Inflation	-0.02	-0.13*	-0.02	0.04	-0.06	-0.20
	(0.05)	(0.07)	(0.06)	(0.09)	(0.13)	(0.12)
Sample	1971-1999	1971-1999	1972-1999	2000-2019	2000-2019	2000-2019

and forward curves as opposed to the monotonically increasing effect shown above for returns. The intuition is that the impact on bond yields equals the effect on a bond's average expected returns over its lifetime. As a result, a *temporary* rise in the compensation for bearing interest rate risk can have a greater impact intermediate-term yields than on long-term yields. Thus, we plot the slope coefficients $\delta_3^{(n)}$ versus maturity n from estimating:

$$y_{t+3}^{(n-3/12)} - y_t^{(n)} = \delta_0^{(n)} + \delta_1^{(1)} L_t + \delta_2^{(1)} S_t + \delta_3^{(3)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t \to t+3}^{(n)}. \quad \text{(A.5)}$$

and

$$f_{t+3}^{(n-3/12)} - f_t^{(n)} = \delta_0^{(n)} + \delta_1^{(n)} L_t + \delta_2^{(n)} S_t + \delta_3^{(n)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t \to t+3}^{(n)}, \quad \text{(A.6)}$$

for $n = 1, 2, ..., 20$ years for both the pre-2000 and post-2000 samples.[6] After 2000, the second plot in Figure A.3 shows that while past increases in the level of short rates forecast a future flattening of the yield curve in post-2000 data, the expected changes in long-term yields are relatively constant beyond 5 years. Turning to forward rates, the bottom plot in Figure A.3 shows that, post 2000, a past increase in short-term rates has a slight humped-shaped effect on the evolution of the forward curve with the peak impact at 4 years.

In summary, the results for different maturities support the view that past increases in short-term rates *temporarily* raise the compensation that investors earn for bearing interest-rate risk.

A.2.5 Trading strategies

As another way of assessing the resulting return predictability documented in Section 2.2 of the paper, we consider simple market-timing strategies in which an investor decides to take either a long or short position in the slope-mimicking portfolio—i.e., in a "curve steepener" trade—every month. Specifically, we consider strategies that take a long (short) position in the slope-mimicking portfolio from month t to month $t+1$ if $L_t < L_{t-h}$ ($L_t > L_{t-h}$). Alternatively, we consider strategies that take a position in the slope-mimicking portfolio from month t to month $t+1$ that is proportional to $-(L_t - L_{t-h})$. Table A.9 computes the annualized Sharpe ratios of these two trading strategies for different choices of h, in the pre- and post-2000 samples. As shown in Table A.9 the implied annualized Sharpe ratios for these strategies range between about 0.35 to 0.65 in the post-2000 sample but were negligible in the pre-2000 sample.

[6] In equation (A.6), $f_t^{(n)} \equiv n y_t^{(n)} - (n-1) y_t^{(n)}$ is the 1-year rate $(n-1)$ years forward as opposed to the instantaneous forward n years forward. Since $f_{t+k}^{(n-k/12)} - f_t^{(n)} = -(rx_{t \to t+k}^{(n)} - rx_{t \to t+k}^{(n-1)})$, there is a tight connection between the coefficients in equations (A.6) and (A.4). Specifically, defining $f_{t+k}^{(1-k/12)} - f_t^{(1)} = -rx_{t \to t+k}^{(1)}$, we have $rx_{t \to t+k}^{(n)} = \sum_{m=1}^{n} (f_t^{(m)} - f_{t+k}^{(m-k/12)})$. Thus, the coefficients in equation (A.4) for maturity n can be recovered by summing up the -1 times coefficients in equation (A.6) for all maturities $m \leq n$. Similarly, the coefficients from equation (A.5) for maturity n are approximately the average of the coefficients in equation (A.6) for all maturities $m \leq n$.

Figure A.3: Predicting returns, the changes in forwards, and the change in yields for various bond maturities n. This figure plots the coefficients $\delta_3^{(n)}$ on the past 6-month change in the level factor versus bond maturity n from estimating equation (A.4) for returns, equation (A.5) for the change in yields, and equation (A.6) for the change in forward rates. The results are shown for $k = 3$-month returns or future changes. Due to the use of overlapping data, we plot 95% confidence intervals using Newey-West (1987) standard errors with a lag truncation parameter of 5. The critical values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

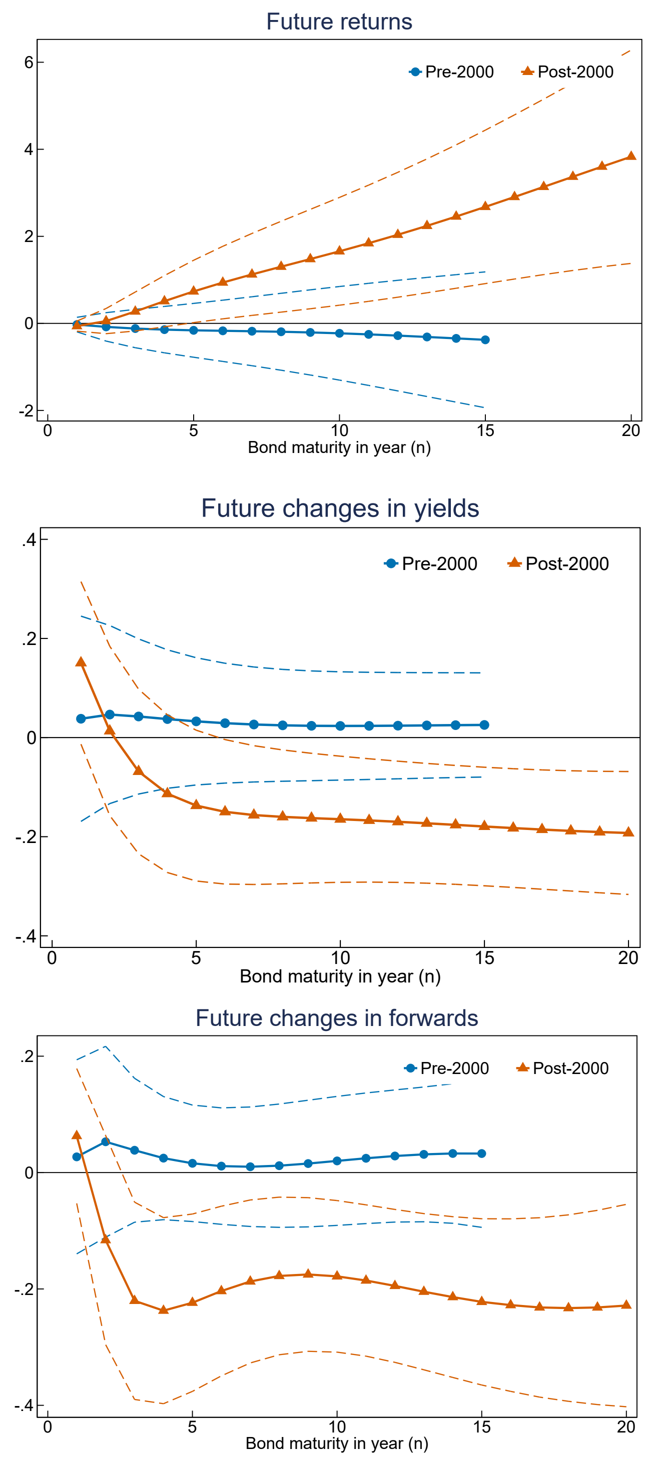


Table A.9: Sharpe ratios for slope-mimicking portfolios This table reports the annualized Sharpe ratios since 2000 of the strategy of going long (short) the slope-mimicking portfolio if the level fell (rose) over the previous h months and also the strategy of taking a position in the slope-mimicking portfolio that is proportional to $-(L_t - L_{t-h})$, and holding the position from t to $t+1$. The position is rebalanced each month. Annualized Sharpe ratios are computed as the sample average monthly excess returns multiplied by $\sqrt{12}$ and divided by the standard deviation of those monthly excess returns.

Strategy with h:	1	3	6	12
Pre-2000				
$2 \times I(L_t - L_{t-h} < 0) - 1$	0.22	0.03	-0.09	-0.00
$-(L_t - L_{t-h})$	0.08	-0.01	0.12	0.09
Post-2000				
$2 \times I(L_t - L_{t-h} < 0) - 1$	0.43	0.53	0.63	0.58
$-(L_t - L_{t-h})$	0.36	0.56	0.43	0.42

A.3 Additional empirical results for Section 4

A.3.1 Reaching for yield

To empirically assess this reaching-for-yield explanation for our findings, we use quarterly data from the Federal Reserve's Financial Accounts on the aggregate net bond acquisitions by insurers (life plus property-casualty), pension funds (private plus state and local), and banks to construct empirical proxies for the bond demand of yield-seeking investors, h_t. We focus on these highly-regulated financial intermediaries since prior research has argued that they are most likely to be concerned about the current yield on their portfolios and, therefore, to reach for yield when interest rates decline.[7] For intermediaries in sector i, we compute the percentage bond flows in quarter t as $\%FLOW_{i,t} = FLOW_{i,t}/HOLD_{i,t-1}$, where $FLOW_{i,t}$ denotes net bond acquisitions by intermediaries in sector i during quarter t and $HOLD_{i,t-1}$ is bond holdings at the end of quarter $t-1$. Bonds here include the sum of U.S. Treasury securities, agency debt and GSE-guaranteed mortgage-backed securities, and corporate bonds. Thus, our construction of these sector-level flows roughly mimics the construction of bond mutual fund flows above.

In Table A.10, we then estimate quarterly regressions that are analogous to equations (4.2) and (4.3) in the main text using these sector-level bond flows $\%FLOW_{i,t}$ as X_t. We report the results separately for the pre-2000 and post-2000 samples. As shown in Panel A of Table A.10, in the post-2000 data, we find little evidence that recent increase in short-term rates lead to a reduction in bond purchases by insurers, pensions, and banks. Furthermore, in Panels B and C, we find

[7] Insurers and banks are generally not required to include changes in the mark-to-market value of their portfolios in their reported earnings, which may give way to yield-seeking behavior. For prior work on reaching-for-yield by insurers, see Becker and Ivashina (2015). For pension funds, see Lu et al. (2019). For banks, see Maddaloni and Peydró (2011), Hanson and Stein (2015), and Drechsler et al. (2018).

little evidence that bond purchases by these intermediaries predict low excess returns on long-term bonds in the following quarter or subsequent yield-curve steepening as would be suggested by a reaching-for-yield explanation for our findings.

In summary, we find little evidence that reaching-for-yield plays a major role in driving the kind of short-lived overreaction of long-term yields to changes in short rates that we see since 2000. To be sure, this lack of evidence does not imply that reaching-for-yield plays an unimportant role in determining financial market risk premia more generally, especially at lower frequencies. This negative conclusion only applies to the ability of reaching-for-yield to explain the sorts of transitory fluctuations in bond risk premia that underpin the horizon-dependent excess sensitivity observed in recent decades.

A.4 Additional empirical results for Section 5

A.4.1 Bond market "conundrums"

Our findings can help explain the rising prevalence of bond market episodes like the one that former Federal Reserve Chairman Greenspan famously called the "conundrum"—the period after June 2004 when the Fed raised short-term rates, but longer-term yields declined. This "conundrum" was first noted in Greenspan (2005) and has been explored in many papers, including Backus and Wright (2007).

Consistent with the weaker low-frequency sensitivity of long-term rates in recent years, "conundrum" episodes—defined as 6-month periods where short- and long-term rates move in *opposite* directions—have grown increasingly common. Specifically, since 2000, 1- and 10-year nominal Treasury yields have moved in the *opposite* direction in 37% of all 6-month periods. By contrast, from 1971 to 1999, the corresponding figure was 18%, and the difference is statistically significant (p-val < 0.001).

Here we show that the non-Markovian dynamics documented in Section 2—the fact that past changes in the level of rates increasingly predict a future flattening of the yield curve—help explain several noteworthy "conundrums." Figure A.4 plots 1-year and 10-year Treasury rates around three widely discussed "conundrums": Greenspan's original 2004 "conundrum," 2008 which was a "conundrum in reverse," and the 2017 "conundrum." In all three cases, 1-year and 10-year yields moved in opposite directions.

Consider Greenspan's original 2004 "conundrum." To draw the link between non-Markovian yield-curve dynamics and this "conundrum," we use the system of predictive equations for level and slope from Table 2 in the main text. Starting in May 2004, we simulate the counterfactual path of 10-year yields that would have prevailed if, in the post-2000 sample, the slope of the yield curve had not responded to past changes in the level. To do so, we take the unrestricted estimates of the predictive equation (2.1b) for slope from column (6) in Table 2 and the restricted estimates from

Table A.10: The role of reaching-for-yield: Evidence from sectoral bond market flows. Data on sectoral-level bond market flows are from the Federal Reserve's Financial Accounts. Bond holdings include the sum of Treasury Securities (Table 210), Agency and GSE-Backed Securities (Instrument Table 211), and Corporate Bonds (Table 213). Our series for "Insurers" combines together data for Property-Casualty Insurance Companies (Table 115) and Life Insurance Companies (Table 116); "Pensions" combines together data for Private Pension Funds (Table 118) and State and Local Government Employee Retirement Funds (Table 120); and "Banks" uses data for U.S.-chartered depository institutions (Table 111). For intermediary sector i, we then compute the percentage bond flow in quarter t as $\%FLOW_{i,t} = FLOW_{i,t}/HOLD_{i,t-1}$, where $FLOW_{i,t}$ denotes net bond acquisitions by intermediaries in sector i during quarter t and $HOLD_{i,t-1}$ is bond holdings at the end of quarter $t-1$. Panel A reports the estimated regression coefficients for equation (4.2) using $X_t = \%FLOW_{i,t}$ for each sector i. We estimate these regressions using quarterly data for the 1971Q3-1999Q4 and 2000Q1-2019Q4 samples. Panels B and C report the estimated coefficients for equation (4.3). We report heteroskedasticity robust standard errors in brackets. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

	Pre-2000			Post-2000		
	(1)	(2)	(3)	(4)	(5)	(6)
Sector (i):	Insurance	Pensions	Banks	Insurance	Pensions	Banks
Dependent Variable: $FLOW_{i,t}$						
L_t	0.24***	0.27***	0.33***	0.23***	-0.17	0.06
	[0.05]	[0.07]	[0.12]	[0.07]	[0.19]	[0.16]
S_t	0.71***	-0.25*	0.67**	0.47***	-0.00	0.42
	[0.11]	[0.15]	[0.28]	[0.11]	[0.29]	[0.29]
$L_t - L_{t-2}$	-0.04	-0.28**	-0.53**	0.16	1.44***	0.05
	[0.09]	[0.11]	[0.26]	[0.17]	[0.42]	[0.41]
Adj.R^2	0.25	0.24	0.12	0.18	0.16	0.01
N	112	112	112	80	80	80
Dependent Variable: $rx^{(10)}_{t\to t+1}$						
L_t	0.39	0.68*	0.68*	0.95**	0.90**	0.85**
	[0.46]	[0.41]	[0.39]	[0.39]	[0.35]	[0.36]
S_t	0.99	1.34*	1.86**	1.83**	1.66***	1.68**
	[0.92]	[0.78]	[0.83]	[0.74]	[0.61]	[0.64]
$FLOW_{t-1\to t}$	0.54	-0.57	-0.50***	-0.56	0.24	-0.20
	[0.59]	[0.40]	[0.15]	[0.73]	[0.18]	[0.32]
Adj.R^2	0.02	0.03	0.06	0.06	0.07	0.06
N	114	114	114	79	79	79
Dependent Variable: $rx^{SLOPE}_{t\to t+1}$						
L_t	0.03	0.04	0.03	0.02	0.01	0.01
	[0.03]	[0.03]	[0.03]	[0.04]	[0.04]	[0.04]
S_t	-0.04	-0.03	-0.04	-0.05	-0.06	-0.06
	[0.09]	[0.08]	[0.08]	[0.07]	[0.06]	[0.06]
$FLOW_{t-1\to t}$	0.02	-0.03	0.02	-0.00	-0.03*	0.03
	[0.05]	[0.03]	[0.02]	[0.07]	[0.02]	[0.03]
Adj.R^2	0.01	0.01	0.02	0.00	0.04	0.02
N	114	114	114	79	79	79
Sample	1971-1999	1971-1999	1971-1999	2000-2019	2000-2019	2000-2019

column (4) which constrain past changes to have no effect ($\delta_{3S} = \delta_{4S} = 0$). Starting in May 2004, we generate the counterfactual path of 10-year yields that would have obtained if $\delta_{3S} = \delta_{4S} = 0$. We hold the level factor at its actual value and use the residuals from the unrestricted regression in column (6), but set the parameters for the slope equation to their estimated values from the *restricted* regression in column (4).

The top panel of Figure A.4 plots the actual 1- and 10-year yields over this 2004 conundrum period along with the 10-year yield under this counterfactual scenario. Had the slope not responded to lagged changes in the level of the yield curve, Figure A.4 shows that, instead of falling, 10-year yields would have risen in 2004. The next two panels repeat this exercise for the 2008 "conundrum in reverse" (starting in December 2007) and the 2017 "conundrum" (starting in November 2016). If the slope had not responded to past changes in level, 10- and 1-year yields would have moved in the same direction in both cases.

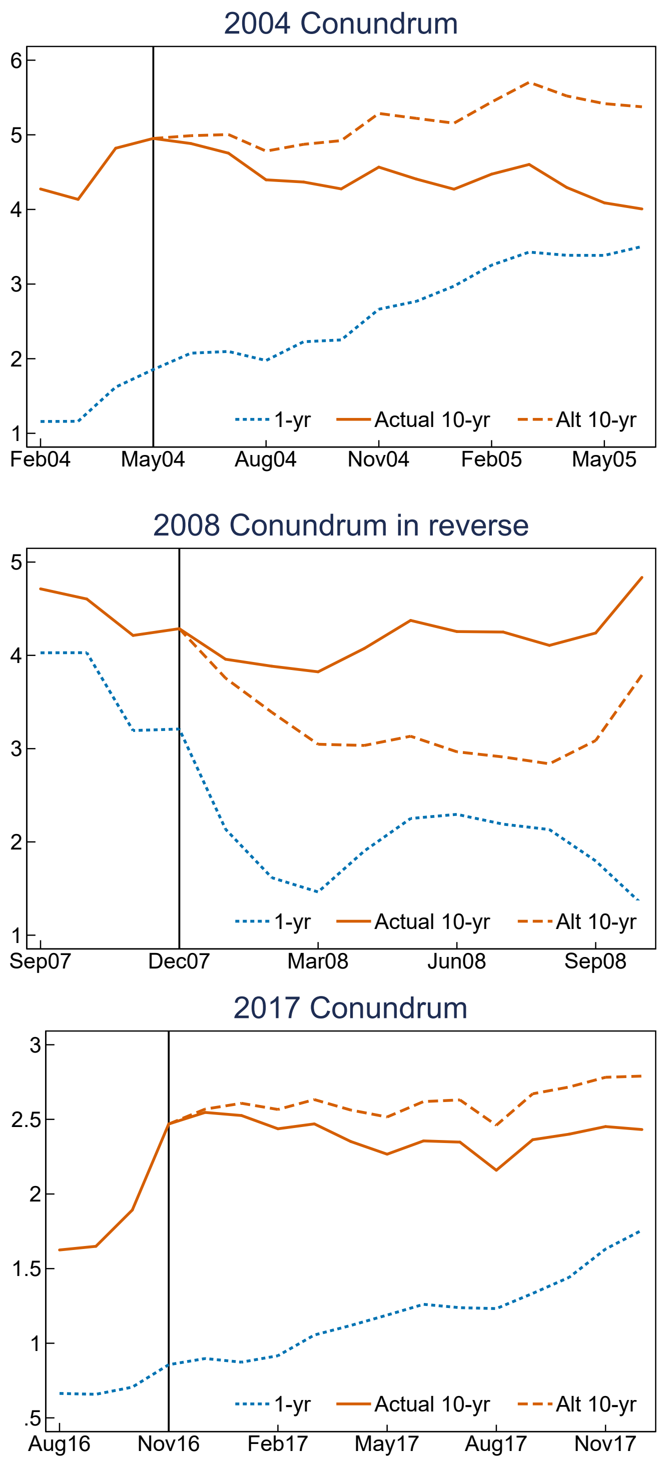
A.4.2 Implications for affine term-structure models

Summary We explore the implications of our results for affine term structure models which are a widely-used, reduced-form tools for understanding the term structure of bond yields (Duffee, 2002; Duffie and Kan, 1996). In these models, the n-year zero coupon yield, $y_t^{(n)}$, takes the affine form: $y_t^{(n)} = \alpha_{0(n)} + \boldsymbol{\alpha}'_{1(n)}\mathbf{x}_t$, where $\mathbf{x}_t$ is a vector of state variables and the $\alpha_{0(n)}$ and $\boldsymbol{\alpha}_{1(n)}$ satisfy a set of recursive equations. Below, we apply the estimation methodology of Adrian et al. (2013) and fit affine term structure models using the first K principal components of 1- to 10-year yields as the state variables $\mathbf{x}_t$. We show that standard affine models—models that are Markovian with respect to these current yield-curve factors—cannot fit our key finding that the sensitivity of long rates to short rates β_h declines so strongly with horizon h in the post-2000 data. Furthermore, we show that this remains so even if we estimate models that include many (e.g., $K = 5$) current yield-curve factors as state variables.

However, we show that our key finding is consistent with non-Markovian term structure models in which past lags of the yield-curve factors are treated as "unspanned state variables." In standard affine models, if the true model is known, one can recover the full set of state variables $\mathbf{x}_t$ by inverting an appropriate set of yields—i.e., the state variables are "spanned" by current yields. An unspanned state variable is a variable that is useful for forecasting future bond yields and returns but that has no impact on the current yield curve. This non-Markovian model allows us to parsimoniously capture our result that past changes in the level of rates are useful for forecasting future bond yields and returns. And, similar models have been considered in Joslin et al. (2013). To be clear, we do not argue that the past increase in the level of rates is *literally* unspanned. Instead, we think this variable is *close* to being unspanned.

Finally, we use a bootstrap procedure to test the hypothesis that each affine model is correctly

Figure A.4: Counterfactual paths of ten-year yields in selected "conundrum" episodes. This figure plots 1- and 10-year yields in the original 2004 "conundrum" episode, the 2008 "conundrum in reverse" episode and the "2017 conundrum." As described in the text, we also plot counterfactual 10-year yields (Alt 10-yr) generated from restricting the slope to depend on lags of level and slope, but not also on lagged changes in level and slope.



specified, using the ratio of yearly to monthly coefficients from equation (1.1) as the test statistic. The test rejects if the observed value of β_{12}/β_1 is too high or low to have been generated by that model. This test is in the spirit of Giglio and Kelly (2018), who test the hypothesis an affine model is correctly specified by checking whether the comovement of yields at different points on the curve is consistent with the model. Using this bootstrap procedure, we conclude that, in the post-2000 sample, the Markovian models are decisively rejected: if these standard models were correctly specified it would be highly unlikely to observe a value of β_{12}/β_1 as small as we do in the data. However, the non-Markovian models are not rejected post-2000. Thus, we conclude that affine models need to include lagged yield-curve factors to match the fact that the sensitivity of long rates declines so sharply with horizon post-2000.

Affine term-structure models A standard discrete-time affine term-structure model (Duffee, 2002; Duffie and Kan, 1996) starts from the assumption that there is a $m \times 1$ state vector $\mathbf{x}_t$ that follows a VAR(1) under the physical or P-measure:

$$\mathbf{x}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_t, \tag{A.7}$$

where the error term is Gaussian with mean zero and identity variance-covariance matrix. The short-term riskless interest rate between time t and $t+1$ is an affine function of the state vector: $i_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$. Meanwhile, the pricing kernel or stochastic discount factor is

$$M_{t+1} = \exp(-i_t - \boldsymbol{\lambda}_t'\boldsymbol{\varepsilon}_{t+1} - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t), \tag{A.8}$$

where the prices of factor risk, $\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t$, are also an affine function of the state vector. The price of an n-period zero-coupon bond, $P_t^{(n)}$, satisfies the recursion

$$P_t^{(n)} = E_t^{\mathrm{P}}[M_{t+1}P_{t+1}^{(n-1)}] = \exp(-i_t)E_t^{\mathrm{Q}}[P_{t+1}^{(n-1)}]. \tag{A.9}$$

Here $E_t^{\mathrm{P}}[\cdot]$ denotes expectations under the physical measure or P-measure and $E_t^{\mathrm{Q}}[\cdot]$ denotes expectations under the risk-neutral pricing measure or Q-measure. (For any random variable X_{t+1}, $E_t^{\mathrm{Q}}[X_{t+1}] = E_t^{\mathrm{P}}[M_{t+1}X_{t+1}]/E_t^{\mathrm{P}}[M_{t+1}]$.) Under the Q-measure, the state variables evolve according to

$$\mathbf{x}_t = \boldsymbol{\mu}^* + \boldsymbol{\Phi}^*\mathbf{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_t, \tag{A.10}$$

where $\boldsymbol{\mu}^* = \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0$ and $\boldsymbol{\Phi}^* = \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1$.

After extensive, but well-known algebra, it follows that

$$P_t^{(n)} = \exp(a_{(n)} + \mathbf{b}_{(n)}'\mathbf{x}_t), \tag{A.11}$$

where $a_{(n)}$ is a scalar and $\mathbf{b}_{(n)}$ is an $m \times 1$ vector that satisfy the recursions:

$$a_{(n+1)} = -\delta_0 + a_{(n)} + \mathbf{b}_{(n)}'\boldsymbol{\mu}^* + \frac{1}{2}\mathbf{b}_{(n)}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathbf{b}_{(n)} \tag{A.12}$$

$$\mathbf{b}_{(n+1)} = \boldsymbol{\Phi}^{*\prime}\mathbf{b}_{(n)} - \boldsymbol{\delta}_1, \tag{A.13}$$

starting from $a_{(1)} = -\delta_0$ and $\mathbf{b}_1 = -\boldsymbol{\delta}_{(1)}$. The continuously compounded yield on an n-period zero-coupon bond, $y_t^{(n)}$, is in turn given by

$$y_t^{(n)} = -n^{-1}\log(P_t^{(n)}) = -n^{-1}a_{(n)} - n^{-1}\mathbf{b}'_{(n)}\mathbf{x}_t. \tag{A.14}$$

Markovian models We now show that standard affine models—models that are Markovian with respect to these current yield-curve factors—cannot fit our key finding that the sensitivity of long rates to short rates β_h declines so strongly with horizon h in the post-2000 data. Applying the estimation methodology of Adrian et al. (2013), we fit affine term-structure models with monthly data using the first K principal components of 1- to 10-year yields as the state variables $\mathbf{x}_t$. We do this in the pre-2000 and post-2000 samples separately for $K = 2$ to 5. We then take the estimated model parameters and work out the model-implied β_h regression coefficients. Specifically, let $\boldsymbol{\Gamma}(j) = E[(\mathbf{x}_{t+j} - E[\mathbf{x}_{t+j}])(\mathbf{x}_t - E[\mathbf{x}_t])']$ denote the autocovariance function of the state vector, which can be obtained from the equations $vec(\boldsymbol{\Gamma}(0)) = (\mathbf{I} - \boldsymbol{\Phi}\otimes\boldsymbol{\Phi})^{-1}vec(\boldsymbol{\Sigma}\boldsymbol{\Sigma}')$ and $\boldsymbol{\Gamma}(j) = \boldsymbol{\Phi}^j\boldsymbol{\Gamma}(0)$ for $j \geq 1$. The population coefficient in a regression of h-month changes in 120-month yields on h-month changes in 12-month yields is then

$$\beta_h = \frac{E[(y_{t+h}^{(120)} - y_t^{(120)})(y_{t+h}^{(12)} - y_t^{(12)})]}{E[(y_{t+h}^{(12)} - y_t^{(12)})^2]} = \frac{1}{10}\frac{\mathbf{b}'_{(120)}[2\boldsymbol{\Gamma}(0) - \boldsymbol{\Gamma}(h) - \boldsymbol{\Gamma}(h)']\mathbf{b}_{(12)}}{\mathbf{b}'_{(12)}[2\boldsymbol{\Gamma}(0) - \boldsymbol{\Gamma}(h) - \boldsymbol{\Gamma}(h)']\mathbf{b}_{(12)}}. \tag{A.15}$$

These model-implied regression coefficients are shown in Panel A of Table A.11. Even if we include a large number of factors as state variables, these standard affine models fail to match the low-frequency decoupling between short- and long-term yields that we observe in the post-2000 data.

Non-Markovian models We next consider an alternate affine model that augments the state vector $\mathbf{x}_t$ to include not just K principal components of yields, but also $L-1$ additional lags of these principal components. Thus, by construction, the model is non-Markovian with respect to the filtration given by the current principal components. We furthermore treat these lagged principal components as unspanned state variables, as described earlier. Formally, this means that if the first K elements of the state vector $\mathbf{x}_t$ are the current principal components, all but the first K elements of $\boldsymbol{\delta}_1$ are zero, and the upper right $K \times K(L-1)$ block of $\boldsymbol{\Phi}^*$ is a matrix of zeros. As Duffee (2011) explains, a state variable will be unspanned if it has *perfectly* offsetting effects on the evolution of future short rates and future term premia.

Economically speaking, this is a rather unusual model. However, it allows us to parsimoniously capture our key finding that past changes in the level of rates are useful for forecasting future yields. In addition, similar models have been considered in Joslin et al. (2013). To be clear, we do not believe that the past increase in the level of rates is *literally* unspanned—i.e., that it has no effect on the current yield curve. Instead, we would argue that this variable is *close* to being unspanned. Specifically, like any factor that has a short-lived impact on bond risk premia, past increases in the level of rates should have only a small effect on the yield curve. Thus, in practice it will likely be quite difficult to recover information about this variable from current yields alone—e.g., because yields are measured with a tiny amount of error or because the true data-generating model evolves over time—so conditioning on this variable will add information beyond that revealed by current yields.[8]

Again, the parameters of this augmented model can be estimated, imposing the restriction that the lagged principal components are unspanned factors as in Adrian et al. (2013). The model-implied β_h regression coefficients can again be derived from equation (A.15). These model-implied coefficients are shown in Panel B of Table A.11. The augmented model is able to get reasonably close to matching the empirical regression coefficients at both high- and low-frequencies and in both samples.

Table A.11: Affine Term Structure Model-Implied coefficients in regression of monthly/yearly changes in 10-year yields on changes in 1-year yields This table reports the slope coefficients in equation (A.15) corresponding to the parameters in an affine term structure model estimated as proposed by Adrian et al. (2013) over August 1971-December 2000 and January 2001-December 2019 subsamples at monthly ($h = 1$) and yearly ($h = 12$) frequencies. The term structure model uses K principal components of yields as state variables in panel A, and adds $L-1$ additional lags of these principal components (for a total of LK state variables) in panel B. p-values are also reported; these are two-sided bootstrap p-values comparing the sample value of β_{12}/β_1 with the bootstrap distribution using that affine model. As memo items the results of the regressions using actual yields are included—these are simply transcribed from Table 1.

		Pre-2000			Post-2000		
		β_1	β_{12}	p-value	β_1	β_{12}	p-value
Panel A: ATSM with K principal components of yields as factors							
$K = 2$		0.42	0.49	0.89	0.74	0.66	0.000
$K = 3$		0.46	0.52	0.65	0.73	0.57	0.003
$K = 4$		0.47	0.52	0.62	0.72	0.52	0.006
$K = 5$		0.47	0.52	0.52	0.69	0.50	0.007
Panel B: ATSM with $L-1$ lags as additional unspanned factors							
$K = 2$	$L = 6$	0.42	0.50	0.84	0.74	0.27	0.65
$K = 3$	$L = 6$	0.46	0.54	1.00	0.74	0.31	0.41
$K = 2$	$L = 12$	0.42	0.51	0.76	0.77	0.24	0.89
$K = 3$	$L = 12$	0.46	0.55	0.90	0.77	0.29	0.60
Memo: Estimates in data (from Table 1)		0.46	0.56		0.64	0.20	

[8]Indeed the model in Section 3, would not feature unspanned variables if we recast the model to have a set of zero-coupon bonds with different maturities. Using the resulting affine model, one could recover the full $(k+2)$-dimensional state vector $\mathbf{x}_t$ from any set of $(k+2)$ yields. However, many of these variables would be close to unspanned—they would have only minimal effects on yields—and, in practice, it would be difficult to extract them from yields.

A bootstrap-based test of model misspecification As another way of looking at this, we use a bootstrap to test the hypothesis that each estimated affine term structure model is correctly specified. Our test uses the ratio of yearly to monthly coefficients (β_{12}/β_1) as the test statistic; the test rejects if this ratio is too low or too high to have been generated by the estimated Q-measure model. This is a test in the spirit of Giglio and Kelly (2018), who test the hypothesis that a given affine model is correctly specified by checking whether the comovement of rates at different points on the term structure is consistent with the estimated Q-measure dynamics.

To implement this test, we simulate the bootstrap distribution of the ratio β_{12}/β_1 for each estimated affine term-structure model. To do this for a given model, we first generate bootstrapped time series of the state vector $\mathbf{x}_t$ using a residual-based bootstrap based on our estimates of the P-measure dynamics in equation (A.7). Using our estimates of $a_{(n)}$ and $\mathbf{b}_{(n)}$ for that same model—which reflect the estimated dynamics under the Q-measure—in combination with a set of bootstrapped draws of the corresponding yield measurement errors, we obtain a bootstrapped time series of yields $y_t^{(n)}$ for $n = 12$ and 120 months. We then compute β_{12}, β_1, and the ratio β_{12}/β_1 for each bootstrapped time series. Repeating this exercise many times, we obtain a bootstrap distribution for the ratio β_{12}/β_1.

For each model, we then compute the two-sided p-value of the observed ratio β_{12}/β_1 with respect to this bootstrap distribution. These p-values are also reported in Table A.11. We find that in the pre-2000 sample, none of the models are rejected. In the post-2000 sample, the Markovian models are decisively rejected: if these standard models were correctly specified it would be highly unlikely to observe a value of β_{12}/β_1 as small as we do in the data. However, the non-Markovian models are not rejected in the post-2000 sample. In this way, we again conclude that a non-Markovian term structure model is required to match the yield-curve dynamics in the post-2000 data.

Conclusion To summarize, our conclusion is that affine term-structure models need to include lagged yield-curve factors to match the frequency-dependent sensitivity of long-term rates we observe in recent years. A large number of static yield curve factors will not do the job.

B Rate-amplification mechanisms: Microfoundations

In this Appendix, we formalize the three supply-and-demand driven channels of rate amplification discussed in the main text: the mortgage refinancing channel, the investor extrapolation channel, and the reaching-for-yield channel. For each channel, we first show how it can be used to microfound rate-amplifying shocks to the net supply of long-term bonds similar to those we introduced in reduced-form in Section 3 of the main text. We then embed each channel our general modelling framework and provide an illustrative calibration.

B.1 The mortgage refinancing channel

The model setup follows Malkhozov et al. (2016).[9] There is a constant face value M of outstanding long-term, *fixed-rate* mortgages with an embedded prepayment option. The primary mortgage rate, denoted y_t^M, equals the long-term bond yield, y_t, plus a constant spread, λ: $y_t^M = y_t + \lambda$. (This constant spread play no role in the resulting analysis and can be set to zero without loss of generality.) Let c_t^M denote the average coupon on outstanding mortgages at the beginning of time t. We assume that c_t^M evolves according to the following law of motion:

$$c_{t+1}^M - c_t^M = -\eta \cdot (c_t^M - y_t^M), \tag{A.16}$$

where $\eta \in [0,1]$. The difference between the beginning-of-period average mortgage coupon c_t^M and the current primary mortgage rate y_t^M is called the "refinancing incentive." Thus, according to equation (A.16), when the refinancing incentive is higher at time t, more households refinance their existing high-coupon mortgages at time t, leading the average mortgage coupon to fall from t to $t+1$. Iterating on equation (A.16) and making use of the fact that $y_t^M = y_t + \lambda$, we obtain:

$$c_t^M = \sum_{j=0}^{\infty} \eta\,(1-\eta)^j\, y_{t-1-j}^M = \sum_{j=0}^{\infty} \eta\,(1-\eta)^j\, y_{t-1-j} + \lambda. \tag{A.17}$$

Thus, the average mortgage coupon is just a backward-looking, geometric average of past long-term yields plus a constant. While clearly a simplification, this is a good empirical description of the average coupon on outstanding mortgages.[10]

We assume the *effective* gross supply of long-bonds that bond investors must hold at time t is

$$s_t = M \cdot DUR_t^M, \tag{A.18}$$

where M is face value of outstanding mortgages and DUR_t^M is the average "duration" or effective maturity of outstanding mortgages at time t.[11] When s_t is high, bond investors must collectively bear greater interest rate risk in equilibrium. We assume that average mortgage duration at time t is

$$DUR_t^M = \overline{DUR}^M - N \cdot \left(c_t^M - y_t^M\right), \tag{A.19}$$

where $N > 0$ is the so-called "negative convexity" of the average mortgage. Intuitively, when the refinancing incentive $(c_t^M - y_t^M)$ is high, many households are likely to refinance their mortgages in the near-term, implying that the average mortgage behaves more like a short-term bond—i.e.,

[9] Hanson (2014) explores the mortgage refinancing channel in a two period model. We follow the modelling approach in Malkhozov et al. (2016) since this allows us to speak to the dynamics which are our primary focus here.

[10] A realistic elaboration would incorporate state-dependence in the elasticity of refinancing with respect to the incentive. For instance, one might assume $c_{t+1}^M - c_t^M = -\eta\left[c_t^M - y_t^M\right] \cdot (c_t^M - y_t^M)$ where $\eta[\cdot] > 0$ and $\eta'[\cdot] > 0$, implying that the average coupon falls more when $c_t^M > y_t^M$ than when $c_t^M < y_t^M$—see e.g., Berger et al. (2018); Eichenbaum et al. (2018)

[11] Formally, duration is the semi-elasticity of a bond's price with respect to its yield. Thus, the longer a bond's duration, the greater is its exposure to movements in interest rates.

DUR_t^M is low and bond investors must bear less interest rate risk. By contrast, when the refinancing incentive is low, households are less likely to refinance and the typical mortgage behaves more like a long-term bond. Again, this is a good empirical description of DUR_t^M (Hanson, 2014; Malkhozov et al., 2016).[12]

Combining equations (A.17), (A.18), and (A.19), the *effective* supply of long-bonds at time t is

$$s_t = M \cdot \overline{DUR}^M + MN \cdot (y_t - \textstyle\sum_{j=0}^{\infty} \eta (1-\eta)^j y_{t-1-j}). \tag{A.20}$$

In other words, bond investors must bear greater interest rate risk when the long-term yield is currently high relative to its backward-looking, geometric average—i.e., when interest rates have recently risen.

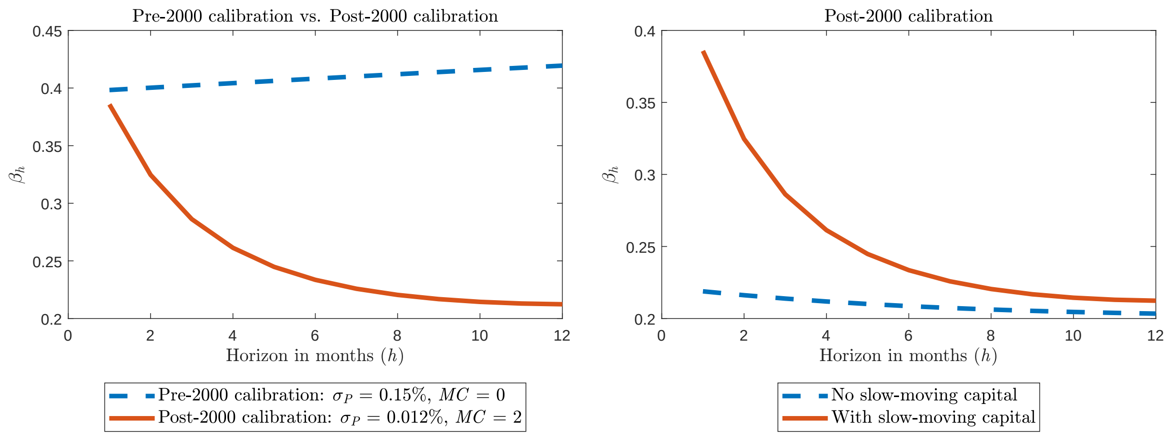
In Internet Appendix C, we solve and calibrate this model of the mortgage refinancing channel. In this version of the model, there are two reasons why shocks to short-term interest rates give rise to transitory movements in the term premium component of long-term yields. First, when $\eta > 0$, mortgage refinancing waves trigger *temporary* shifts in the effective supply of long-term bonds—i.e., these effective supply shocks are less persistent than the underlying shocks to short-term interest rates. Second, these supply shocks are met by a slow-moving arbitrage response. This combination of transitory supply shocks and a slow-moving arbitrage response creates short-lived imbalances in the market for long-term bonds, leading long-term yields to temporarily overreact to short-term rates.[13]

Naturally, this version of the model can match the key stylized fact we have documented, namely that $\beta_h = \beta_h = Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]/Var\left[i_{t+h} - i_t\right]$ is a sharply declining function of h in the post-2000 data but not in the pre-2000 data. As an illustrative calibration, we assume that $MN = 2$ in the post-2000 data and $MN = 0$ in the pre-2000 calibration. In other words, we assume that the mortgage refinancing channel is operative in the post-2000 period, but was not operative in the

[12] As detailed in Hanson (2014), there are two key reasons why movements in expected mortgage refinancing temporarily alters the aggregate amount of interest rate risk that specialized bond investors must bear. First, households only *gradually* refinance their mortgages following a decline in primary mortgage rates. Second, household borrowers do not alter their asset-side holdings of long-term bonds to hedge the time-varying interest rate risk they are bearing on the liability side. In combination, these features mean that households are effectively borrowing shorter term during refinancing waves when the refinancing incentive, $(c_t^M - y_t^M)$, is high. As a result, households bear greater interest rate risk during refinancing waves, while bond investors bear less risk. In summary, refinancing waves function like shocks to the effective supply of long-term bonds because risk sharing between households and specialized bond investors is imperfect and varies over time.

[13] One simplification of this model is that all bond investors hold mortgage-backed securities (MBS) and, thus, bear a time-varying amount interest rate risk. In practice, two different kinds of investors own MBS. One set of MBS investors—e.g., mortgage banks and the government sponsored enterprises—"delta-hedge" the embedded prepayment option and, thus, bear a (relatively) constant amount of interest rate risk over time. Other MBS investors do not delta-hedge and bear a time-varying amount of risk. As discussed in Hanson (2014), in the first instance, it does not matter whether some MBS holders delta-hedge the prepayment option since the relevant hedging flows correspond one-for-one with changes in the aggregate quantity of duration risk. However, a slow-moving arbitrage response to refinancing waves arguably becomes more relevant to the extent that some MBS investors delta-hedge their time-varying interest rate exposure.

Figure A.5: Illustrative calibration of mortgage refinancing model.



pre-2000 period. We assume $\eta = 0.15$ in both periods. The values of all other model parameters, including $q = 0.30$ and $k = 12$, are the same as those in the calibrations in Section 3.

B.2 Investor extrapolation channel

Recalling that $i_t = i_{P,t} + i_{T,t}$, we assume that diagnostic investors make biased forecasts of the persistent and transitory components of short-term interest rates. Following Maxted (2020), we assume the expectations of diagnostic investors are given by:

$$E_t^D[i_{P,t+1}] = \bar{\imath} + \rho_P(i_{P,t} - \bar{\imath}) + \theta \cdot m_{P,t}, \quad \text{(A.21a)}$$

$$E_t^D[i_{T,t+1}] = \rho_T i_{T,t} + \theta \cdot m_{T,t}, \quad \text{(A.21b)}$$

where

$$m_{P,t} = \kappa_P m_{P,t-1} + \varepsilon_{P,t} = (i_{P,t} - \bar{\imath}) - (\rho_P - \kappa_P)\sum_{j=0}^{\infty} \kappa_P^j (i_{P,t-j-1} - \bar{\imath}), \quad \text{(A.22a)}$$

$$m_{T,t} = \kappa_T m_{T,t-1} + \varepsilon_{T,t} = i_{T,t} - (\rho_T - \kappa_T)\sum_{j=0}^{\infty} \kappa_T^j i_{T,t-j-1}, \quad \text{(A.22b)}$$

$\theta \geq 0$, $\kappa_P \in [0, \rho_P]$, and $\kappa_T \in [0, \rho_T]$. When $\theta = 0$, diagnostic expectations coincide with rational expectations, which we continue to denote using $E_t[\cdot]$. When $\theta > 0$, equations (A.21) and (A.22) imply that diagnostic investors tend to overestimate future short-term rates when short rates have recently risen. And, the κ_P and κ_T parameters govern the persistence of their mistaken beliefs about short rates.[14] While diagnostic investors make biased forecasts of short rates, we assume for simplicity that they form rational forecasts of all other relevant state variables.

[14] As shown in Maxted (2020), these κ parameters are a simple way of parameterizing the "background context" that diagnostic investors use to assess the "representativeness" of incoming data for future states. Specifically, in the limit where κ_P and $\kappa_T \to 0$, the background context when making forecasts at time t is what diagnostic investors knew at time $t-1$ as in Bordalo et al. (2017) and the resulting expectational errors are very short-lived. In the opposite limit where $\kappa_P \to \rho_P$ and $\kappa_T \to \rho_T$, the background context at time t is the unconditional distribution of short rates as in D'Arienzo (2020) and the resulting expectational errors are far more persistent.

A mass f of bond investors have diagnostic expectations and their demand for long-term bonds is:

$$h_t = \tau \frac{E_t^D [rx_{t+1}]}{Var_t^D [rx_{t+1}]} = \tau \frac{E_t^D [rx_{t+1}]}{Var_t [rx_{t+1}]}, \tag{A.23}$$

where $E_t^D [rx_{t+1}]$ denotes diagnostic investors' biased expectation of bond excess returns.[15] There is a mass $(1-f)$ of a bond investors with rational expectations. Of these rational investors, fraction q are fast-moving with demands $b_t = \tau (E_t [rx_{t+1}] / Var_t [rx_{t+1}])$ and fraction $(1-q)$ are slow-moving and only rebalance the portfolios every k periods. The demand for long-term bonds from the subset of slow-moving investors who are active at time t is $d_t = \tau(E_t[\sum_{j=1}^k rx_{t+j}]/Var_t[\sum_{j=1}^k rx_{t+j}])$. We assume the *gross supply* of long-term bonds is *constant* over time and equal to $\bar{s}$. Thus, the market clearing condition for long-term bonds at time t is:

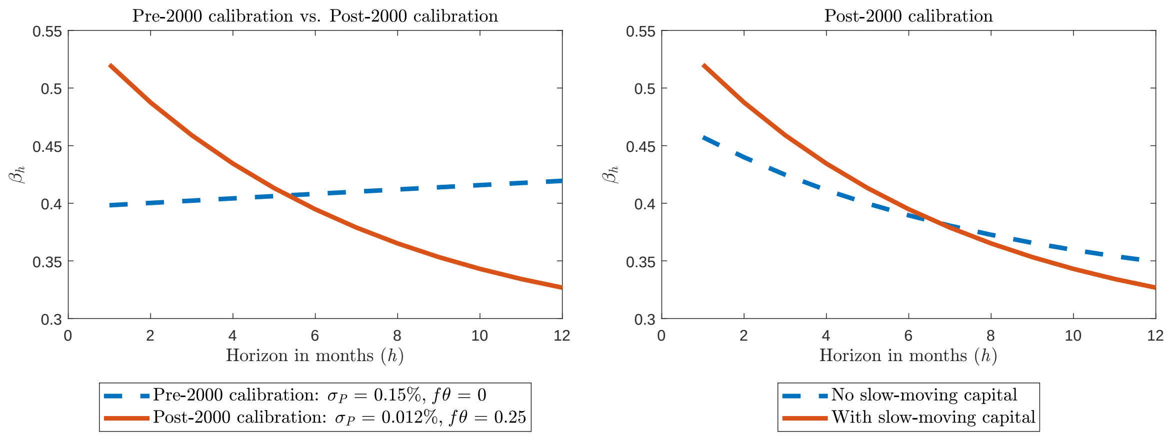
$$\overbrace{fh_t + (1-f)\, qb_t + (1-f)\,(1-q)\, k^{-1} d_t}^{\text{Active demand}} = \overbrace{\bar{s} - (1-f)\,(1-q)k^{-1} \sum_{i=1}^{k-1} d_{t-i}}^{\text{Active supply}}. \tag{A.24}$$

In Internet Appendix C, we solve for equilibrium in this setting. When short rates have recently fallen, diagnostic investors underestimate future short-term interest rates and, as a result, want to hold more long-term bonds. To accommodate this induced demand shock, rational investors must reduce their holdings of long-term bonds, pushing down the term premium compensation required by rational investors. Since the biases of diagnostic investors are tied to recent changes in short rates, this model is nearly isomorphic to our reduced-form specification where the short-rate-driven shocks to bond supply are more transient than short rates. In particular, this investor extrapolation channel leads long-term rates to *temporarily* overreact to movements in short rates due to the combination of (i) transitory shifts in non-fundamental demand for long-term bonds that are triggered by short rate shocks and (ii) a slow-moving arbitrage response to these non-fundamental demand shifts.

As shown in the illustrative calibration below, this model in which investors extrapolate changes in short-term interest rates can match the key stylized facts we document. In the post-2000 period, we assume that $f = 50\%$ of investors have diagnostic expectations with parameter $\theta = 0.5$ (we set $\kappa_P = \kappa_T = 0.8$) and that $q = 0.15$ and $k = 18$, so there is a fairly slow-moving arbitrage response to the resulting non-fundamental shifts in demand for long-term bonds. In the pre-2000 period, we assume that $f = 0$. As discussed in the main text, the rise in f is meant to capture the growing importance of extrapolation-prone bond fund investors in the U.S. bond market in recent decades. The values of all other model parameters are the same as those in the calibrations in Section 3.

[15] We have $Var_t^D [rx_{t+1}] = Var_t [rx_{t+1}]$ since, as shown by Maxted (2020), diagnostic investors perceive the same conditional variance of future short-term interest rates as rational investors.

Figure A.6: Illustrative calibration of the investor extrapolation model.



B.3 Reaching-for-yield channel

We assume that fraction f of bond investors are "yield-seeking" and have non-standard preferences as in Hanson and Stein (2015). The idea is that, for either frictional or behavioral reasons, these investors care about the *current yield* on their portfolios over and above expected portfolio returns. Specifically, yield-seeking investors' demand for long-term bonds is:

$$h_t = \tau \frac{y_t - i_i}{V^{(1)}}. \quad \text{(A.25)}$$

Since $E_t[rx_{t+1}] = (y_t - i_i) - (\phi/(1-\phi)) \cdot E_t[y_{t+1} - y_t]$, equation (A.25) implies that yield-seeking investors are only concerned with the current income or carry from holding long-term bonds and neglect any expected capital gains and losses from holding long-term bonds. A mass $(1-f)$ of a bond investors are expected-return-oriented and have standard mean-variance preferences. Of these expected-return-oriented investors, fraction q are fast-moving investors with demands $b_t = \tau(E_t[rx_{t+1}]/Var_t[rx_{t+1}])$ and fraction $(1-q)$ are slow-moving. The demand for long-term bonds from the subset of slow-moving investors who are active at time t is $d_t = \tau(E_t[\sum_{j=1}^{k} rx_{t+j}]/Var_t[\sum_{j=1}^{k} rx_{t+j}])$. We assume the *gross supply* of long-term bonds is *constant* over time and equal to $\overline{s}$. Thus, the market clearing condition for long-term bonds at time t is the same as in equation (A.24).

In Internet Appendix C, we solve for equilibrium in this setting. To build intuition, first consider the case where there is no slow-moving capital. In this case where $q = 1$, our model is simply an infinite-horizon version of the 2-period model in Hanson and Stein (2015). Because expected mean reversion in short rates implies that the yield curve is steep when short rates are low, yield-seeking investors' demand for long-term bonds is higher when short rates are lower.[16] To accommodate this induced demand shock, expected-return-oriented investors must reduce their holdings of long-term bonds when short rates are low, pushing down the term premium compensation they require.

[16] Formally, $E_t[y_{t+1} - y_t] > 0$ when i_t is low, so $(y_t - i_i) > E_t[rx_{t+1}]$ when i_t is low. As a result, $h_t = \tau((y_t - i_i)/Var_t[rx_{t+1}]) > \tau(E_t[rx_{t+1}]/Var_t[rx_{t+1}]) = b_t$ when i_t is low. Conversely, $h_t < b_t$ when i_t is high.

Thus, in the absence of slow-moving capital, long-term rates are excessively sensitive to short rates because short rates and term premium move in the same direction. However, without slow-moving capital, this excess sensitivity is *not* greater at short horizons—i.e., changes in short rates do not create temporary market imbalances.

When $q < 1$, our model adds a slow-moving arbitrage response to the price-pressure created by yield-seeking investors. This means that the excess sensitivity of long-term rates to short-term rates will be greatest at short horizons. The intuition is simple. Suppose there is an decline in short rates which steepens the yield curve, thereby boosting yield-seeking investors' demand for long-term bonds. In the short-run, the only expected-return-oriented investors can absorb this induced demand shock for long-term bonds are the fast-moving ones and the slow-moving ones who initially happen to be active. However, the mass of slow-moving investors who can absorb this induced demand shock grows over time. As a result, the excess sensitivity of long-term rates to movements in short-term rates is greatest at high frequencies and diminishes at lower frequencies.

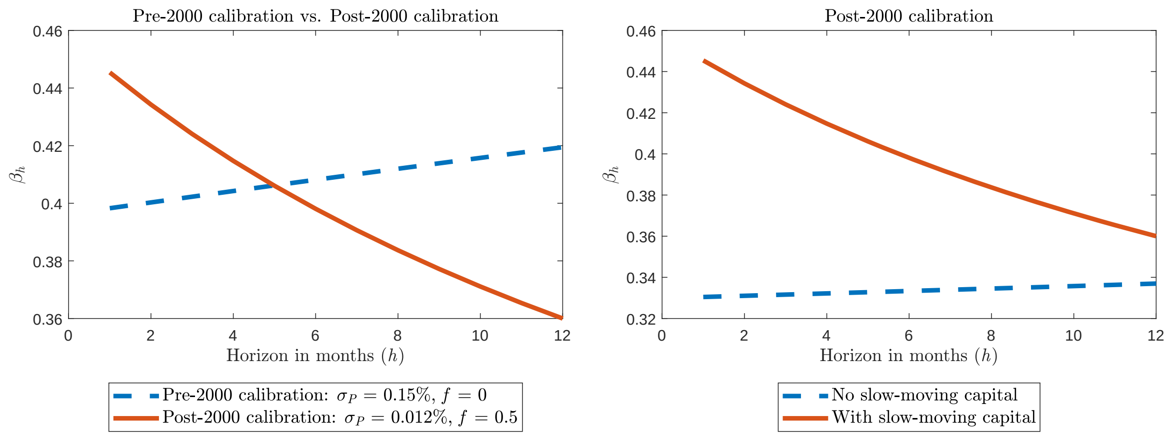
This model of the reaching-for-yield channel can qualitatively match the key stylized facts we have documented. Specifically, in the post-2000 period, we assume that $f = 0.5$, $q = 0.1$, and $k = 24$—i.e., we assume a good deal of reaching for yield and a fairly sluggish arbitrage response. By contrast, we assume that $f = 0$ in the pre-2000 period. The rise in f is consistent with the idea that yield-seeking investor behavior has become stronger in recent decades. The values of all other model parameters are the same as those in the calibrations in Section 3. As shown below, we see that β_h is decreasing in h in the post-2000 calibration, but is increasing in h—and far less variable—in the pre-2000 calibration.

While the combination of reaching-for-yield and slow-moving capital generates horizon-dependent sensitivity, our calibrations struggle to *quantitatively* match the profile of β_h seen in the post-2000 data. Specifically, comparing the calibration of the reaching-for-yield model in Figure A.7 with those for the mortgage refinancing and investor extrapolation models in Figures A.5 and A.6, respectively, we see that the former model struggles generate quantitatively match the steep post-2000 profile of β_h. This is because the reaching-for-yield channel generates highly persistent shifts rate-amplifying net supply, whereas the refinancing and extrapolation channels generate transitory shifts in net supply. And, as we have emphasized throughout, strongly horizon-dependent excess sensitivity is most likely to arise when transitory rate-amplifying supply-and-demand shocks are met by a slow-moving arbitrage response.

C Solution of the baseline model

This Appendix provides additional details on our economic model. In particular, we provide additional details on how we solve for the rational expectations equilibrium of our model.

Figure A.7: Illustrative calibration of the investor reaching-for-yield model.



C.1 Long-term nominal bonds

In this subsection, we derive the Campbell and Shiller (1988) approximation to the return on a default-free perpetuity.

Consider a perpetual default-free nominal bond which pay a nominal coupon of K each period. Let P_t denote the nominal price of the long-term bond at time t. Thus, the nominal return on the long-term bond from t to $t+1$ is

$$1 + R_{t+1} = \frac{P_{t+1} + K}{P_t}. \tag{A.26}$$

Defining $\phi \equiv 1/(1+K) < 1$, the one-period log return on the bond from time t to $t+1$ is approximately

$$r_{t+1} \equiv \ln(1 + R_{t+1}) \approx \overbrace{\frac{1}{1-\phi}}^{D} y_t - \overbrace{\frac{\phi}{1-\phi}}^{D-1} y_{t+1}, \tag{A.27}$$

where y_t is the log yield-to-maturity at time t and

$$D = \frac{1}{1-\phi} = \frac{K+1}{K} \tag{A.28}$$

is the Macaulay duration when the bond is trading at par. The log-linear approximation for default-free coupon-bearing bonds in equation (A.27) appears in Chapter 10 of Campbell et al. (1996).

To derive this approximation, note that the Campbell-Shiller (1988) approximation of the 1-period log return on the long-term bond is

$$r_{t+1} = \ln(P_{t+1} + K) - p_t \approx \varphi + \phi p_{t+1} + (1-\phi) k - p_t, \tag{A.29}$$

where $p_t = \log(P_t)$ is the log price, $k = \log(K)$ is the log coupon, and where $\phi = 1/(1 + \exp(k - \overline{p}))$ and $\varphi = -\log(\phi) - (1-\phi)\log(\phi^{-1} - 1)$ are parameters of the log-linearization. Iterating equation

(A.29) forward, we find that the log bond price is

$$p_t = (1-\phi)^{-1}\varphi + k - \sum_{i=0}^{\infty}\phi^i E_t\left[r_{t+i+1}\right]. \tag{A.30}$$

Applying this approximation to the *yield-to-maturity*, defined as the *constant return* that equates bond price and the discounted value of promised cashflows, we obtain

$$p_t = (1-\phi)^{-1}\varphi + k - (1-\phi)^{-1} y_t. \tag{A.31}$$

Equation (A.27) then follows by substituting the expression for p_t in equation (A.31) into the Campbell-Shiller return approximation in equation (A.29).

Assuming the steady-state price of the bonds is par ($\overline{p}=0$), we have $\phi = 1/(1+K)$. Thus, bond duration is $D = -\partial p_t/\partial y_t = (1-\phi)^{-1} = (1+K)/K$. Since $-\partial p_t/\partial y_t = -(\partial P_t/\partial Y_t)((1+Y_t)/P_t) = (Y_t^{\cdot}+1)/Y_t$ this corresponds to Macaulay duration when the bonds are trading at par ($Y_t = K$).

Let i_t denote the interest rate on short-term nominal bonds from t to $t+1$ and let $rx_{t+1} \equiv r_{t+1} - i_t$ denote the excess return on long-term nominal bonds over short-term nominal bonds from t to $t+1$. Then, iterating equation (A.27) forward and taking expectations, the yield on long-term nominal bonds is given by:

$$y_t = (1-\phi)\sum_{j=0}^{\infty}\phi^j E_t\left[i_{t+j} + rx_{t+j+1}\right]. \tag{A.32}$$

C.2 Bond market participants

There are two types of risk-averse arbitrageurs in the model, each with identical risk tolerance τ, who differ solely in the frequency with which they can rebalance their bond portfolios.

The first group of arbitrageurs are fast-moving arbitrageurs who are free to adjust their holdings of long-term and short-term bonds each period. Fast-moving arbitrageurs are present in mass q and we denote their demand for long-term bonds at time t by b_t. Fast-moving arbitrageurs have mean-variance preferences over 1-period portfolio log returns. Thus, their demand for long-term bonds at time t is given by

$$b_t = \tau \frac{E_t\left[rx_{t+1}\right]}{Var_t\left[rx_{t+1}\right]}, \tag{A.33}$$

where

$$rx_{t+1} \equiv r_{t+1} - i_t = \frac{1}{1-\phi}y_t - \frac{\phi}{1-\phi}y_{t+1} - i_t \tag{A.34}$$

is the excess return on long-term bonds from t to $t+1$.

The second group of arbitrageurs is a set of slow-moving arbitrageurs who can only adjust their holdings of long-term and short-term bonds every k periods. Slow-moving arbitrageurs are present in mass $1-q$. A fraction $1/k$ of these slow-moving arbitrageurs is active each period and can reallocate their portfolios. However, they must then maintain this same portfolio allocation for the next k periods. As in Duffie (2010), this is a reduced-form way to model the frictions that limit the speed of capital flows. Since they only rebalance their portfolios every k periods, slow-moving arbitrageurs have mean-variance preferences over their k-period *cumulative* portfolio excess return. Thus, the demand for long-term bonds from the subset of slow-moving arbitrageurs who are active at time t is given by

$$d_t = \tau \frac{E_t[\sum_{j=1}^{k} rx_{t+j}]}{Var_t[\sum_{j=1}^{k} rx_{t+j}]}. \tag{A.35}$$

C.3 Risk factors

Short-term nominal interest rates: Short-term nominal bonds are available in perfectly elastic supply. At time t, arbitrageurs learn that short-term bonds will earn a riskless log return of i_t in nominal terms between time t and $t+1$. We assume that the short-term nominal interest rate is the sum of a highly persistent component $i_{P,t}$ and a more transient component $i_{T,t}$:

$$i_t = i_{P,t} + i_{T,t}. \tag{A.36}$$

We assume that the persistent component $i_{P,t}$ follows an exogenous AR(1) process:

$$i_{P,t+1} = \bar{\imath} + \rho_P \left(i_{P,t} - \bar{\imath}\right) + \varepsilon_{P,t+1}, \tag{A.37}$$

where $0 < \rho_P < 1$ and $Var_t\left[\varepsilon_{P,t+1}\right] = \sigma_P^2$. Similarly, we assume that the transient component $i_{T,t}$ follows an exogenous AR(1) process:

$$i_{T,t+1} = \rho_T i_{T,t} + \varepsilon_{T,t+1}, \tag{A.38}$$

where $0 < \rho_T \leq \rho_P < 1$ and $Var_t\left[\varepsilon_{T,t+1}\right] = \sigma_T^2$.

Supply of long-term bonds: We assume that the long-term nominal bond is available in an exogenous, time-varying *net supply* s_t that must be held in equilibrium by fast arbitrageurs and slow-moving arbitrageurs. This net supply equals the *gross supply* of long-term bonds *minus the demand* for long-term bonds from any unmodeled agents who have inelastic demand for these bonds. Formally, we assume that s_t follows an AR(1) process:

$$s_{t+1} = \bar{s} + \rho_s \left(s_t - \bar{s}\right) + \varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1}, \tag{A.39}$$

where $0 < \rho_s \leq \rho_T$, $C > 0$, and $Var_t\left[\varepsilon_{s,t+1}\right] = \sigma_s^2$. The $\varepsilon_{s,t+1}$ shocks in equation (A.39) capture other forces that are unrelated to short rates which also impact the net supply of long-term bonds. While the model can be solved for any arbitrary correlation structure between the $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$ shocks, we assume, for simplicity, that these three underlying shocks are mutually orthogonal.

To understand the implied process for net bond supply, let L denote the time-series lag operator and note

$$\begin{aligned}(1-\rho_s L)(s_t-\overline{s}) &= \varepsilon_{s,t}+C\varepsilon_{P,t}+C\varepsilon_{T,t} \\ &= \varepsilon_{s,t}+C(1-\rho_P L)(i_{P,t}-\overline{\imath})+C(1-\rho_T L)\,i_{T,t}.\end{aligned} \tag{A.40}$$

Working out the lag polynomial, we see that

$$\begin{aligned}s_t = \overline{s}+C[(i_{P,t}-\overline{\imath})-(\rho_P-\rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j(i_{P,t-j-1}-\overline{\imath})] \\ +C[i_{T,t}-(\rho_T-\rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j i_{T,t-j-1}]+[\textstyle\sum_{j=0}^{\infty}\rho_s^j\varepsilon_{s,t-j}].\end{aligned} \tag{A.41}$$

which follows from the fact that

$$\begin{aligned}(1-\rho_s L)^{-1}(1-\rho_x L)\,x_t &= \textstyle\sum_{j=0}^{\infty}\rho_s^j L^j(1-\rho_x L)\,x_t \\ &= x_t-\rho_x x_{t-1}+\rho_s x_{t-1}-\rho_s\rho_x x_{t-2}+\rho_s^2 x_{t-2}-\rho_s^3\rho_x x_{t-2}+\cdots \\ &= x_t-(\rho_x-\rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j x_{t-j-1}.\end{aligned} \tag{A.42}$$

C.4 Equilibrium Conjecture

For the sake of concreteness, suppose that $k=4$. We conjecture that equilibrium yields take the form

$$y_t=\alpha_0+\boldsymbol{\alpha}_1'\mathbf{x}_t, \tag{A.43}$$

and that the demands of active slow-moving arbitrageurs are of the form

$$d_t=\delta_0+\boldsymbol{\delta}_1'\mathbf{x}_t, \tag{A.44}$$

where the $k+2$ dimensional state vector is

$$\mathbf{x}_t=\begin{bmatrix} i_{P,t}-\bar{i} \\ i_{T,t} \\ s_t-\overline{s} \\ d_{t-1}-\delta_0 \\ d_{t-2}-\delta_0 \\ d_{t-3}-\delta_0 \end{bmatrix}. \tag{A.45}$$

These assumptions imply that the state vector follows an AR(1) process. Critically, the transition matrix Γ is a function of the parameters $\boldsymbol{\delta}_1$ governing slow-moving arbitrageur demand so we write $\Gamma=\Gamma(\boldsymbol{\delta}_1)$. Specifically, we have

$$\begin{aligned}\mathbf{x}_{t+1} &= \Gamma(\delta)\,\mathbf{x}_t+\boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_s & 0 & 0 & 0 \\ \delta_P & \delta_T & \delta_s & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_{P,t}-\bar{i} \\ i_{T,t} \\ s_t-\overline{s} \\ d_{t-1}-\delta_0 \\ d_{t-2}-\delta_0 \\ d_{t-3}-\delta_0 \end{bmatrix}+\begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ \varepsilon_{s,t+1}+C\varepsilon_{P,t+1}+C\varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}\end{aligned} \tag{A.46}$$

where $\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right]$. Since we have assumed that $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_P^2 & 0 & C\sigma_P^2 & 0 & 0 & 0 \\ 0 & \sigma_T^2 & C\sigma_\pi^2 & 0 & 0 & 0 \\ C\sigma_P^2 & C\sigma_T^2 & C^2\sigma_P^2 + C^2\sigma_T^2 + \sigma_s^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.47}$$

We adopt the convention that $\mathbf{e}$ is the vector with a 1 corresponding to $i_{P,t} - \bar{i}$ and $i_{T,t}$ and 0s elsewhere, i.e., $\mathbf{e} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}'$; $\mathbf{e}_s$ is the basis vector with a 1 corresponding to $s_t - \bar{s}$ and 0s elsewhere, i.e., $\mathbf{e}_s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}'$; and $\mathbf{e}_d$ is a 1 corresponding to $d_{t-1} - \delta_0$, $d_{t-2} - \delta_0$, $\cdots$, $d_{t-(k-1)} - \delta_0$ and 0s elsewhere, i.e., $\mathbf{e}_d = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}'$.

Finally, we denote $\mathbf{C}_{[t+i,t+j]} = Cov\left[\mathbf{x}_{t+i}, \mathbf{x}_{t+j} | \mathbf{x}_t\right]$ and note that

$$\mathbf{C}_{[t+i,t+j]} = \sum\nolimits_{s=1}^{\min\{i,j\}} [\boldsymbol{\Gamma}^{i-s}]\boldsymbol{\Sigma}[\boldsymbol{\Gamma}^{j-s}]', \tag{A.48}$$

so $\mathbf{C}_{[t+j,t+i]} = \mathbf{C}'_{[t+i,t+j]}$.

C.5 Arbitrageur demands

Fast-moving arbitrageurs' demand: Given this conjecture, we work out fast-moving arbitrageurs' demand for long-term bonds. Given the conjectured form of equilibrium yields, the realized 1-period excess returns on long bonds from t to $t+1$ is

$$\begin{aligned} rx_{t+1} &= \frac{1}{1-\phi} y_t - \frac{\phi}{1-\phi} y_{t+1} - i_t \\ &= \left(\alpha_0 - \bar{i}\right) + \left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' \mathbf{x}_t - \left(\frac{\phi}{1-\phi}\boldsymbol{\alpha}_1\right)' \mathbf{x}_{t+1}, \end{aligned} \tag{A.49}$$

which implies

$$E_t\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' \mathbf{x}_t - \left(\frac{\phi}{1-\phi}\boldsymbol{\alpha}_1\right)' \boldsymbol{\Gamma}\mathbf{x}_t \tag{A.50}$$

and

$$Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1 \tag{A.51}$$

Thus, fast-moving arbitrageurs' demand for long-term bonds is

$$b_t = \tau \frac{E_t\left[rx_{t+1}\right]}{Var_t\left[rx_{t+1}\right]} = \left[\tau \frac{\alpha_0 - \bar{i}}{\left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1}\right] + \left[\tau \frac{\left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' - \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1' \boldsymbol{\Gamma}}{\left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1}\right] \mathbf{x}_t. \tag{A.52}$$

Slow-moving arbitrageurs' demand: We next work out slow-moving arbitrageurs' demand for long-term bonds. Given our conjecture, the realized k-period cumulative excess returns on long bonds from t to $t+k$ is

$$\begin{aligned}\textstyle\sum_{j=1}^{k} rx_{t+j} &= \textstyle\sum_{j=0}^{k-1}\left(y_{t+j}-i_{t+j}\right)-\frac{\phi}{1-\phi}\left(y_{t+k}-y_{t}\right) \\ &= k\left(\alpha_{0}-\bar{i}\right)+\left(\boldsymbol{\alpha}_{1}-\mathbf{e}\right)^{\prime}\left(\textstyle\sum_{j=0}^{k-1}\mathbf{x}_{t+j}\right)-\frac{\phi}{1-\phi}\boldsymbol{\alpha}_{1}^{\prime}\left(\mathbf{x}_{t+k}-\mathbf{x}_{t}\right)\end{aligned} \tag{A.53}$$

Thus, expected k-period cumulative returns are

$$E_{t}[\textstyle\sum_{j=1}^{k} rx_{t+j}]=k\left(\alpha_{0}-\bar{i}\right)+\left(\left(\boldsymbol{\alpha}_{1}-\mathbf{e}\right)^{\prime}\left(\mathbf{I}-\boldsymbol{\Gamma}\right)^{-1}+\frac{\phi}{1-\phi}\boldsymbol{\alpha}_{1}^{\prime}\right)\left(\mathbf{I}-\boldsymbol{\Gamma}^{k}\right)\mathbf{x}_{t}, \tag{A.54}$$

and the variance of k-period cumulative excess returns is

$$\begin{aligned}Var_{t}[\textstyle\sum_{j=1}^{k} rx_{t+j}] &= Var_{t}[\left(\boldsymbol{\alpha}_{1}-\mathbf{e}\right)^{\prime}\left(\textstyle\sum_{j=1}^{k-1}\mathbf{x}_{t+j}\right)-\left(\frac{\phi}{1-\phi}\right)\boldsymbol{\alpha}_{1}^{\prime}\mathbf{x}_{t+k}] \\ &= \left(\boldsymbol{\alpha}_{1}-\mathbf{e}\right)^{\prime}\left(\textstyle\sum_{l=1}^{k-1}\sum_{j=1}^{k-1}\mathbf{C}_{[t+l,t+j]}\right)\left(\boldsymbol{\alpha}_{1}-\mathbf{e}\right)+\left(\frac{\phi}{1-\phi}\right)^{2}\boldsymbol{\alpha}_{1}^{\prime}\mathbf{C}_{[t+k,t+k]}\boldsymbol{\alpha}_{1} \\ &\quad -2\left(\frac{\phi}{1-\phi}\right)\left(\boldsymbol{\alpha}_{1}-\mathbf{e}\right)^{\prime}\textstyle\sum_{j=1}^{k-1}\mathbf{C}_{[t+j,t+k]}\boldsymbol{\alpha}_{1}.\end{aligned} \tag{A.55}$$

Slow-moving arbitrageurs' demand long long-term bonds is

$$d_{t}=\tau\frac{E_{t}[\sum_{j=1}^{k} rx_{t+j}]}{Var_{t}[\sum_{j=1}^{k} rx_{t+j}]}. \tag{A.56}$$

Thus, given our conjectures, slow-moving arbitrageurs demands will indeed take a linear form. Specifically, we have

$$\delta_{0}=\tau\frac{k\left(\alpha_{0}-\bar{i}\right)}{V^{(k)}} \tag{A.57}$$

where $V^{(k)}=Var_{t}[\sum_{j=1}^{k} rx_{t+j}]$ and

$$\boldsymbol{\delta}_{1}^{\prime}=\tau\frac{\left(\left(\boldsymbol{\alpha}_{1}-\mathbf{e}\right)^{\prime}\left(\mathbf{I}-\boldsymbol{\Gamma}\right)^{-1}+\frac{\phi}{1-\phi}\boldsymbol{\alpha}_{1}^{\prime}\right)}{V^{(k)}}\left(\mathbf{I}-\boldsymbol{\Gamma}^{k}\right) \tag{A.58}$$

C.6 Equilibrium solution

To solve for the equilibrium, we need to clear the market for bonds in a way that is consistent with optimization on the part of fast-moving arbitrageurs and slow-moving arbitrageurs. The market-clearing condition is

$$\overbrace{(1-q)k^{-1}d_{t}+qb_{t}}^{\text{Active demand}}=\overbrace{s_{t}-(1-q)(k^{-1}\textstyle\sum_{i=1}^{k-1}d_{t-i})}^{\text{Active supply}}. \tag{A.59}$$

Letting $V^{(1)} = Var_t[rx_{t+1}] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1$, denote the variance of 1-period excess returns, active demand is

$$\begin{aligned}(1-q)k^{-1}d_t + qb_t & \qquad \text{(A.60)} \\ = \left[(1-q)k^{-1}\delta_0 + q\tau\frac{(\alpha_0 - \bar{i})}{V^{(1)}}\right] + \left[(1-q)k^{-1}\boldsymbol{\delta}_1' + q\tau\frac{\left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' - \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\boldsymbol{\Gamma}}{V^{(1)}}\right]\mathbf{x}_t\end{aligned}$$

Active supply is

$$\begin{aligned}s_t - (1-q)k^{-1}\textstyle\sum_{i=1}^{k-1} d_{t-i} & \qquad \text{(A.61)} \\ = \left[\bar{s} - (1-q)\frac{(k-1)}{k}\delta_0\right] + \left[\left(\mathbf{e}_s - (1-q)k^{-1}\mathbf{e}_d\right)'\right]\mathbf{x}_t.\end{aligned}$$

Matching constants terms, we obtain

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) \qquad \text{(A.62)}$$

Matching slope coefficients, we have

$$\begin{aligned}\boldsymbol{\alpha}_1 &= (1-\phi)\left[\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\mathbf{e} + (1-\phi)\frac{V^{(1)}}{\tau q}\left[\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left[\mathbf{e}_s - k^{-1}(1-q)\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right] \qquad \text{(A.63)} \\ &= \frac{1-\phi}{1-\phi\rho_P}\mathbf{e}_P + \frac{1-\phi}{1-\phi\rho_T}\mathbf{e}_T + \frac{V^{(1)}}{\tau q}\left[\frac{1-\phi}{1-\phi\rho_s}\mathbf{e}_s - k^{-1}(1-q)(1-\phi)\left[\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right]\end{aligned}$$

where $\mathbf{e}_P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$ and $\mathbf{e}_T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}'$.

Thus, equilibrium yields take the form

$$\begin{aligned}y_t &= \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t \qquad \text{(A.64)} \\ &= \overbrace{\bar{i} + \frac{1-\phi}{1-\phi\rho_P}\left(i_{P,t} - \bar{i}\right) + \frac{1-\phi}{1-\phi\rho_T}i_{P,t}}^{\text{Expected future short real rates}} \\ &+ \overbrace{\frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right)}^{\text{Unconditional term premia}} \\ &+ \overbrace{\left[\frac{V^{(1)}}{\tau q}\left(\frac{1-\phi}{1-\phi\rho_s}(s_t - \bar{s}) - (1-\phi)(1-q)k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)'\left[\mathbf{I} - \phi\boldsymbol{\Gamma}\right]^{-1}\mathbf{x}_t\right)\right]}^{\text{Conditional term premia}}.\end{aligned}$$

Equilibrium excess returns are given by

$$E_t[rx_{t+1}] = \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) + \frac{V^{(1)}}{\tau q}\left[(s_t - \bar{s}) - (1-q)k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)'\mathbf{x}_t\right] \qquad \text{(A.65)}$$

C.7 Equilibrium existence and uniqueness

A rational expectations equilibrium of our model is a fixed point of a specific operator involving the "price-impact" coefficients, $(\boldsymbol{\alpha}_1')$, which show how bond supply and inactive slow-moving arbitrageur demand impact bond yields, and the "demand-impact" coefficients, $(\boldsymbol{\delta}_1')$, which show how bond supply and inactive demand impact the demand of active slow-moving arbitrageurs. Specifically, let $\boldsymbol{\omega} = (\boldsymbol{\alpha}_1', \boldsymbol{\delta}_1')'$ and consider the operator $\mathbf{f}(\boldsymbol{\omega}_0)$ which gives (i) the price-impact coefficients that will clear the market for long-term bonds and (ii) the demand-impact coefficients consistent with optimization on the part of active slow-moving arbitrageurs when arbitrageurs conjecture that $\boldsymbol{\omega} = \boldsymbol{\omega}_0$ at all future dates. Thus, a rational expectations equilibrium of our model is a fixed point $\boldsymbol{\omega}^* = \mathbf{f}(\boldsymbol{\omega}^*)$.

Specifically, an equilibrium solves the following system of equations

$$\boldsymbol{\alpha}_1 = (1-\phi)\left[\mathbf{I} - \phi\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)'\right]^{-1}\left[\mathbf{e} + \frac{V^{(1)}(\boldsymbol{\alpha}_1)}{\tau q}\left(\mathbf{e}_s - k^{-1}(1-q)(\mathbf{e}_d + \boldsymbol{\delta}_1)\right)\right] \tag{A.66}$$

and

$$\boldsymbol{\delta}_1' = \tau\frac{\left((\boldsymbol{\alpha}_1 - \mathbf{e})'(\mathbf{I} - \Gamma(\boldsymbol{\delta}_1))^{-1} + \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\right)}{V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)}\left(\mathbf{I} - \boldsymbol{\Gamma}(\boldsymbol{\delta}_1)^k\right) \tag{A.67}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\Gamma(\boldsymbol{\delta}_1)$ to emphasize that the transition matrix depends on $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)$ to emphasize that the k-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. We can write this system of non-linear equations more compactly as

$$\boldsymbol{\alpha}_1 = \mathbf{f}_{\boldsymbol{\alpha}_{A1}}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1) \text{ and } \boldsymbol{\delta}_1 = \mathbf{f}_{\boldsymbol{\delta}_1}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1) \tag{A.68}$$

or simply as $\boldsymbol{\omega} = \mathbf{f}(\boldsymbol{\omega})$ where $\boldsymbol{\omega} = (\boldsymbol{\alpha}_1', \boldsymbol{\delta}_1')'$.

This is a system of $2(k+1)$ equations in $2(k+1)$ unknowns. However, in any rational expectations equilibrium of our model, bond yields always reflect the expected path of future short rates. As a result, equilibrium bond holdings do not depend directly on short rates. Formally, it is easy to see that, in any equilibrium, active slow-moving arbitrageur demand does not depend on $i_{P,t}$ and $i_{T,t}$, so the first two elements of $\boldsymbol{\delta}_1'$ are zeros and the first two elements are $\boldsymbol{\alpha}_1'$ are $(1-\phi)/(1-\phi\rho_P)$ and $(1-\phi)/(1-\phi\rho_T)$, respectively. This implies that an equilibrium of our model is a solution to a system of $2k$ nonlinear equations in $2k$ unknowns. Specifically, we need to determine how equilibrium yields and active slow-moving demand respond to shifts in the supply of bonds: this generates 2 unknowns and 2 corresponding equations. We also need to determine how equilibrium yields and active slow-moving demand respond to the holdings of inactive slow-moving arbitrageurs: this generates $2(k-1)$ unknowns and $2(k-1)$ corresponding equations.

We solve the relevant system of $2k$ nonlinear equations numerically using the Powell hybrid algorithm which performs a quasi-Newton search to find solutions to a system of nonlinear equations starting from an initial guess. To find all of the solutions, we apply this algorithm by sampling over $10,000$ different initial guesses. Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for α_0 and δ_0 using

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) \text{ and } \delta_0 = \tau \frac{k\left(\alpha_0 - \bar{i}\right)}{V^{(k)}}, \tag{A.69}$$

which yields

$$\alpha_0 = \bar{i} + \frac{\bar{s}}{\tau\left[q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}\right]} \text{ and } \delta_0 = \frac{\frac{k}{V^{(k)}}}{q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}} \times \bar{s}. \tag{A.70}$$

When asset supply is stochastic, an equilibrium solution only exists if arbitrageurs are sufficiently risk tolerant (i.e., for τ sufficiently large). When an equilibrium exists, there are multiple equilibrium solutions. Equilibrium non-existence and multiplicity of this sort arise in overlapping-generations, rational-expectations models such as ours where risk-averse arbitrageurs with finite investment horizons trade an infinitely-lived asset that is subject to supply shocks.[17] Different equilibria correspond to different self-fulfilling beliefs that arbitrageurs can hold about the price-impact of supply shocks and, hence, the risks associated with holding long-term bonds. See Greenwood et al. (2018) for an extensive discussion of these issues.

The intuition for equilibrium multiplicity can be understood most clearly in the simple case when there are only fast-moving arbitrageurs. If arbitrageurs are sufficiently risk tolerant there are two equilibria in this special case: a low price impact (or low return volatility) equilibrium and a high price impact (or high return volatility) equilibrium. If arbitrageurs believe that supply shocks will have a large impact on long-term bonds prices, they will perceive bonds as being highly risky. As a result, arbitrageurs will only absorb a positive supply shock if they are compensated by a large decline in bond prices, making the initial belief self-fulfilling. However, if arbitrageurs believe that bond prices will be less sensitive to supply shocks, they will perceive bond as being less risky and will absorb a supply shock even if they are only compensated by a modest decline in bond prices.

Things are slightly more complicated in our general model with slow-moving capital. Specifically, the introduction of slow-moving capital can give rise to additional unstable equilibria. However, we always find a unique equilibrium that is stable in the sense that equilibrium is robust to a small perturbation in arbitrageurs' beliefs regarding the equilibrium that will prevail in the future. Formally, letting $\boldsymbol{\omega}^{(1)} = \boldsymbol{\omega}^* + \boldsymbol{\xi}$ for some small $\boldsymbol{\xi}$ and defining $\boldsymbol{\omega}^{(n)} = \mathbf{f}(\boldsymbol{\omega}^{(n-1)})$, an equilibrium $\boldsymbol{\omega}^*$ is stable if $\lim_{n\to\infty}\boldsymbol{\omega}^{(n)} = \boldsymbol{\omega}^*$ and is unstable if $\lim_{n\to\infty}\boldsymbol{\omega}^{(n)} \neq \boldsymbol{\omega}^*$. Let $\{\lambda_i\}$ denote the eigenvalues of the Jacobian $\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}\left(\boldsymbol{\omega}^*\right)$. If $\max_i |\lambda_i| < 1$, then $\boldsymbol{\omega}^*$ is stable; if $\max_i |\lambda_i| > 1$, then $\boldsymbol{\omega}^*$ is unstable.

Consistent with Samuelson's (1947) "correspondence principle," which says that the comparative statics of stable equilibria have certain properties, this unique stable equilibrium has com-

[17] For previous treatments of these issues, see Spiegel (1998), Bacchetta and van Wincoop (2003), Watanabe (2008), Banerjee (2011), Greenwood and Vayanos (2014), Albagli (2015), and Greenwood, Hanson, and Liao (forthcoming).

parative statics that accord with standard economic intuition. By contrast, the unstable equilibria have comparative statics that run contrary to standard intuition.[18] We focus on this unique stable equilibrium in our numerical illustrations.

C.8 Behavior of β_h in the baseline model

Consider the model-implied counterpart of the empirical regression coefficient in equation (1.1). In the model, the coefficient β_h from a regression of $y_{t+h} - y_t$ on $i_{t+h} - i_t$ is:

$$\beta_h = \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} = \frac{\boldsymbol{\alpha}_1'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}, \tag{A.71}$$

where $\mathbf{V} = Var\left[\mathbf{x}_t\right]$ denotes the variance of the state vector $\mathbf{x}_t$ and $\mathbf{e}$ denotes the $(k+2)\times 1$ vector with ones in the first and second positions and zeros elsewhere.

To derive this expression, note that $y_{t+h} - y_t = \boldsymbol{\alpha}_1'\left(\mathbf{x}_{t+h} - \mathbf{x}_t\right)$ and $i_{t+h} - i_t = \mathbf{e}'\left(\mathbf{x}_{t+h} - \mathbf{x}_t\right)$. Since the state-vector $\mathbf{x}_t$ follows a VAR(1) process $\mathbf{x}_{t+1} = \boldsymbol{\Gamma}\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1}$ with $\Sigma = Var\left[\boldsymbol{\epsilon}_{t+1}\right]$, we have $vec(\mathbf{V}) = (I - \boldsymbol{\Gamma}\otimes\boldsymbol{\Gamma})^{-1}vec(\boldsymbol{\Sigma})$. Noting that $Cov\left[\mathbf{x}_{t+j}, \mathbf{x}_t'\right] = \boldsymbol{\Gamma}^j\mathbf{V}$ and $Cov[\mathbf{x}_t, \mathbf{x}_{t+j}'] = \mathbf{V}\left(\boldsymbol{\Gamma}'\right)^j$, we have $Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] = 2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h$ and the result follows.

We can then demonstrate the following result:

Proposition 1. *The dependence of the coefficient β_h on time horizon h is governed by (i) the persistence ρ_x of the three shocks $x \in \{s, T, P\}$, (ii) the volatilities of the two short-rate shocks, σ_T and σ_P, (iii) the strength of the rate-amplification mechanisms C, and (iv) the degree to which capital is slow moving q.*

1. *When there are no rate-amplifying net supply shocks ($C = 0$), changes in term premia are unrelated to shifts in short rates and long-term yields do not exhibit excess sensitivity. Furthermore,*
   - *(a) if $\rho_T = \rho_P$, β_h is independent of h, σ_T, and σ_P.*
   - *(b) if $\rho_T < \rho_P$, β_h is* increasing *in h; the level of β_h falls with σ_T and rises with σ_P for all h.*
2. *When there are rate-amplifying net supply shocks ($C > 0$), changes in term premia are positively correlated with changes in short rates and long-term yields exhibit excess sensitivity. Furthermore,*
   - *(a) if $\rho_s = \rho_T = \rho_P$, and all capital is fast-moving ($q = 1$), then β_h is independent of h;*
   - *(b) if $\rho_s \le \rho_T = \rho_P$ and either (i) supply shocks are transient ($\rho_s < \rho_T$) or (ii) capital is slow-moving ($q < 1$), then β_h is decreasing in h;*
   - *(c) if $\rho_s \le \rho_T < \rho_P$, β_h can be non-monotonic in h.*

[18] For instance, in the special case where there is no slow-moving capital, the low price-impact equilibrium is stable and the high price-impact equilibrium is unstable. At the stable equilibrium, an increase in the volatility of short rates or the volatility of supply shocks is associated with an increase in the price-impact coefficient and an increase in the volatility of returns. By contrast, these comparitive statics take the opposite sign at the unstable equilibrium.

Proof: To demonstrate this result, it suffices to consider two special cases. First, we consider the case where there is no slow-moving capital ($q = 1$). We use this case to establish part 1., part 2.(a), and part 2.(c) of the Proposition. To establish part 2.(b), we study the case where $q < 1$, $k = 2$, and $\rho_T = \rho_P \equiv \rho_i$. The arguments given in this special case generalize naturally to the case where $k > 2$.

C.8.1 Solution in the special case without slow-moving capital ($q = 1$)

We first solve the model the special case in which there is no slow-moving capital ($q = 1$). In this special case, the equilibrium yield on long-term bonds is

$$y_t = \overbrace{\left(\bar{\imath} + \tau^{-1}V^{(1)}\bar{s}\right)}^{\alpha_0} + \overbrace{\frac{1-\phi}{1-\rho_P\phi}}^{\alpha_P}\left(i_{P,t} - \bar{\imath}\right) + \overbrace{\frac{1-\phi}{1-\rho_T\phi}}^{\alpha_T} i_{T,t} + \overbrace{\tau^{-1}V^{(1)}\frac{1-\phi}{1-\rho_s\phi}}^{\alpha_s}\left(s_t - \bar{s}\right), \tag{A.72}$$

where

$$\begin{aligned} V^{(1)} &= Var_t\left[\frac{\phi}{1-\phi}y_{t+1}\right] \qquad \text{(A.73)} \\ &= Var_t\left[\frac{\phi}{1-\rho_T\phi}\varepsilon_{P,t+1} + \frac{\phi}{1-\rho_T\phi}\varepsilon_{T,t+1} + \tau^{-1}V^{(1)}\frac{\phi}{1-\rho_s\phi}\left(\varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1}\right)\right] \end{aligned}$$

is the smaller root of the following quadratic equation:

$$\begin{aligned} 0 = &\left[\left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}\sigma_s\right)^2 + \left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_P\right)^2 + \left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_T\right)^2\right] \times \left(V^{(1)}\right)^2 \qquad \text{(A.74)} \\ &+ \left[2\left(\frac{\phi}{1-\rho_P\phi}\sigma_P\right)\left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_P\right) + 2\left(\frac{\phi}{1-\rho_T\phi}\sigma_T\right)\left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_T\right) - 1\right] \times V^{(1)} \\ &+ \left[\left(\frac{\phi}{1-\rho_P\phi}\sigma_P\right)^2 + \left(\frac{\phi}{1-\rho_T\phi}\sigma_T\right)^2\right]. \end{aligned}$$

In this case, the model-implied regression coefficient is

$$\begin{aligned} \beta_h &= \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} \qquad \text{(A.75)} \\ &= \frac{\alpha_P Var\left[\Delta_h i_{P,t}\right] + \alpha_T Var\left[\Delta_h i_{T,t}\right] + \alpha_s\left(Cov\left[\Delta_h i_{P,t}, \Delta_h s_t\right] + Cov\left[\Delta_h i_{T,t}, \Delta_h s_t\right]\right)}{Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]}. \end{aligned}$$

where for $X \in \{P, T\}$ we have $Var\left[\Delta_h i_{X,t}\right] = 2\left[(1-\rho_X^h)/\left(1-\rho_X^2\right)\right]\sigma_X^2$ and $Cov\left[\Delta_h i_{X,t}, \Delta_h s_t\right] = C\left[\left(2-\rho_s^h-\rho_X^h\right)/\left(1-\rho_s\rho_X\right)\right]\sigma_X^2$.

We first consider the *level* of β_h irrespective of horizon h. Inspecting equation (A.75), it is easy to see that:

- **When $C = 0$ and $\rho_T < \rho_P$, the *level* of β_h is increasing in σ_P for all h.** An increase in σ_P raises the fraction of total short-rate variation at all horizons that is due to movements in the more persistent component (i.e., raises $Var[\Delta_h i_{P,t+h}] / (Var[\Delta_h i_{P,t+h}] + Var[\Delta_h i_{T,t+h}])$ for all h). Since shocks to the more persistent component of short rates have larger impact on long-term yields via a straightforward expectations hypothesis channel (i.e., since $\alpha_P > \alpha_T$), an increase in σ_P raises the level of β_h at all horizons.

We next consider the way β_h behaves as a function of horizon h. Again, using equation (A.75), it is easy to show that:

- **When $C = 0$ and $\rho_T = \rho_P$, β_h is a constant that is independent of h.** These assumptions imply that the expectations hypothesis holds—i.e., there is no excess sensitivity—and that all shocks to short rates have the same persistence. In this benchmark case, $\beta_h = \alpha_P = \alpha_T$ for all h—i.e., the sensitivity of long rates to short rates is the same at all horizons.[19] Furthermore, β_h is independent of σ_T and σ_P.

- **When $C = 0$ and $\rho_T < \rho_P$, β_h is an increasing function of h.** These assumptions imply that the expectations hypothesis holds, but there are now transient and persistent shocks to short rates. In this case, β_h rises with h since (i) movements in the more persistent component of short rates are associated with larger movement in long-term yields (i.e., $\alpha_P > \alpha_T$) and (ii) because the persistent component dominates changes in short rates at longer horizons (i.e., $Var[\Delta_h i_{P,t+h}] / (Var[\Delta_h i_{P,t+h}] + Var[\Delta_h i_{T,t+h}])$ rises with h when $\rho_T < \rho_P$).

- **When $C > 0$ and $\rho_s = \rho_T = \rho_P$, β_h is a constant that is independent of h.** In this case, there is excess sensitivity—shifts in short rates lead to shifts in the term premium on long-term bonds—but the excess sensitivity is the same irrespective of horizon h. This is because $\Delta_h s_{t+h} = C\Delta_h i_{P,t+h} + C\Delta_h i_{T,t+h}$ when $\rho_s = \rho_T = \rho_P$ (see equation (A.41)) and $Var[\Delta_h i_{P,t+h}] / (Var[\Delta_h i_{P,t+h}] + Var[\Delta_h i_{T,t+h}]) = \sigma_P^2 / \left(\sigma_P^2 + \sigma_T^2\right)$ when $\rho_T = \rho_P$.

- **When $C > 0$ and $\rho_s < \rho_T = \rho_P$, β_h is a decreasing function of h.** In this case, long-term interest rates exhibit excess sensitivity to movements in short rates that declines with horizon h. Intuitively, if the supply shocks induced by shocks to short rates are more transient than

[19] To see this, note that (treating h as continuous), we have

$$\frac{\partial}{\partial h}\left(\frac{Var[\Delta_h i_{P,t+h}]}{Var[\Delta_h i_{P,t+h}] + Var[\Delta_h i_{T,t+h}]}\right) = \frac{\partial}{\partial h}\left(\frac{\frac{1-\rho_P^h}{1-\rho_P^2}\sigma_P^2}{\frac{1-\rho_T^h}{1-\rho_T^2}\sigma_T^2 + \frac{1-\rho_P^h}{1-\rho_P^2}\sigma_P^2}\right) = \sigma_P^2\sigma_T^2 \frac{\left(1-\rho_T^h\right)\left(1-\rho_P^h\right)}{\left(1-\rho_P^2\right)\left(1-\rho_T^2\right)} \frac{-\frac{\ln(\rho_P)\rho_P^h}{1-\rho_P^h} + \frac{\ln(\rho_T)\rho_T^h}{1-\rho_T^h}}{\left(\frac{1-\rho_T^h}{1-\rho_T^2}\sigma_T^2 + \frac{1-\rho_P^h}{1-\rho_P^2}\sigma_P^2\right)^2}.$$

Since $\rho_T < \rho_P$, then result then follows from the fact that $-\frac{\ln(\rho)\rho^h}{1-\rho^h}$ is increasing in ρ for $\rho \in [0,1)$. To see this last fact note that

$$-\frac{\partial}{\partial \rho}\frac{\ln(\rho)\rho^h}{1-\rho^h} = -\frac{1-\rho^h+\ln(\rho)h}{\left(1-\rho^h\right)^2}\rho^{h-1} \propto -\left(1-\rho^h+\ln(\rho)h\right) \equiv f(\rho).$$

We have $f(1) = 0$ and $\lim_{\rho\to 0} f(\rho) = \infty$ and $f'(\rho) = -h\left(1-\rho^h\right)/\rho < 0$. Thus, we have $f(\rho) > 0$ for all $\rho \in [0,1)$.

the underlying shocks to short rates, then term premia will react more in the short run than in the long run. Thus, there will be greater excess sensitivity in the short run.[20]

- **When $C > 0$ and $\rho_s < \rho_T < \rho_P$, β_h can be a globally non-monotonic function of h.** Decompose the long-term yield into an expectations hypothesis component eh_t that reflects expected future short rates and a term premium piecetp_t that reflects expected future bond risk premium: $y_t = eh_t + tp_t$. Thus, we have $\beta_h = \beta_h^{eh} + \beta_h^{tp}$ where $\beta_h^{eh} = Cov\left[eh_{t+h} - eh_t, i_{t+h} - i_t\right]/Var\left[i_{t+h} - i_t\right]$ and $\beta_h^{tp} = Cov\left[tp_{t+h} - tp_t, i_{t+h} - i_t\right]/Var\left[i_{t+h} - i_t\right]$. In this case, β_h^{eh} is an increasing an concave function of h (the logic here is the same as $C = 0$ and $\rho_T < \rho_P$). And, β_h^{tp} is a decreasing and convex function of h (the logic here is the same as the case where $C > 0$ and $\rho_s < \rho_T = \rho_P$). Depending on which effect dominates, $\beta_t = \beta_h^{eh} + \beta_h^{tp}$ can either be monotonically increasing (e.g., if ρ_P is much larger than ρ_T and ρ_s is near ρ_T), monotonically decreasing (e.g., if ρ_P is near ρ_T and ρ_s is much smaller than ρ_T), U-shaped, or inverse-U-shaped.

C.8.2 Special case where $q < 1$, $k = 2$, and $\rho_T = \rho_P \equiv \rho_i$

Summary: Suppose that $C > 0$, $q < 1$, $k = 2$, and $\rho_s \leq \rho_T = \rho_P \equiv \rho_i$. Then we always have $\beta_h - \beta_{h-1} < 0$ for $h \leq 2$. Furthermore, β_h is globally increasing in the sense that $\beta_1 > \lim_{h\to\infty} \beta_h$. However, it is possible for β_h to oscillate non-monotonically for $h > 2$.

The solution: Here we have $y_t = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t$ and $d_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$. The state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_t - \bar{i} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \end{bmatrix}, \tag{A.76}$$

and its dynamics are given by

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\delta)\,\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \qquad \text{(A.77)} \\ &= \begin{bmatrix} \rho_i & 0 & 0 \\ 0 & \rho_s & 0 \\ 0 & \delta_s & \delta_d \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ C\varepsilon_{i,t+1} + \varepsilon_{s,t+1} \\ 0 \end{bmatrix}, \end{aligned}$$

where $\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right]$, $\delta_s > 0$, and $\delta_d < 0$. Assuming for simplicity that $\varepsilon_{i,t+1}$ and $\varepsilon_{s,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_i^2 & C\sigma_i^2 & 0 \\ C\sigma_i^2 & C^2\sigma_i^2 + \sigma_s^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A.78}$$

[20]To show this, it suffices to show that $\left(2 - \rho_s^h - \rho_T^h\right)/\left(2 - \rho_T^h - \rho_T^h\right) > 0$ is decreasing in h when $\rho_s < \rho_T$. To see this note that

$$\frac{\partial}{\partial h}\left(\frac{2 - \rho_s^h - \rho_T^h}{2 - \rho_T^h - \rho_T^h}\right) = \frac{2\left(1 - \rho_s^h\right)\left(1 - \rho_T^h\right)\left(-\frac{\rho_s^h \ln \rho_s}{1 - \rho_s^h} + \frac{\rho_T^h \ln \rho_T}{1 - \rho_T^h}\right)}{\left(2 - \rho_T^h - \rho_T^h\right)^2}.$$

Since $\rho_s < \rho_T$, then result then follows from the fact that $-\frac{\ln(\rho)\rho^h}{1-\rho^h}$ is increasing in ρ for $\rho \in [0, 1)$.

Using the face that $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$, we can show that

$$\mathbf{V} = Var \begin{bmatrix} i_t - \bar{i} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \end{bmatrix} = \begin{bmatrix} \frac{\sigma_i^2}{1-\rho_i^2} & \frac{C\sigma_i^2}{1-\rho_i\rho_s} & \frac{\rho_i\delta_s}{1-\delta_d\rho_i}\frac{C\sigma_i^2}{1-\rho_i\rho_s} \\ \frac{C\sigma_i^2}{1-\rho_i\rho_s} & \frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} & \frac{\delta_s\rho_s}{1-\delta_d\rho_s}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} \\ \frac{\rho_i\delta_s}{1-\delta_d\rho_i}\frac{C\sigma_i^2}{1-\rho_i\rho_s} & \frac{\delta_s\rho_s}{1-\delta_d\rho_s}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} & \frac{\delta_s^2(1+\delta_d\rho_s)}{(1-\delta_d^2)(1-\delta_d\rho_s)}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} \end{bmatrix}. \tag{A.79}$$

Equilibrium excess returns are given by

$$E_t[rx_{t+1}] = \frac{V^{(1)}}{\tau q}(\bar{s} - (1-q)\delta_0) + \frac{V^{(1)}}{\tau q}\left[(s_t - \bar{s}) - \frac{1}{2}(1-q)\left(\delta_s(s_t - \bar{s}) + (1+\delta_d)(d_{t-1} - \delta_0)\right)\right] \tag{A.80}$$

Equilibrium yields are given by

$$y_t = \overbrace{\bar{i} + \frac{1-\phi}{1-\phi\rho_i}\left(i_{i,t} - \bar{i}\right)}^{\text{Expected future short real rates}} + \overbrace{\frac{V^{(1)}}{\tau q}(\bar{s} - (1-q)\delta_0)}^{\text{Unconditional term premia}} \tag{A.81}$$

$$+ \overbrace{\left[\frac{V^{(1)}}{\tau q}\left(\frac{1-\phi}{1-\phi\rho_s}(s_t - \bar{s}) - \frac{1}{2}(1-q)\left(\frac{1-\phi}{1-\phi\rho_s}\frac{1+\phi}{1-\phi\delta_d}\delta_s(s_t - \bar{s}) + \frac{1-\phi}{1-\phi\delta_d}(1+\delta_d)(d_{t-1} - \delta_0)\right)\right)\right]}^{\text{Conditional term premia}}.$$

Characterizing the solution: Here we show that:

1. $\alpha_s > 0$ (an increase in supply s_t is associated with an increase in long-term yields y_t);
2. $\delta_s > 0$ (an increase in supply s_t is associated with an increase in bond purchases d_t by active slow investors);
3. $\alpha_d < 0$ (an increase in the holdings of inactive slow investors d_{t-1} is associated with a decline in long-term yields y_t); and
4. $-1 < \delta_d < 0$ (an increase in the holdings of inactive slow investors d_{t-1} is associated with a decline in bond purchases d_t by active slow investors).

To begin, note that the market-clearing condition is this case is:

$$s_t - (1-q)\frac{1}{2}d_{t-1} = qb_t + (1-q)\frac{1}{2}d_t. \tag{A.82}$$

Plugging in the relevant expressions for $b_t = \left(\tau/V^{(1)}\right) \times E_t[(y_t - i_t) - (\phi/(1-\phi))(y_{t+1} - y_t)]$ and d_t, we have

$$\begin{aligned} & s_t - (1-q)\frac{1}{2}d_{t-1} \\ = \; & q\frac{\tau}{V^{(1)}}\begin{pmatrix} \left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\left(\alpha_s(s_t - \bar{s}) + \alpha_d(d_{t-1} - \delta_0)\right) \\ -\frac{\phi}{1-\phi}\left[\alpha_s\rho_s(s_t - \bar{s}) + \alpha_d\left[\delta_s(s_t - \bar{s}) + \delta_d(d_{t-1} - \delta_0)\right]\right] \end{pmatrix} \\ & +(1-q)\frac{1}{2}\left(\delta_0 + \delta_s(s_t - \bar{s}) + \delta_d(d_{t-1} - \delta_0)\right). \end{aligned} \tag{A.83}$$

(Here we have use the solution for α_i and the fact that $\delta_i = 0$). Similarly, we have

$$\begin{aligned} d_t - \delta_0 &= \delta_s (s_t - \overline{s}) + \delta_d (d_{t-1} - \delta_0) \qquad \text{(A.84)} \\ &= \frac{\tau}{V^{(2)}} \left(\begin{array}{c} E_t[(y_t - i_t) + (y_{t+1} - i_{t+1}) - (\phi/(1-\phi))(y_{t+2} - y_t)] \\ -E[(y_t - i_t) + (y_{t+1} - i_{t+1}) - (\phi/(1-\phi))(y_{t+2} - y_t)] \end{array} \right) \\ &= \frac{\tau}{V^{(2)}} \left(\begin{array}{c} [\alpha_s (s_t - \overline{s}) + \alpha_d (d_{t-1} - \delta_0)] \\ + [\alpha_s \rho_s (s_t - \overline{s}) + \alpha_d [\delta_s (s_t - \overline{s}) + \delta_d (d_{t-1} - \delta_0)]] \\ -\frac{\phi}{1-\phi} \left(\begin{array}{c} [\alpha_s \rho_s^2 (s_t - \overline{s}) + \alpha_d [(\delta_s (\delta_d + \rho_s))(s_t - \overline{s}) + \delta_d^2 (d_{t-1} - \delta_0)]] \\ - [\alpha_s (s_t - \overline{s}) + \alpha_d (d_{t-1} - \delta_0)] \end{array} \right) \end{array} \right) \end{aligned}$$

Differentiating these two equations with respect to s_t and d_{t-1}, we obtain the following four conditions in four unknowns $(\alpha_s, \alpha_d, \delta_s, \delta_d)$:

$$\begin{aligned} 1 &= q \frac{\tau}{V^{(1)}[\alpha_s]} \left(\frac{1-\phi\rho_s}{1-\phi} \alpha_s - \frac{\phi}{1-\phi} \alpha_d \delta_s \right) + (1-q) \frac{1}{2} \delta_s, \qquad \text{(A.85a)} \\ -(1-q)\frac{1}{2} &= q \frac{\tau}{V^{(1)}[\alpha_s]} \left(\frac{1-\phi\delta_d}{1-\phi} \alpha_d \right) + (1-q) \frac{1}{2} \delta_d, \qquad \text{(A.85b)} \\ \delta_s &= \frac{\tau}{V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]} \left(\frac{1-\phi\rho_s}{1-\phi} ((\rho_s + 1)\alpha_s + \alpha_d \delta_s) - \frac{\phi(1+\delta_d)}{1-\phi} \alpha_d \delta_s \right), \qquad \text{(A.85c)} \\ \delta_d &= \frac{\tau}{V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]} \frac{1-\phi\delta_d}{1-\phi} (\delta_d + 1) \alpha_d, \qquad \text{(A.85d)} \end{aligned}$$

where we write $V^{(1)}[\alpha_s]$ to emphasize that $V^{(1)} = Var_t[rx_{t+1}] > 0$ depends on α_s and $V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]$ to emphasize that $V^{(2)} = Var_t[rx_{t+1} + rx_{t+2}] > 0$ depends on $(\alpha_s, \alpha_d, \delta_s, \delta_d)$.

- First, we show that $\delta_d < 0$. Combining (A.85b) and (A.85d), we have

$$\delta_d = \frac{\tau}{V^{(2)}} (\delta_d + 1) \frac{1-\phi\delta_d}{1-\phi} \alpha_d = -\frac{1}{2} \frac{1-q}{q} \frac{V^{(1)}}{V^{(2)}} (\delta_d + 1)^2 < 0. \qquad \text{(A.86)}$$

- Next we want to show that $-1 < \delta_d$. We have

$$(1+\delta_d) = 1 - \frac{1}{2} \frac{1-q}{q} \frac{V^{(1)}[\alpha_s]}{V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]} (1+\delta_d)^2. \qquad \text{(A.87)}$$

  We have $(1+\delta_d) > 0$ or $\delta_d > -1$ in the stable solution.

- Next, since $-1 < \delta_d < 0$, equation (A.85d) implies that $\alpha_d < 0$.

- To show that $\delta_s > 0$, we combine (A.85c), (A.85d), and (A.85a) to obtain

$$\begin{aligned} \delta_s &= \frac{\tau}{V^{(2)}} \left(\frac{1-\phi\rho_s}{1-\phi} ((\rho_s + 1)\alpha_s + \alpha_d \delta_s) - \phi \delta_s \left(\frac{1+\delta_d}{1-\phi} \alpha_d \right) \right) \qquad \text{(A.88)} \\ &= \tau \frac{(\rho_s + 1)}{V^{(2)}} \frac{1-\phi\rho_s}{1-\phi} \alpha_s + \frac{\tau}{V^{(2)}} \frac{1-\phi\rho_s}{1-\phi} \alpha_d \delta_s - \delta_s \frac{\phi\delta_d}{1-\phi\delta_d} \text{ [using (A.85d)]} \\ &= \frac{(\rho_s + 1)}{\tau V^{(2)}} \left(\frac{\tau V^{(1)}}{q} \left(1 - (1-q)\frac{1}{2}\delta_s \right) + \frac{\phi}{1-\phi} \alpha_d \delta_s \right) \\ &\quad + \frac{\tau}{V^{(2)}} \frac{1-\phi\rho_s}{1-\phi} \alpha_d \delta_s - \delta_s \frac{\phi\delta_d}{1-\phi\delta_d} \text{ [using (A.85a)]} \\ &= \frac{(\rho_s + 1)}{q} \frac{V^{(1)}}{V^{(2)}} \left(1 - (1-q)\frac{1}{2}\delta_s \right) + \tau \frac{\alpha_d \delta_s}{V^{(2)}} \frac{1+\phi}{1-\phi} - \delta_s \frac{\phi\delta_d}{1-\phi\delta_d}. \end{aligned}$$

Rearranging, we obtain

$$\delta_s = (1 - \phi\delta_d)\left[\frac{(\rho_s + 1)}{q}\frac{V^{(1)}[\alpha_s]}{V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]}\left(1 - (1 - q)\frac{1}{2}\delta_s\right) + \tau\frac{\alpha_d\delta_s}{V^{(2)}}\frac{1 + \phi}{1 - \phi}\right]. \tag{A.89}$$

Since $\delta_d < 0$ and $\alpha_d < 0$, this equation implies that we must have $\delta_s > 0$ in equilibrium. Specifically, if $\delta_s < 0$, then the right-hand-side of this equation is positive and the left-hand-side is negative. Thus, we cannot have $\delta_d < 0$ in equilibrium and must instead have $\delta_s > 0$ in equilibrium.

- Finally, we want to show that $\alpha_s > 0$. Combining (A.85a) and (A.85b), we obtain

$$\alpha_s = \frac{\tau^{-1}V^{(1)}}{q}\frac{1 - \phi}{1 - \phi\rho_s}\left(1 - \frac{1}{2}(1 - q)\,\delta_s\frac{1 + \phi}{1 - \phi\delta_d}\right). \tag{A.90}$$

  Since $\delta_d < 0$, we have $(1 + \phi)\,/\,(1 - \phi\delta_d) < 1$ and it suffices to show that $2 \geq (1 - q)\,\delta_s$ for all $q \in [0, 1]$. When $q = 1$, we have $(1 - q)\,\delta_s = 0$. When $q = 0$, we have $(1 - q)\,\delta_s = 2$. Since $(1 - q)\,\delta_s$ is monotonically increasing in q in the model's stable equilibrium, it follows that $2 \geq (1 - q)\,\delta_s$ for all $q \in [0, 1]$. Therefore, we have $2 > (1 - q)\,\delta_s\,[(1 + \phi)\,/\,(1 - \phi\delta_d)]$ and hence $\alpha_s > 0$ for all $q \in [0, 1]$.

Computing β_h: We now compute and characterize β_h. We have

$$\beta_h = \frac{Cov\,[y_{t+h} - y_t, i_{t+h} - i_t]}{Var\,[i_{t+h} - i_t]} = \frac{\boldsymbol{\alpha}_1'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}. \tag{A.91}$$

Using the fact that

$$\boldsymbol{\Gamma}^h = \begin{bmatrix} \rho_i^h & 0 & 0 \\ 0 & \rho_s^h & 0 \\ 0 & \delta_s\frac{\rho_s^h - \delta_d^h}{\rho_s - \delta_d} & \delta_d^h \end{bmatrix}, \tag{A.92}$$

we have

$$Var\,[\mathbf{x}_{t+h} - \mathbf{x}_t] = 2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h \tag{A.93}$$

$$= \begin{bmatrix} 2\frac{1-\rho_i^h}{1-\rho_i^2}\sigma_i^2 & C\frac{2-\rho_s^h-\rho_i^h}{1-\rho_s\rho_i}\sigma_i^2 & \frac{C\sigma_i^2}{1-\rho_i\rho_s}\frac{\rho_i\delta_s\left(2-\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)}{1-\rho_i\delta_d} \\ C\frac{2-\rho_s^h-\rho_i^h}{1-\rho_s\rho_i}\sigma_i^2 & \frac{2\left(1-\rho_s^h\right)\left(C^2\sigma_i^2+\sigma_s^2\right)}{1-\rho_s^2} & \frac{\delta_s\left(\rho_s\frac{2-\rho_s^h-\delta_d^h}{1-\delta_d\rho_s}-\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)\left(C^2\sigma_i^2+\sigma_s^2\right)}{1-\rho_s^2} \\ \frac{C\sigma_i^2}{1-\rho_i\rho_s}\frac{\rho_i\delta_s\left(2-\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)}{1-\rho_i\delta_d} & \frac{\delta_s\left(\rho_s\frac{2-\rho_s^h-\delta_d^h}{1-\delta_d\rho_s}-\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)\left(C^2\sigma_i^2+\sigma_s^2\right)}{1-\rho_s^2} & \frac{2\delta_s^2\left(\left(1-\delta_d^h\right)\frac{1+\delta_d\rho_s}{(1-\delta_d^2)}-\rho_s\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)}{(1-\delta_d\rho_s)}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} \end{bmatrix}$$

Thus, we have

$$\beta_h = \alpha_i + \alpha_s \times R_{i,s}(h) + \alpha_d \times R_{i,d}(h), \tag{A.94}$$

where $\alpha_i > 0$, $\alpha_s > 0$, $\alpha_d < 0$, and

$$R_{i,s}(h) = \frac{Cov\,[i_{t+h} - i_t, s_{t+h} - s_t]}{Var\,[i_{t+h} - i_t]} = C\frac{1 - \rho_i^2}{1 - \rho_s\rho_i}\frac{2 - \rho_s^h - \rho_i^h}{2\left(1 - \rho_i^h\right)}, \tag{A.95}$$

$$R_{i,d}(h) = \frac{Cov\,[i_{t+h} - i_t, d_{t+h-1} - d_{t-1}]}{Var\,[i_{t+h} - i_t]} = C\frac{1 - \rho_i^2}{1 - \rho_s\rho_i}\frac{\rho_i\delta_s}{1 - \rho_i\delta_d}\frac{2 - \rho_i^h - \delta_d^h - \frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}}{2\left(1 - \rho_i^h\right)}.$$

Proof that $\beta_2 < \beta_1$: We know that $R_{i,s}(h)$ is strictly decreasing in h when $\rho_s < \rho_i$ and is constant when $\rho_s = \rho_i$. Thus, since $\alpha_s > 0$, it follows that the second term $(\alpha_{1,s} \times R_{i,s}(h))$ is weakly decreasing in h and is strictly decreasing when $\rho_s < \rho_i$. Since $\alpha_d < 0$ when $q < 1$, if we can show that $R_{i,d}(2) > R_{i,d}(1)$, then we have $\alpha_d \times R_{i,d}(2) < \alpha_d \times R_{i,d}(1)$ and we are done. We have

$$R_{i,d}(h) = C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d}\left(1+\frac{\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}}{2\left(1-\rho_i^h\right)}\right) = C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d}\left(1+F\left(h,\rho_s\right)\right) \tag{A.96}$$

where

$$F\left(h,\rho_s\right) = \frac{\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}}{2\left(1-\rho_i^h\right)}. \tag{A.97}$$

We want to show that $F(2,\rho_s) > F(1,\rho_s)$. We have

$$F\left(2,\rho_s\right)-F\left(1,\rho_s\right) = \frac{1}{2\rho_i}\left(1+\rho_i-\frac{\delta_d\left(1-\rho_s\rho_i\right)}{\left(1-\rho_i^2\right)}-\frac{\rho_s-\rho_i^3}{1-\rho_i^2}\right). \tag{A.98}$$

Note that $F(2,\rho_s) - F(1,\rho_s)$ is decreasing in ρ_s when $\rho_s \in [0,\rho_i]$. Thus, setting $\rho_s = \rho_i$, we have

$$F\left(2,\rho_s\right)-F\left(1,\rho_s\right) \geq F\left(2,\rho_i\right)-F\left(1,\rho_i\right) = \frac{1}{2\rho_i}\left(1-\delta_d\right) > 0. \tag{A.99}$$

Thus, we have $F(2,\rho_s) > F(1,\rho_s)$ and thus $R_{i,d}(2) > R_{i,d}(1)$.

Proof that β_h is globally increasing in the sense that $\beta_1 > \lim_{h\to\infty}\beta_h$: To show that is globally increasing, it suffices to show that $\lim_{h\to\infty} R_{i,d}(h) > R_{i,d}(1)$. We have

$$\lim_{h\to\infty} R_{i,d}(h) = C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d} > 0 \tag{A.100}$$

since $\rho_i \in (0,1)$, $\rho_s \in (0,1)$ and $\delta_d \in (-1,0)$. We also have

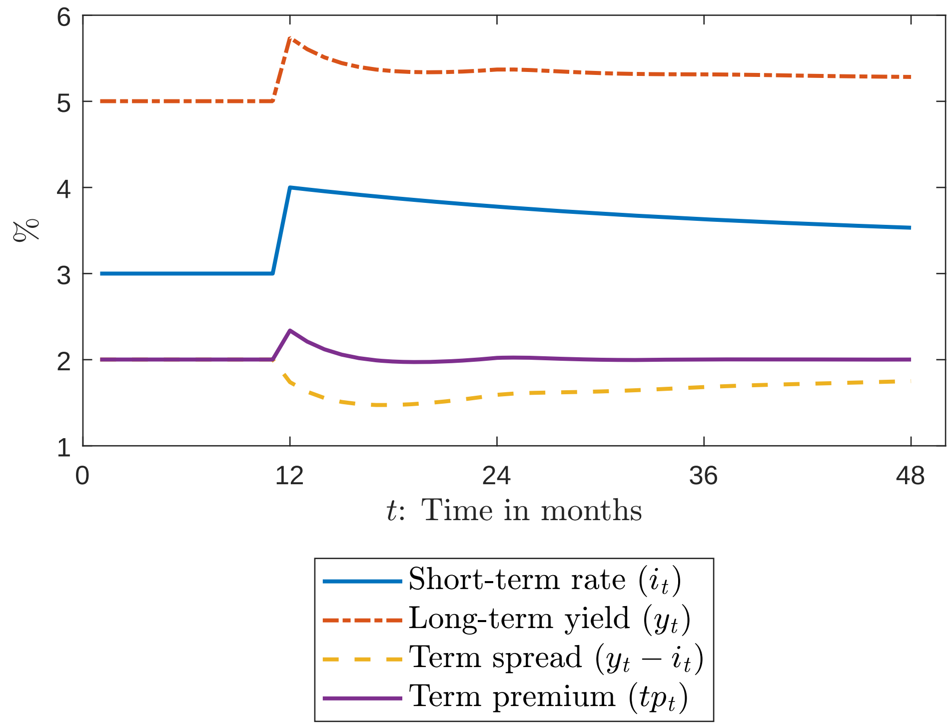
$$R_{i,d}(1) = -C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d}\frac{1}{2\rho_i}\left(1-\rho_i\right) < 0. \tag{A.101}$$

Thus, we conclude that $\beta_1 > \lim_{h\to\infty}\beta_h$.

C.9 Model-implied impulse response to a short-rate shock

Figure A.8 shows the model-implied impulse response functions in the post-2000 calibration following a 100 bp shock to short rates that lands in month $t = 12$. (We assume there is a 50 bp shock to both the persistent and transient components of the short rate.) The long-term yield is the sum of an expectations-hypothesis component and a term premium component: $y_t = eh_t + tp_t$. Thus, the term spread is $y_t - i_t = (eh_t - i_t) + tp_t$. The figure shows impulse responses for short-term rates (i_t), long-term yields (y_t), the term spread $(y_t - i_t)$, and the term premium (tp_t).

Figure A.8: Model-implied impulse response functions for the post-2000 calibration. For the post-2000 calibration, we show the response of short-term and long-term interest rates following a one-time shock to short-term interest rates. We plot short-term nominal interest rates (i_t), long-term nominal yields (y_t), the term spread ($y_t - i_t$), and the term premium (tp_t). Initially, short-term nominal rates are at their steady-state level of $\bar{\imath} = 3\%$ and the term premium on long-term nominal bonds is at a steady-level of 2%. We then assume there is a 50 bp shock to both the persistent and transient components of the short rate that lands at $t = 12$, leading short-term nominal rates to jump from 3% to 4%.



The initial shock to short rates leads to a rise in term premia. Thus, relative to the expectations-hypothesis, long-term rates are excessively sensitive to short rates. However, the rise in term premia wears off quickly, explaining our key finding that β_h declines sharply with horizon h. Nonetheless, the impulse to short rates causes the yield curve to flatten on impact as in the data. This is because $(eh_t - i_t)$ falls on impact and this flattening due to the expectations hypothesis outweighs the steepening due to the rise in term premia. However, the initial rise in short rates predicts additional yield curve flattening—and predictable reversals in long-term yields—over the following months.

D Modelling rate-amplifying mechanisms

D.1 The mortgage refinancing channel

D.1.1 General model solution

We now we drop $s_t - \overline{s}$ from the state vector $\mathbf{x}_t$ and add $c_t^M - \overline{c}^M = c_t^M - (\alpha_0 + \lambda)$ to the state vector. As always, we conjecture that $y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$ and $d_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t$. Let $\mathbf{e}_c' \mathbf{x}_t = c_t^M - \overline{c}^M$, we then have

$$s_t = M \times \overline{DUR}^M + MN \times \left(y_t^M - c_t^M\right) = M \times \overline{DUR}^M + MN \times (\boldsymbol{\alpha}_1 - \mathbf{e}_c)' \mathbf{x}_t. \tag{A.102}$$

The law of motion for c_t^M is

$$c_{t+1}^M - \overline{c}^M = (1-\eta)\left(c_t^M - \overline{c}^M\right) + \eta\left(y_t^M - \overline{y}^M\right). \tag{A.103}$$

Making use of the fact that $\left(y_t^M - \overline{y}^M\right) = (y_t - \overline{y}) = \boldsymbol{\alpha}_1' \mathbf{x}_t$, we can write

$$\mathbf{e}_c' \mathbf{x}_{t+1} = (1-\eta)\, \mathbf{e}_c' \mathbf{x}_t + \eta \boldsymbol{\alpha}_1' \mathbf{x}_t. \tag{A.104}$$

Thus, our assumptions imply that the state vector follows an AR(1) process. Critically, the transition matrix Γ is a function of (i) the parameters $\boldsymbol{\delta}_1$ governing slow-moving arbitrageur demand and (ii) the parameters $\boldsymbol{\alpha}_1$ governing yields, so we write $\Gamma = \Gamma(\boldsymbol{\delta}_1, \boldsymbol{\alpha}_1)$. Specifically, for $k = 4$, we have

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\boldsymbol{\delta}_1, \boldsymbol{\alpha}_1)\, \mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 \\ \eta\alpha_P & \eta\alpha_T & (1-\eta)+\eta\alpha_c & \eta\alpha_{d_1} & \eta\alpha_{d_2} & \eta\alpha_{d_3} \\ \delta_P & \delta_T & \delta_c & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{P,t} - \overline{i} \\ i_{T,t} \\ c_t^M - \overline{c}^M \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{A.105}$$

where $\Sigma \equiv Var_t[\boldsymbol{\epsilon}_{t+1}]$. Since $\varepsilon_{P,t+1}$ and $\varepsilon_{T,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_P^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.106}$$

To solve for the equilibrium, we need to clear the market for bonds in a way that is consistent with optimization on the part of fast-moving arbitrageurs and slow-moving arbitrageurs. The market-clearing condition is

$$\overbrace{(1-q)k^{-1}d_t + qb_t}^{\text{Active demand}} = \overbrace{M \times [\overline{DUR}^M + N \times \left(y_t^M - c_t^M\right)] - (1-q)(k^{-1} \textstyle\sum_{i=1}^{k-1} d_{t-i})}^{\text{Active supply}}. \tag{A.107}$$

Letting $V^{(1)} = Var_t[rx_{t+1}] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1$, denote the variance of 1-period excess returns, active demand is

$$\begin{aligned}
&(1-q)k^{-1}d_t + qb_t \qquad \text{(A.108)}\\
&= \left[(1-q)k^{-1}\delta_0 + q\tau\frac{(\alpha_0 - \bar{i})}{V^{(1)}}\right] + \left[(1-q)k^{-1}\boldsymbol{\delta}_1' + q\tau\frac{\left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' - \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\boldsymbol{\Gamma}}{V^{(1)}}\right]\mathbf{x}_t
\end{aligned}$$

Active supply is

$$M \times [\overline{DUR}^M + N \times (y_t^M - c_t^M)] - (1-q)k^{-1}\textstyle\sum_{i=1}^{k-1} d_{t-i} \qquad \text{(A.109)}$$

$$= \left[M \times \overline{DUR}^M - (1-q)\frac{(k-1)}{k}\delta_0\right] + \left[MN \times (\boldsymbol{\alpha}_1 - \mathbf{e}_c) - (1-q)k^{-1}\mathbf{e}_d\right]' \mathbf{x}_t. \qquad \text{(A.110)}$$

Matching constant terms, we obtain

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}(M \times \overline{DUR}^M - (1-q)\delta_0). \qquad \text{(A.111)}$$

Matching slope coefficients and solving we find

$$\boldsymbol{\alpha}_1 = (1-\phi)\left[\left(1 - (1-\phi)\frac{MN}{q\tau}V^{(1)}\right)\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left[\mathbf{e} - \frac{V^{(1)}}{q\tau}\left(MN\mathbf{e}_c + (1-q)\,k^{-1}\,(\mathbf{e}_d + \boldsymbol{\delta}_1)\right)\right]. \qquad \text{(A.112)}$$

In summary, an equilibrium in this model extension solves the following system of equations

$$\boldsymbol{\alpha}_1 = (1-\phi)\left[\left(1 - (1-\phi)\frac{MN}{q\tau}V^{(1)}(\boldsymbol{\alpha}_1)\right)\mathbf{I} - \phi\left[\boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)\right]'\right]^{-1}\left[\mathbf{e} - \frac{V^{(1)}(\boldsymbol{\alpha}_1)}{q\tau}\left(MN\mathbf{e}_c + (1-q)\,k^{-1}\,(\mathbf{e}_d + \boldsymbol{\delta}_1)\right)\right. \qquad \text{(A.113)}$$

and

$$\boldsymbol{\delta}_1' = \tau\frac{\left((\boldsymbol{\alpha}_1 - \mathbf{e})'\,(\mathbf{I} - \boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1))^{-1} + \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\right)}{V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)}\left(\mathbf{I} - \boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)^k\right), \qquad \text{(A.114)}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)$ to emphasize that the transition matrix depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)$ to emphasize that the k-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. Unlike in our baseline model, the first two elements of $\boldsymbol{\delta}_1$ are going to be positive, since slow-moving arbitrageurs will buy more long-term bonds when interest rates rise.[21] Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for α_0 and δ_0 using

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}(\bar{s} - (1-q)\delta_0) \text{ and } \delta_0 = \tau\frac{k\left(\alpha_0 - \bar{i}\right)}{V^{(k)}}, \qquad \text{(A.115)}$$

[21] If we were to add independent shocks to the supply of long-term bonds to the model, then the mortgage refinancing channel would also lead long-term yields to temporarily over-react to those bond supply shocks.

which yields

$$\alpha_0 = \bar{i} + \frac{\bar{s}}{\tau\left[q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}\right]} \text{ and } \delta_0 = \frac{\frac{k}{V^{(k)}}}{q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}} \times \bar{s}. \tag{A.116}$$

D.1.2 Proof of Proposition 2

Proposition 2. ***Mortgage refinancing model.*** *For simplicity suppose $\rho_T = \rho_P$. When $MN > 0$, long-term yields are excessively sensitive to short rates. When $MN > 0$ and $\eta = 0$, this excess sensitivity is only horizon-dependent—i.e., the model-implied regression coefficients β_h in equation (A.71) only decline with horizon h—when arbitrage capital is slow moving ($q < 1$). By contrast, when $MN > 0$ and $\eta > 0$, β_h declines with horizon h even if all arbitrage capital is fast-moving ($q = 1$).*

Proof: To demonstrate this result, it suffices to consider two special cases. First, we consider the case where there is no slow-moving capital ($q = 1$) and where and $\rho_T = \rho_P \equiv \rho_i$. Next, we study the case where $q < 1$, $k = 2$, and $\rho_T = \rho_P \equiv \rho_i$. The arguments given in this special case generalize naturally to the case where $k > 2$.

Case #1: $q = 1$ and $\rho_T = \rho_P \equiv \rho_i$: In this case, we have

$$y_t = \alpha_0 + \alpha_1' \mathbf{x}_t = \alpha_0 + \alpha_i \left(i_t - \bar{i}\right) + \alpha_c \left(c_t^M - \bar{c}^M\right). \tag{A.117}$$

The law of motion for the state vector is

$$\mathbf{x}_{t+1} = \Gamma(\boldsymbol{\alpha})\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} = \begin{bmatrix} \rho_i & 0 \\ \eta\alpha_i & (1-\eta)+\eta\alpha_c \end{bmatrix} \begin{bmatrix} i_t - \bar{i} \\ c_t^M - \bar{c}^M \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \end{bmatrix}, \tag{A.118}$$

where

$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_i^2 & 0 \\ 0 & 0 \end{bmatrix}. \tag{A.119}$$

Thus, we have

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 (\alpha_i)^2 \sigma_i^2. \tag{A.120}$$

Letting

$$Z = MN\tau^{-1}V^{(1)} = MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2 (\alpha_i)^2 \sigma_i^2, \tag{A.121}$$

we have:

$$\begin{aligned}
\begin{bmatrix} \alpha_i \\ \alpha_c \end{bmatrix} &= (1-\phi)\left(\left((1-(1-\phi)MN\tau^{-1}V^{(1)}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \phi\begin{bmatrix} \rho_i & \alpha_i\eta \\ 0 & (1-\eta)+\eta\alpha_c \end{bmatrix}\right)^{-1}\begin{bmatrix} 1 \\ -(\tau)^{-1}V^{(1)}MN \end{bmatrix} \\
&= \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i-(1-\phi)Z}\left(1 - \frac{\frac{\phi}{1-\phi}Z\alpha_i\eta}{1-Z+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\right) \\ -\frac{Z}{1-Z+\frac{\phi}{1-\phi}\eta(1-\alpha_c)} \end{bmatrix}.
\end{aligned} \tag{A.122}$$

Thus, the full fixed point problem is:

$$\begin{bmatrix} \alpha_i \\ \alpha_c \end{bmatrix} = \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i-(1-\phi)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}\left(1-\frac{\eta\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^3(\alpha_i)^3\sigma_i^2}{1-\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\right) \\ -\frac{MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}{1-MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)} \end{bmatrix}. \tag{A.123}$$

We now characterize the solution and β_h in two special limiting cases. We then characterize the solution in general.

Limit where $\tau^{-1}MN \to 0$: In the limit, where the mortgage financing channel disappears—i.e., where $\tau^{-1}MN \to 0$, we have $\alpha_i \to (1-\phi)/(1-\phi\rho_i)$ and $\alpha_c \to 0$. Thus, we have $\beta_h = (1-\phi)/(1-\phi\rho_i)$ for all h—i.e., the expectations hypothesis holds and there is no excess sensitivity.

Limit where $\tau^{-1}MN > 0$ and $\eta = 0$: In the limit where $\tau^{-1}MN > 0$ but where $\eta = 0$, the fixed point problem simplifies to

$$\begin{bmatrix} \alpha_i \\ \alpha_c \end{bmatrix} = \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i-(1-\phi)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2} \\ -\frac{MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}{1-MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2} \end{bmatrix}. \tag{A.124}$$

The condition for α_i then implies that

$$\alpha_i = g(\alpha_i) = \frac{1-\phi}{1-\phi\rho_i}\left(1+\tau^{-1}\sigma_i^2 MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^3\right) \tag{A.125}$$

There are at most 3 solutions, $\alpha_i^* = g(\alpha_i^*)$, to this cubic equation. Since $g(0) > 0$ and $g'(\alpha_i) > 0$ for all α_i, there will always be a single negative solution $\alpha_i^* < 0$ that is unstable in the sense that $g'(\alpha_i^*) > 1$. When $\tau^{-1}\sigma_i^2 MN(\phi/(1-\phi))^2$ is large there will be no positive solutions. When $\tau^{-1}\sigma_i^2 MN(\phi/(1-\phi))^2$ is small, there are two positive solutions, the smaller of which is stable in the sense that $g'(\alpha_i^*) < 1$. We are always interested in the stable solution of this fixed point problem. Focusing on this stable solution, it is then trivially the case that $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$. When $\eta = 0$, we have $c_t^M = \bar{c}^M$ and thus $\alpha_c\left(c_t^M - \bar{c}\right) = 0$. In other words, α_c is not relevant in equilibrium. Thus, when $q = 1$, $\tau^{-1}MN > 0$, and $\eta = 0$, we have $\beta_h = \alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ for all h. In other words, the expectations hypothesis fails and there is excess sensitivity, but this excess sensitivity is not horizon dependent.

General case where $\tau^{-1}MN > 0$ and $\eta > 0$: Here we have

$$\begin{bmatrix} \alpha_i \\ \alpha_c \end{bmatrix} = \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i-(1-\phi)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}\left(1-\frac{\eta\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^3(\alpha_i)^3\sigma_i^2}{1-\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\right) \\ -\frac{MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}{1-MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)} \end{bmatrix}. \tag{A.126}$$

Relative to the case where $\eta = 0$, α_i is reduced because mortgage refinancing dynamics—i.e., the fact that the average mortgage coupon evolves over time—mean that the induced duration supply shocks are expected to be less persistent. Analogously, α_c is reduced in absolute value for the same reason.

Characterizing the solution: We first show that $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ and $\alpha_c^* < 0$ in any stable solution. We can rewrite the first condition for α_i to obtain

$$\alpha_i = \frac{1-\phi}{1-\phi\rho_i - \eta\phi\alpha_c}\left(1 + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2 (\alpha_i)^3 \sigma_i^2\right) \tag{A.127}$$

Thus, for a fixed $\alpha_c < 0$ the solution when $\eta > 0$ is lower than the solution when $\eta = 0$. We want to show that we still have $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ in any stable solution. It suffices to show that the fixed point operator returns $\alpha_i > (1-\phi)/(1-\phi\rho_i)$ if we initially suppose that $\alpha_i = (1-\phi)/(1-\phi\rho_i)$. Concretely, suppose that a stable solution exists and guess that the solution is $\alpha_i = (1-\phi)/(1-\phi\rho_i)$. Then, using the condition for α_i, we have

$$\alpha_c = -\frac{1}{\eta}\tau^{-1}MN\frac{\phi(1-\phi)}{(1-\phi\rho_i)^2}\sigma_i^2. \tag{A.128}$$

Now plug this into the condition for α_c and solve for α_i to obtain

$$(\alpha_i)^2 = \frac{1 + \frac{\phi}{1-\phi}\eta + \tau^{-1}MN\frac{\phi^2}{(1-\phi\rho_i)^2}\sigma_i^2}{\eta\frac{\phi}{1-\phi}\frac{(1-\phi\rho_i)^2}{(\phi-1)^2} + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2\sigma_i^2} \tag{A.129}$$

Since

$$\begin{aligned} &\frac{1 + \frac{\phi}{1-\phi}\eta + \tau^{-1}MN\frac{\phi^2}{(1-\phi\rho_i)^2}\sigma_i^2}{\eta\frac{\phi}{1-\phi}\frac{(1-\phi\rho_i)^2}{(\phi-1)^2} + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2\sigma_i^2} - \left(\frac{1-\phi}{1-\phi\rho_i}\right)^2 \\ = \;& \frac{\tau}{\phi}\frac{(1-\phi)^3}{MN\phi(1-\phi)\sigma_i^2 + \tau\eta(1-\phi\rho_i)^2} > 0, \end{aligned} \tag{A.130}$$

this implies that we must have $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ in any stable solution.

We want to show that $\alpha_c < 0$ in any stable solution. It suffices to show that the fixed point operator returns $\alpha_c < 0$ if we initially suppose that $\alpha_c = 0$. Suppose $\alpha_c = 0$. Then, α_i solves

$$\alpha_i = \frac{1-\phi}{1-\phi\rho_i}\left(1 + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^3\sigma_i^2\right) > \frac{1-\phi}{1-\phi\rho_i}. \tag{A.131}$$

Assuming this solution is stable, we have

$$Z = \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2 < 3\left(\frac{1-\phi}{1-\phi\rho_i}\right)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2 < 1. \tag{A.132}$$

Then α_c solves

$$-\frac{\phi}{1-\phi}\eta\left[\alpha_c^2\right] + \left[(1-Z) + \frac{\phi}{1-\phi}\eta\right]\alpha_c + Z = 0. \tag{A.133}$$

The relevant stable solution is the smaller solution. And the smaller solution here is negative. Thus, we have $\alpha_c^* < 0$ in any stable solution.

Calculating β_h: To calculate β_h we make the following observations. First, using the fact that $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$, we have

$$\mathbf{V} = \begin{bmatrix} \frac{\sigma_i^2}{1-\rho_i^2} & \frac{\eta\alpha_i\sigma_i^2\rho_i}{\left(1-\rho_i^2\right)(1-\rho_i+\eta\rho_i(1-\alpha_c))} \\ \frac{\eta\alpha_i\sigma_i^2\rho_i}{\left(1-\rho_i^2\right)(1-\rho_i+\eta\rho_i(1-\alpha_c))} & \frac{\eta\alpha_i^2\sigma_i^2(1+\rho_i-\eta\rho_i(1-\alpha_c))}{(1-\alpha_c)\left(1-\rho_i^2\right)(1+\eta\alpha_c+1-\eta)(1-\rho_i+\eta\rho_i(1-\alpha_c))} \end{bmatrix} \tag{A.134}$$

Next, using the fact that

$$\mathbf{\Gamma}^h = \begin{bmatrix} (\rho_i)^h & 0 \\ \eta\alpha_i \frac{(\rho_i)^h - ((1-\eta)+\eta\alpha_c)^h}{\rho_i - ((1-\eta)+\eta\alpha_c)} & ((1-\eta)+\eta\alpha_c)^h \end{bmatrix} \tag{A.135}$$

we have

$$Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] = 2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h \tag{A.136}$$

$$= \begin{bmatrix} 2\left(1-\rho_i^h\right) Var\left[i_t\right] & \begin{array}{c}\left(2-\rho_i^h-((1-\eta)+\eta\alpha_c)^h\right) Cov\left[i_t,c_t\right] \\ -\eta\alpha_i \frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)} Var\left[i_t\right]\end{array} \\ \begin{array}{c}\left(2-\rho_i^h-((1-\eta)+\eta\alpha_c)^h\right) Cov\left[i_t,c_t\right] \\ -\eta\alpha_i \frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)} Var\left[i_t\right]\end{array} & 2\left(\begin{array}{c}\left(1-((1-\eta)+\eta\alpha_c)^h\right) Var\left[c_t\right] \\ -\eta\alpha_i \frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)} Cov\left[i_t,c_t\right]\end{array}\right) \end{bmatrix}$$

Thus, we have

$$\begin{aligned} \beta_h &= \alpha_i + \alpha_c \frac{\left(2-2\rho_i^h+\rho_i^h-((1-\eta)+\eta\alpha_c)^h\right) Cov\left[i_t,c_t\right] - \eta\alpha_i \frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)} Var\left[i_t\right]}{2\left(1-\rho_i^h\right) Var\left[i_t\right]} \\ &= \alpha_i + \alpha_c \frac{Cov\left[i_t,c_t\right]}{Var\left[i_t\right]} + \alpha_c \left(\frac{Cov\left[i_t,c_t\right]}{Var\left[i_t\right]} - \frac{\eta\alpha_i}{\rho_i-((1-\eta)+\eta\alpha_c)}\right) \frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{2\left(1-\rho_i^h\right)} \\ &= \alpha_i \left(\frac{1-\rho_i(1-\eta)}{1-\rho_i+\eta\rho_i(1-\alpha_c)}\right) \\ &\quad + \left[-\frac{1-\rho_i^2}{(\rho_i-((1-\eta)+\eta\alpha_c))(1-\rho_i((1-\eta)+\eta\alpha_c))}\alpha_c\alpha_i\eta\right] \left\{\frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{2\left(1-\rho_i^h\right)}\right\}. \end{aligned} \tag{A.137}$$

There are two cases to consider:

1. If $\rho_i > (1-\eta)+\eta\alpha_c$, the term in square brackets is positive. And, since $\rho_i > (1-\eta)+\eta\alpha_c$, the term in curly braces is decreasing in h. Thus, β_h is decreasing in h.[22]

2. Alternately, if $\rho_i < (1-\eta)+\eta\alpha_c$, the term in square brackets is negative. And, since $\rho_i < (1-\eta)+\eta\alpha_c$, the term in curly braces is increasing in h. Thus, β_h is decreasing in h.

Thus, β_h is decreasing in h when $\eta > 0$, $\tau^{-1}MN > 0$, and $q = 1$.

[22]To show that the term in curly braces is decreasing in h when $\rho_i > \rho_c \equiv (1-\eta)+\eta\alpha_c$, recall that we showed above that $\left(2-\rho_i^h-\rho_c^h\right)/\left(2-\rho_i^h-\rho_i^h\right)$ exceeds 1 and is decreasing in h when $\rho_i > \rho_c$. However, we have $\left(\rho_i^h-\rho_c^h\right)/\left(2\left(1-\rho_i^h\right)\right) = \left(2-\rho_i^h-\rho_c^h\right)/\left(2-\rho_i^h-\rho_i^h\right) - 1$. Thus, the term in curly brackets is positive and decreasing in h when $\rho_i > \rho_c$. Conversely, the term in curly brackets is negative and increasing in h when $\rho_i < \rho_c$.

Summary: When there is no slow-moving capital ($q = 1$), the mortgage refinancing model generate excess sensitivity when $\tau^{-1}MN > 0$. Furthermore, this excess sensitivity is horizon dependent—i.e., it is most pronounced at high frequencies—when $\eta > 0$.

Case #2: $q < 1$, $k = 2$, and $\rho_T = \rho_P \equiv \rho_i$

Solution: In this case, we have

$$y_t = \alpha_0 + \alpha_1' \mathbf{x}_t = \alpha_0 + \alpha_i \left(i_t - \bar{i}\right) + \alpha_c \left(c_t^M - \bar{c}^M\right) + \alpha_d \left(d_{t-1} - \delta_0\right). \tag{A.138}$$

The law of motion for the state vector is

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma\left(\boldsymbol{\alpha}\right)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \qquad \text{(A.139)} \\ &= \begin{bmatrix} \rho_i & 0 & 0 \\ \eta\alpha_i & (1-\eta) + \eta\alpha_c & \eta\alpha_d \\ \delta_i & \delta_c & \delta_d \end{bmatrix} \begin{bmatrix} i_t - \bar{i} \\ c_t^M - \bar{c}^M \\ d_{t-1} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \\ 0 \end{bmatrix}, \end{aligned}$$

where

$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_i^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A.140}$$

Thus, we have

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \left(\alpha_i\right)^2 \sigma_i^2. \tag{A.141}$$

Defining

$$Z = \frac{MN}{q\tau} V^{(1)} = \frac{MN}{q\tau}\left(\frac{\phi}{1-\phi}\right)^2 \left(\alpha_i\right)^2 \sigma_i^2 \tag{A.142}$$

(we have defined Z to depend on q), the solution is

$$\begin{aligned} \boldsymbol{\alpha}_1 &= (1-\phi)\left[\left(1-(1-\phi)Z\right)\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left[\mathbf{e} - \left(Z\mathbf{e}_c + \frac{Z}{MN}(1-q)k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right)\right] \qquad \text{(A.143)} \\ &= (1-\phi)\left[\left(1-(1-\phi)Z\right)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \phi\begin{bmatrix} \rho_i & \eta\alpha_i & \delta_i \\ 0 & (1-\eta)+\eta\alpha_c & \delta_c \\ 0 & \eta\alpha_d & \delta_d \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 - \frac{Z}{MN}\frac{\delta_i(1-q)}{2} \\ -Z - \frac{Z}{MN}\delta_c\frac{1}{2}(1-q) \\ -\frac{Z}{MN}\left(\delta_d + 1\right)\frac{1}{2}(1-q) \end{bmatrix} \end{aligned}$$

The solution does not readily simplify because Γ is no longer lower triangular. In all other cases, Γ is effectively lower triangular, which makes the expressions much nicer.

Characterizing the solution: As above, one can show that we have $\alpha_i > (1-\phi)/(1-\phi\rho_i)$, $\alpha_c < 0$, and $\alpha_d < 0$ in any stable solution. Similarly, we have $\delta_i > 0$, $\delta_c < 0$, and $-1 < \delta_d < 0$ in any stable solution.

Computing β_h: Using $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$, we have

$$\mathbf{V} = \begin{bmatrix} V_i & C_{i,c} & C_{i,d} \\ C_{i,c} & V_c & C_{c,d} \\ C_{i,d} & C_{c,d} & V_d \end{bmatrix} \tag{A.144}$$

where

$$V_i = \frac{\sigma_i^2}{1-\rho_i^2}, \tag{A.145}$$

$$C_{i,c} = \frac{\sigma_i^2}{1-\rho_i^2} \rho_i \frac{\eta\alpha_i (1-\delta_d\rho_i) + \eta\alpha_d\delta_i\rho_i}{1 - ((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i (1 - ((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \tag{A.146}$$

$$C_{i,d} = \frac{\sigma_i^2}{1-\rho_i^2} \rho_i \frac{\delta_i (1 - ((1-\eta)+\eta\alpha_c)\rho_i) + \delta_c\eta\alpha_i\rho_i}{1 - ((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i (1 - ((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \tag{A.147}$$

Since $C_{i,c} \equiv Cov\left[i_t, c_t^M\right] > 0$ and $C_{i,d} \equiv Cov\left[i_t, d_{t-1}\right] > 0$ in any stable equilibrium when $\eta > 0$ and $q < 1$, we have

$$\alpha_i (1-\delta_d\rho_i) + \alpha_d\delta_i\rho_i > 0, \tag{A.148}$$

$$\delta_i (1 - ((1-\eta)+\eta\alpha_c)\rho_i) + \delta_c\eta\alpha_i\rho_i > 0, \tag{A.149}$$

$$1 - ((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i (1 - ((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2 > 0. \tag{A.150}$$

Next, letting

$$\mathbf{\Gamma}^h = \begin{bmatrix} \rho_i & 0 & 0 \\ \gamma_i & \gamma_c & \gamma_d \\ \delta_i & \delta_c & \delta_d \end{bmatrix}^h \equiv \begin{bmatrix} a_h & 0 & 0 \\ b_h & d_h & f_h \\ c_h & e_h & g_h \end{bmatrix}, \tag{A.151}$$

we have

$$\begin{aligned} & Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] \\ = \;& 2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h \\ = \;& \left[\begin{matrix} 2(1-a_h)V_i & (2-d_h-a_h)C_{c,i} - f_hC_{i,d} - b_hV_i & (2-a_h- \\ (2-d_h-a_h)C_{c,i} - f_hC_{i,d} - b_hV_i & 2(1-d_h)V_c - 2f_hC_{c,d} - 2b_hC_{c,i} & (2-g_h-d_h)C_{c,d} \\ (2-a_h-g_h)C_{i,d} - e_hC_{i,c} - c_hV_i & (2-g_h-d_h)C_{c,d} - b_hC_{d,i} - c_hC_{c,i} - e_hV_c - V_df_h & 2(1-g_h \end{matrix} \right. \end{aligned}$$

Combing these elements, we have

$$\begin{aligned} \beta_h &= \alpha_i + \alpha_c \frac{(2-a_h-d_h)C_{i,c} - f_hC_{i,d} - b_hV_i}{2(1-a_h)V_i} + \alpha_d \frac{(2-a_h-g_h)C_{i,d} - e_hC_{i,c} - c_hV_i}{2(1-a_h)V_i} \\ &= \left[\alpha_i + \alpha_c\frac{C_{i,c}}{V_i} + \alpha_d\frac{C_{i,d}}{V_i}\right] \\ &\quad + \left[\alpha_c \left\{\frac{(a_h-d_h)C_{i,c} - f_hC_{i,d} - b_hV_i}{2(1-a_h)V_i}\right\} + \alpha_d\left\{\frac{(a_h-g_h)C_{i,d} - e_hC_{i,c} - c_hV_i}{2(1-a_h)V_i}\right\}\right] \end{aligned} \tag{A.152}$$

Since $\lim_{h\to\infty}\mathbf{\Gamma}^h=\mathbf{0}$, we have

$$\begin{aligned}
\lim_{h\to\infty}\beta_h &= \alpha_i+\alpha_c\frac{C_{i,c}}{V_i}+\alpha_d\frac{C_{i,d}}{V_i} \\
&= \alpha_i+\alpha_c\rho_i\frac{\eta\alpha_i\left(1-\delta_d\rho_i\right)+\eta\alpha_d\delta_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&\quad +\alpha_d\rho_i\frac{\delta_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)+\delta_c\eta\alpha_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&= \frac{\left(\alpha_i-\delta_d\alpha_i\rho_i+\alpha_d\delta_i\rho_i\right)\left(1-\rho_i\left(1-\eta\right)\right)}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&= \frac{\left(1-\rho_i\left(1-\eta\right)\right)}{\rho_i\eta}\rho_i\frac{\eta\alpha_i\left(1-\delta_d\rho_i\right)+\eta\alpha_d\delta_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&= \frac{\left(1-\rho_i\left(1-\eta\right)\right)}{\rho_i\eta}\frac{C_{i,c}}{V_i}>0.
\end{aligned} \tag{A.153}$$

We also have

$$\beta_h=\lim_{h\to\infty}\beta_h+\alpha_c\mathcal{R}_{i,c}\left(h\right)+\alpha_d\mathcal{R}_{i,d}\left(h\right) \tag{A.154}$$

where

$$\begin{aligned}
\mathcal{R}_{i,c}\left(h\right) &= \frac{Cov\left[i_{t+h}-i_t,c_{t+h}^M-c_t^M\right]}{Var\left[i_{t+h}-i_t\right]}-\frac{Cov\left[i_t,c_t^M\right]}{Var\left[i_t\right]} \\
&= \frac{\left(a_h-d_h\right)C_{ci}-f_hC_{id}-b_hV_i}{2\left(1-a_h\right)V_i} \\
&= \frac{\begin{array}{c}\left(a_h-d_h\right)\rho_i\frac{\eta\alpha_i(1-\delta_d\rho_i)+\eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -f_h\rho_i\frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i)+\delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2}-b_h\end{array}}{2\left(1-a_h\right)}
\end{aligned} \tag{A.155}$$

and

$$\begin{aligned}
\mathcal{R}_{i,d}\left(h\right) &= \frac{Cov\left[i_{t+h}-i_t,d_{t+h-1}-d_{t-1}\right]}{Var\left[i_{t+h}-i_t\right]}-\frac{Cov\left[i_t,d_{t-1}\right]}{Var\left[i_t\right]} \\
&= \frac{\left(a_h-g_h\right)C_{i,d}-e_hC_{i,c}-c_hV_i}{2\left(1-a_h\right)V_i} \\
&= \frac{\begin{array}{c}\left(a_h-g_h\right)\rho_i\frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i)+\delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -e_h\rho_i\frac{\eta\alpha_i(1-\delta_d\rho_i)+\eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2}-c_h\end{array}}{2\left(1-a_h\right)}
\end{aligned} \tag{A.156}$$

Proof that $\lim_{h\to\infty}\beta_h<\beta_1$: We have

$$\begin{aligned}
& \beta_1-\lim_{h\to\infty}\beta_h \\
=\ & \alpha_c\mathcal{R}_{i,c}\left(1\right)+\alpha_d\mathcal{R}_{i,d}\left(1\right) \\
=\ & \frac{1}{2}\left(\rho_i+1\right)\frac{\left[-\alpha_d\right]\left(\delta_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)+\delta_c\eta\alpha_i\rho_i\right)+\left[-\alpha_c\right]\eta\left(\alpha_i\left(1-\delta_d\rho_i\right)+\alpha_d\delta_i\rho_i\right)}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
>\ & 0.
\end{aligned} \tag{A.157}$$

We know that the denominator is positive. Furthermore, since $\alpha_d < 0$ and $(\delta_i (1 - ((1-\eta) + \eta\alpha_c)\rho_i) + \delta_c\eta\alpha_i\rho_i) > 0$ and $\alpha_c < 0$ and $(\alpha_i (1 - \delta_d\rho_i) + \alpha_d\delta_i\rho_i) > 0$, the numerator is also positive.

Proof that $\beta_2 < \beta_1$: We want to show that

$$\beta_2 - \lim_{h\to\infty} \beta_h < \beta_1 - \lim_{h\to\infty} \beta_h.$$

We have

$$\begin{bmatrix} a_2 & 0 & 0 \\ b_2 & d_2 & f_2 \\ c_2 & e_2 & g_2 \end{bmatrix} = \begin{bmatrix} \rho_i & 0 & 0 \\ \eta\alpha_i & (1-\eta) + \eta\alpha_c & \eta\alpha_d \\ \delta_i & \delta_c & \delta_d \end{bmatrix}^2 \tag{A.158}$$

$$= \begin{bmatrix} \rho_i^2 & 0 & 0 \\ \eta\alpha_i (1 - \eta + \eta\alpha_c) + \eta\alpha_i\rho_i + \eta\alpha_d\delta_i & ((1-\eta) + \eta\alpha_c)^2 + \eta\alpha_d\delta_c & \eta\alpha_d ((1-\eta) + \eta\alpha_c + \delta_d) \\ \delta_d\delta_i + \delta_i\rho_i + \eta\delta_c\alpha_i & \delta_c ((1-\eta) + \eta\alpha_c + \delta_d) & \delta_d^2 + \eta\alpha_d\delta_c \end{bmatrix}$$

We have

$$\begin{aligned}
\beta_2 - \lim_{h\to\infty} \beta_h &= \alpha_c \frac{\begin{array}{c} \left(\rho_i^2 - \left(((1-\eta) + \eta\alpha_c)^2 + \eta\alpha_d\delta_c\right)\right) \rho_i \frac{\eta\alpha_i(1-\delta_d\rho_i) + \eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \\ -\eta\alpha_d ((1-\eta) + \eta\alpha_c + \delta_d) \rho_i \frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i) + \delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \\ -(\eta\alpha_i (1 - \eta + \eta\alpha_c) + \eta\alpha_i\rho_i + \eta\alpha_d\delta_i) \end{array}}{2\left(1-\rho_i^2\right)} \\
&\quad + \alpha_d \frac{\begin{array}{c} \left(\rho_i^2 - \left(\delta_d^2 + \eta\alpha_d\delta_c\right)\right) \rho_i \frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i) + \delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \\ -\delta_c ((1-\eta) + \eta\alpha_c + \delta_d) \rho_i \frac{\eta\alpha_i(1-\delta_d\rho_i) + \eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \\ -(\delta_d\delta_i + \delta_i\rho_i + \eta\delta_c\alpha_i) \end{array}}{2\left(1-\rho_i^2\right)}
\end{aligned} \tag{A.159}$$

Comparing this with $\beta_1 - \lim_{h\to\infty} \beta_h$, we have

$$\begin{aligned}
\beta_1 - \beta_2 &= \frac{1}{2}\left\{\frac{(1-\rho_i)\delta_i(\alpha_d\delta_d - \alpha_d)}{1 - ((1-\eta) + \eta\alpha_c)\rho_i - \delta_d\rho_i(1 - ((1-\eta) + \eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2}\right\} \\
&\quad + \frac{\eta}{2}\left\{\frac{\begin{array}{c} \eta\alpha_i\left[\alpha_c^2 - \alpha_c - \delta_d\alpha_c^2\rho_i + \alpha_c(\delta_d + \alpha_d\delta_c)\rho_i\right] \\ + \left[\alpha_c\alpha_d\delta_i + \alpha_d\delta_c\alpha_i(1-\rho_i) - \alpha_d\delta_i\rho_i - \alpha_c\alpha_d\delta_d\delta_i\rho_i + \alpha_d(\delta_d + \alpha_d\delta_c)\delta_i\rho_i\right] \end{array}}{1 - ((1-\eta) + \eta\alpha_c)\rho_i - \delta_d\rho_i(1 - ((1-\eta) + \eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2}\right\} \\
&> 0.
\end{aligned} \tag{A.160}$$

The first term in curly braces is positive. We just need to consider the second term in curly braces. The denominator is positive. Finally, since $(\delta_d + \alpha_d\delta_c) < 0$, all the terms in the numerator are positive. Thus, the second term in curly braces is also positive and we conclude that $\beta_1 > \beta_2$.

D.2 The investor extrapolation channel

D.2.1 General model solution

We we drop $s_t - \overline{s}$ from the state vector $\mathbf{x}_t$ and now add $m_{P,t}$ and $m_{T,t}$ to the state vector. For the sake of concreteness, suppose that $k = 4$. We conjecture that equilibrium yields take the form $y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$, and that the demands of active slow-moving arbitrageurs are of the form $d_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t$, where the $k+3$ dimensional state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \\ m_{P,t} \\ m_{T,t} \end{bmatrix}. \tag{A.161}$$

Rational investors believe the law motion for the state vector is:

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\boldsymbol{\delta})\, \mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 & 0 \\ \delta_P & \delta_T & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} & \delta_{m_P} & \delta_{m_P} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_P & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_T \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \\ m_{P,t} \\ m_{T,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \\ \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \end{bmatrix}, \end{aligned} \tag{A.162}$$

where

$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_P^2 & 0 & 0 & 0 & 0 & \sigma_P^2 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 & 0 & \sigma_T^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_P^2 & 0 & 0 & 0 & 0 & \sigma_P^2 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 & 0 & \sigma_T^2 \end{bmatrix}. \tag{A.163}$$

By contrast, diagnostic investors believe the law of motion is:

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma_D(\boldsymbol{\delta})\, \mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & \theta & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 & \theta \\ \delta_P & \delta_T & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} & \delta_{m_P} & \delta_{m_T} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_P & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_T \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \\ m_{P,t} \\ m_{T,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \\ \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \end{bmatrix}. \end{aligned} \tag{A.164}$$

Thus, both rational investors and diagnostic investors properly forecast the evolution of diagnostic investors' time-varying biases $m_{P,t}$ and $m_{T,t}$. However, diagnostic investors' forecasts of future short rates depend on $m_{P,t}$ and $m_{T,t}$, while rational investors' short rate forecasts do not depend on $m_{P,t}$ and $m_{T,t}$. Intuitively, the biases of diagnostic investors ($m_{P,t}$ and $m_{T,t}$) are like time-varying "signals" about future short rates. Diagnostic investors incorrectly believe that these signals are informative about future short rates. Rational investors know that these signals are irrelevant for forecasting future short rates (given what they already know), but they still need to keep track of these signals and forecast their evolution because these signals influence the future demands of diagnostic investors.

Rational investors and diagnostic investors agree about the risk of holding long-term bonds until the next period, but disagree about expected returns. Specifically, both rational and diagnostic investors believe the variance of 1-period excess returns is

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1. \tag{A.165}$$

Rational investors form unbiased forecasts of expected returns given by:

$$E_t\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t. \tag{A.166}$$

Thus, the demands of fast-moving rational investors are:

$$b_t = \tau \frac{E_t\left[rx_{t+1}\right]}{V^{(1)}} = \tau \frac{\left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t}{V^{(1)}}. \tag{A.167}$$

By contrast, diagnostic investors form biased forecasts of expected returns given by:

$$E_t^D\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}_D\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t.$$

Thus, the demands of diagnostic investors are:

$$h_t = \tau \frac{E_t^D\left[rx_{t+1}\right]}{V^{(1)}} = \tau \frac{\left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}_D\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t}{V^{(1)}}. \tag{A.168}$$

It is easy to see that

$$E_t^D\left[rx_{t+1}\right] = E_t\left[rx_{t+1}\right] - \frac{\phi}{1-\phi}\theta\left(\alpha_P m_{P,t} + \alpha_T m_{T,t}\right), \tag{A.169}$$

where $\alpha_P = (1-\phi)/(1-\phi\rho_P)$ and $\alpha_T = (1-\phi)/(1-\phi\rho_T)$ are first and second elements of $\boldsymbol{\alpha}_1$, respectively. Thus, diagnostic investors underestimate the expected return to holding long-term bonds when short rates have recently rise—i.e., when $m_{P,t}$ and $m_{T,t}$ are positive—and therefore demand fewer long-term bonds that fast-moving rational investors.

Active demand at time t is

$$\begin{aligned}
& f h_t + (1-f) q b_t + (1-f)(1-q) k^{-1} d_t \\
&= \left[\tau\left(f + (1-f) q\right)\frac{\left(\alpha_0 - \bar{i}\right)}{V^{(1)}} + (1-f)(1-q) k^{-1}\delta_0\right] \\
&+ \left[\tau f \frac{\frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}_D\right) - \mathbf{e}'}{V^{(1)}} + \tau(1-f) q \frac{\frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}\right) - \mathbf{e}'}{V^{(1)}} + (1-f)(1-q) k^{-1}\boldsymbol{\delta}_1'\right]\mathbf{x}_t
\end{aligned} \tag{A.170}$$

and active supply is

$$\begin{aligned} & \bar{s}-(1-f)(1-q)k^{-1}\sum\nolimits_{i=1}^{k-1} d_{t-i} \\ = & \left[\bar{s}-(1-f)(1-q)\frac{(k-1)}{k}\delta_0\right]+\left[-(1-f)(1-q)k^{-1}\mathbf{e}_d'\right]\mathbf{x}_t. \end{aligned} \tag{A.171}$$

Matching constants terms, we obtain

$$\alpha_0=\bar{i}+\frac{V^{(1)}}{\tau}\frac{\bar{s}-(1-f)(1-q)\delta_0}{f+(1-f)q} \tag{A.172}$$

Matching slope coefficients on $\mathbf{x}_t$, we obtain

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\mathbf{I}-\phi\left(\frac{f\boldsymbol{\Gamma}_D+(1-f)q\boldsymbol{\Gamma}}{f+(1-f)q}\right)'\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)k^{-1}(\mathbf{e}_d+\boldsymbol{\delta}_1)}{f+(1-f)q}\right] \tag{A.173}$$

In summary, an equilibrium in this model extension solves the following system of equations

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\mathbf{I}-\phi\left(\frac{f\boldsymbol{\Gamma}_D(\boldsymbol{\delta}_1)+(1-f)q\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)}{f+(1-f)q}\right)'\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}(\boldsymbol{\alpha}_1)}{\tau}\frac{(1-f)(1-q)k^{-1}(\mathbf{e}_d+\boldsymbol{\delta}_1)}{f+(1-f)q}\right] \tag{A.174}$$

and

$$\boldsymbol{\delta}_1'=\tau\frac{\left((\boldsymbol{\alpha}_1-\mathbf{e})'(\mathbf{I}-\boldsymbol{\Gamma}(\boldsymbol{\delta}_1))^{-1}+\frac{\theta}{1-\theta}\boldsymbol{\alpha}_1'\right)}{V^{(k)}(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)}\left(\mathbf{I}-\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)^k\right), \tag{A.175}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)$ and $\boldsymbol{\Gamma}_D(\boldsymbol{\delta}_1)$ emphasize that the true and perceived transition matrices depend on $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)$ to emphasize that the k-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. As in our baseline model, the first two elements of $\boldsymbol{\delta}_1$ are going to be zero, since, in the absence of mistakes by diagnostic investors, rational investors don't want to adjust their holdings as a function of short rates. With diagnostic expectations, it is not the level of short rates that governs investors' biased perceptions. Instead, what matters is the recent changes in short rates as summarized by $m_{P,t}$ and $m_{T,t}$. Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for α_0 and δ_0 using

$$\alpha_0=\bar{i}+\frac{V^{(1)}}{\tau}\frac{\bar{s}-(1-f)(1-q)\delta_0}{f+(1-f)q}\text{ and }\delta_0=\tau\frac{k\left(\alpha_0-\bar{i}\right)}{V^{(k)}}, \tag{A.176}$$

which yields

$$\alpha_0 = \bar{i}+\frac{\bar{s}/\tau}{(f+(1-f)q)\frac{1}{V^{(1)}}+(1-f)(1-q)\frac{k}{V^{(k)}}}\text{ and} \tag{A.177}$$

$$\delta_0 = \frac{\frac{k}{V^{(k)}}}{(f+(1-f)q)\frac{1}{V^{(1)}}+(1-f)(1-q)\frac{k}{V^{(k)}}}\times\bar{s}. \tag{A.178}$$

D.2.2 Proof of Proposition 3

Proposition 3. ***Investor extrapolation model.*** *For simplicity suppose $\rho_T = \rho_P$ and $\kappa_T = \kappa_P$. When $f\theta > 0$, long rates are excessively sensitive to short rates. When $f\theta > 0$ and $\kappa_T = \rho_T$, this excess sensitivity is only horizon-dependent—i.e., the regression coefficients β_h only decline with horizon h—when unbiased arbitrage capital is slow moving ($q < 1$). By contrast, when $f\theta > 0$ and $\kappa_P < \rho_T$, β_h declines with horizon h even if all arbitrage capital is fast-moving ($q = 1$).*

Proof: To demonstrate this result, it suffices to consider two special cases. We suppose throughout that $\rho_T = \rho_P \equiv \rho_i$ and $\kappa_T = \kappa_P \equiv \kappa_i$. First, we consider the case where there is no slow-moving capital ($q = 1$). Next, we study the case where $q < 1$ and $k = 2$. The arguments given in this special $k = 2$ case generalize naturally to the case where $k > 2$.

Case #1: No slow-moving capital ($q = 1$), $\rho_T = \rho_P \equiv \rho_i$, and $\kappa_T = \kappa_P \equiv \kappa_i$ We assume that $\rho_T = \rho_P \equiv \rho_i$, and $\kappa_T = \kappa_P \equiv \kappa_i$. In the absence of slow-moving capital ($q = 1$), the model with extrapolative investors can be solved analytically. Specifically, when $q = 1$, we have

$$\boldsymbol{\alpha}_1 = \begin{bmatrix} \alpha_i \\ \alpha_{m_i} \end{bmatrix} = (1-\phi)\left[\mathbf{I} - \phi\left(f\boldsymbol{\Gamma}_D + (1-f)\,\boldsymbol{\Gamma}\right)'\right]^{-1}\mathbf{e} = \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i} \\ f\theta\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} \end{bmatrix}. \tag{A.179}$$

In this case, the model-implied coefficient β_h from regression of $y_{t+h} - y_t$ on $i_{t+h} - i_t$ is

$$\begin{aligned} \beta_h &= \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} \\ &= \frac{\alpha_i Var\left[\Delta_h i_t\right] + \alpha_{m_i} Cov\left[\Delta_h m_{i,t}, \Delta_h i_t\right]}{Var\left[\Delta_h i_{t+h}\right]}, \end{aligned} \tag{A.180}$$

where $Var\left[\Delta_h i_t\right] = 2\left[(1-\rho_i^h)/\left(1-\rho_i^2\right)\right]\sigma_i^2$ and $Cov\left[\Delta_h m_{i,t}, \Delta_h i_t\right] = \left[\left(2-\kappa_i^h-\rho_i^h\right)/\left(1-\kappa_i\rho_i\right)\right]\sigma_i^2$.

We then can demonstrate the following results:

- When $f\theta = 0$, then β_h is a constant that is independent of h..
- When $f\theta > 0$ and $\kappa_i = \rho_i$, then β_h is a constant that is independent of h. Specifically, we have

  $$\beta_h = \frac{1-\phi}{1-\phi\rho_i}\left(1 + f\theta\frac{\phi}{1-\phi\rho_i}\right) > \frac{1-\phi}{1-\phi\rho_i}. \tag{A.181}$$

  Thus, there is excess sensitivity relative to the expectations hypothesis, but this sensitivity is not horizon dependent.
- When $f\theta > 0$ and $\kappa_i < \rho_i$, β_h is a decreasing function of h. In other words, there is horizon-dependent excess sensitivity.

- When $f\theta > 0$, we have
$$E_t\left[rx_{t+1}\right] = \frac{\tau^{-1}V^{(1)}}{1-f}\left(\overline{s} - fh_t\right). \tag{A.182}$$
Thus, we have $\partial E_t\left[rx_{t+1}\right]/\partial h_t < 0$—i.e., the expected returns on long-term bonds are low when the demand of extrapolative agents is high.

Case #2: $q < 1$, $k = 2$, $\rho_T = \rho_P \equiv \rho_i$, and $\kappa_T = \kappa_P \equiv \kappa_i$

Solution: We conjecture that equilibrium yields take the form $y_t = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t$, and that the demands of active slow-moving arbitrageurs are of the form $d_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$, where the state vector is
$$\mathbf{x}_t = \begin{bmatrix} i_t - \overline{i} \\ d_{t-1} - \delta_0 \\ m_t \end{bmatrix}. \tag{A.183}$$

Rational investors believe the law motion for the state vector is:
$$\begin{aligned}\mathbf{x}_{t+1} &= \Gamma\left(\boldsymbol{\delta}\right)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_i & 0 & 0 \\ 0 & \delta_d & \delta_m \\ 0 & 0 & \kappa \end{bmatrix}\begin{bmatrix} i_t - \overline{i} \\ d_{t-1} - \delta_0 \\ m_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \\ \varepsilon_{i,t+1} \end{bmatrix},\end{aligned} \tag{A.184}$$
where
$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_i^2 & 0 & \sigma_i^2 \\ 0 & 0 & 0 \\ \sigma_i^2 & 0 & \sigma_i^2 \end{bmatrix}. \tag{A.185}$$
By contrast, diagnostic investors believe the law of motion is:
$$\begin{aligned}\mathbf{x}_{t+1} &= \Gamma_D\left(\boldsymbol{\delta}\right)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_i & 0 & \theta \\ 0 & \delta_d & \delta_m \\ 0 & 0 & \kappa_i \end{bmatrix}\begin{bmatrix} i_t - \overline{i} \\ d_{t-1} - \delta_0 \\ m_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \\ \varepsilon_{i,t+1} \end{bmatrix}.\end{aligned} \tag{A.186}$$

The solution for $\boldsymbol{\alpha}_1$ takes the form:
$$\begin{aligned}\boldsymbol{\alpha}_1 &= \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i} \\ 0 \\ \frac{\theta f}{f+(1-f)q}\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} \end{bmatrix} \\ &\quad -\frac{1}{2}\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)}{f+(1-f)q}\begin{bmatrix} 0 \\ \frac{1-\phi}{1-\phi\delta_d}\left(1+\delta_d\right) \\ \frac{1-\phi}{(1-\kappa_i\phi)(1-\phi\delta_d)}\left(\delta_d\left(1-\phi\delta_d\right)+\phi\delta_m\left(1+\delta_d\right)\right) \end{bmatrix}\end{aligned} \tag{A.187}$$

Thus, we have

$$\alpha_i = \frac{1-\phi}{1-\phi\rho_i} > 0 \tag{A.188}$$

$$\alpha_d = -\frac{1}{2}\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)}{f+(1-f)q}\frac{1-\phi}{1-\phi\delta_d}(1+\delta_d) < 0 \tag{A.189}$$

$$\begin{aligned} \alpha_m &= \frac{\theta f}{f+(1-f)q}\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} \\ &\quad + \left[-\frac{1}{2}\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)}{f+(1-f)q}\frac{1-\phi}{(1-\phi\delta_d)}(1+\delta_d)\right]\frac{\delta_d(1-\phi\delta_d)+\phi\delta_m(1+\delta_d)}{(1-\kappa_i\phi)(1+\delta_d)} \\ &= \frac{\theta f}{f+(1-f)q}\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} + \alpha_d\frac{\delta_d(1-\phi\delta_d)+\phi\delta_m(1+\delta_d)}{(1-\kappa_i\phi)(1+\delta_d)} > 0. \end{aligned} \tag{A.190}$$

In this case, we have

$$E_t[rx_{t+1}] = \frac{\tau^{-1}V^{(1)}}{(1-f)q}\left[\bar{s}-(1-f)(1-q)(d_t+d_{t-1})/2 - fh_t\right]. \tag{A.191}$$

Thus, large purchases by extrapolative investors are associated with low expected excess returns on long-term bonds in the time series

$$\frac{Cov[rx_{t+1},h_t]}{Var[h_t]} = -\frac{\tau^{-1}V^{(1)}}{(1-f)q}\left[f+(1-f)(1-q)\frac{Cov[(d_t+d_{t-1})/2,h_t]}{Var[h_t]}\right] < 0. \tag{A.192}$$

The inequality follows because the term in square brackets is positive—i.e., since slow-moving investors do not fully offset the demand shocks from extrapolative investors.

Characterizing the solution: As above, we can show that $\alpha_i > 0$, $\delta_i = 0$, $\alpha_m > 0$, $\delta_m > 0$, $\alpha_d < 0$, and $-1 < \delta_d < 0$.

Computing β_h: Now we compute

$$\beta_h = \frac{Cov[y_{t+h}-y_t, i_{t+h}-i_t]}{Var[i_{t+h}-i_t]} = \frac{\boldsymbol{\alpha}_1'(2\mathbf{V}-\boldsymbol{\Gamma}^h\mathbf{V}-\mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V}-\boldsymbol{\Gamma}^h\mathbf{V}-\mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}. \tag{A.193}$$

Using the fact that $vec(\mathbf{V}) = (\mathbf{I}-\boldsymbol{\Gamma}\otimes\boldsymbol{\Gamma})^{-1}vec(\boldsymbol{\Sigma})$, we have

$$\mathbf{V} = Var[\mathbf{x}_t] = \begin{bmatrix} \frac{\sigma_i^2}{1-\rho_i^2} & \sigma_i^2\delta_m\frac{\rho_i}{(1-\kappa_i\rho_i)(1-\delta_d\rho_i)} & \frac{\sigma_i^2}{1-\kappa_i\rho_i} \\ \sigma_i^2\delta_m\frac{\rho_i}{(1-\kappa_i\rho_i)(1-\delta_d\rho_i)} & \sigma_i^2\frac{\delta_m^2(\kappa_i\delta_d+1)}{(1-\kappa_i^2)(1-\delta_d^2)(1-\kappa_i\delta_d)} & \kappa_i\sigma_i^2\frac{\delta_m}{(1-\kappa_i^2)(1-\kappa_i\delta_d)} \\ \frac{\sigma_i^2}{1-\kappa_i\rho_i} & \kappa_i\sigma_i^2\frac{\delta_m}{(1-\kappa_i^2)(1-\kappa_i\delta_d)} & \frac{\sigma_i^2}{1-\kappa_i^2} \end{bmatrix}. \tag{A.194}$$

Since

$$\boldsymbol{\Gamma}^h = \begin{bmatrix} \rho_i^h & 0 & 0 \\ 0 & \delta_d^h & \delta_m\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d} \\ 0 & 0 & \kappa_i^h \end{bmatrix}, \tag{A.195}$$

we have

$$
\begin{aligned}
& Var\left[\mathbf{x}_{t+h}-\mathbf{x}_t\right] \qquad \text{(A.196)} \\
= \; & 2\mathbf{V}-\mathbf{\Gamma}^h\mathbf{V}-\mathbf{V}(\mathbf{\Gamma}')^h \\
= \; & \begin{bmatrix}
2\sigma_i^2\frac{1-\rho_i^h}{1-\rho_i^2} & \delta_m\frac{\sigma_i^2\left(\rho_i\frac{2-\delta_d^h-\rho_i^h}{1-\delta_d\rho_i}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i\rho_i} & \frac{2-\kappa_i^h-\rho_i^h}{1-\kappa_i\rho_i}\sigma_i^2 \\
\delta_m\frac{\sigma_i^2\left(\rho_i\frac{2-\delta_d^h-\rho_i^h}{1-\delta_d\rho_i}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i\rho_i} & 2\sigma_i^2\frac{\delta_m^2\left(\frac{(\kappa_i-\delta_d)(1-\delta_d^h)}{\kappa_i-\delta_d}\frac{(\kappa_i\delta_d+1)}{1-\delta_d^2}-\kappa_i\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{\left(1-\kappa_i^2\right)(1-\kappa_i\delta_d)} & \delta_m\frac{\sigma_i^2\left(\kappa_i\frac{2-\kappa_i^h-\delta_d^h}{1-\kappa_i\delta_d}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i^2} \\
\frac{2-\kappa_i^h-\rho_i^h}{1-\kappa_i\rho_i}\sigma_i^2 & \delta_m\frac{\sigma_i^2\left(\kappa_i\frac{2-\kappa_i^h-\delta_d^h}{1-\kappa_i\delta_d}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i^2} & 2\sigma_i^2\frac{1-\kappa_i^h}{1-\kappa_i^2}
\end{bmatrix}
\end{aligned}
$$

Thus, we have

$$\beta_h=\alpha_i+\alpha_m\times R_{i,m}\left(h\right)+\alpha_d\times R_{i,d}\left(h\right) \qquad \text{(A.197)}$$

where $\alpha_i>0$, $\alpha_m>0$, $\alpha_d<0$, and

$$R_{i,m}\left(h\right) = \frac{Cov\left[i_{t+h}-i_t,m_{t+h}-m_t\right]}{Var\left[i_{t+h}-i_t\right]}=\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{2-\kappa_i^h-\rho_i^h}{2\left(1-\rho_i^h\right)}, \qquad \text{(A.198)}$$

$$R_{i,d}\left(h\right) = \frac{Cov\left[i_{t+h}-i_t,d_{t+h-1}-d_{t-1}\right]}{Var\left[i_{t+h}-i_t\right]}=\delta_m\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{\rho_i\frac{2-\delta_d^h-\rho_i^h}{1-\delta_d\rho_i}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}}{2\left(1-\rho_i^h\right)}. \qquad \text{(A.199)}$$

Proof that $\beta_1>\beta_2$ and that $\beta_1>\lim_{h\to\infty}\beta_h$: Since $\alpha_m>0$ and $R_{i,m}\left(h\right)$ is decreasing in h when $\kappa_i<\rho_i$, it follows that $\alpha_m\times R_{i,m}\left(h\right)$ is decreasing in h when $\kappa_i<\rho_i$.

Since $\alpha_d<0$, it then suffices to show that $R_{i,d}\left(1\right)<R_{i,d}\left(2\right)$ and that $R_{i,d}\left(1\right)<\lim_{h\to\infty}R_{i,d}\left(h\right)$. We have

$$R_{i,d}\left(1\right) = -\frac{\delta_m}{2}\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{1}{1-\delta_d\rho_i}<0. \qquad \text{(A.200)}$$

$$R_{i,d}\left(2\right) = \frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{\kappa_i^2-\delta_d^2}{\kappa_i-\delta_d}\right) \qquad \text{(A.201)}$$

$$\lim_{h\to\infty}R_{i,d}\left(h\right) = \delta_m\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{\rho_i}{1-\delta_d\rho_i}>0. \qquad \text{(A.202)}$$

We have

$$
\begin{aligned}
R_{i,d}\left(2\right)-R_{i,d}\left(1\right) &= \frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{\kappa_i^2-\delta_d^2}{\kappa_i-\delta_d}\right)+\frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\frac{1-\rho_i^2}{1-\delta_d\rho_i} \qquad \text{(A.203)} \\
&= \frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{\kappa_i^2-\delta_d^2}{\kappa_i-\delta_d}+\frac{1-\rho_i^2}{1-\delta_d\rho_i}\right) \\
&> \frac{\delta_m}{2}\frac{1}{1-\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{1-\delta_d^2}{1-\delta_d}+\frac{1-\rho_i^2}{1-\delta_d\rho_i}\right) \\
&= \frac{\delta_m}{2}\frac{\rho_i^2+2\rho_i-\delta_d}{1-\delta_d\rho_i}>0
\end{aligned}
$$

where the inequality uses the fact that

$$\frac{\partial}{\partial \kappa_i} \frac{\kappa_i^2 - \delta_d^2}{\kappa_i - \delta_d} = \frac{2\kappa_i (\kappa_i - \delta_d) - \left(\kappa_i^2 - \delta_d^2\right)}{(\kappa_i - \delta_d)^2} = 1 \tag{A.204}$$

and the fact that $\kappa_i < 1$.

D.3 Reaching for yield channel

D.3.1 General model solution

We we drop $s_t - \bar{s}$ from the state vector $\mathbf{x}_t$. For the sake of concreteness, suppose that $k = 4$. We conjecture that equilibrium yields take the form $y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$, and that the demands of active slow-moving arbitrageurs are of the form $d_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t$, where the $k+1$ dimensional state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix}. \tag{A.205}$$

These assumptions imply that the state vector follows an AR(1) process. Critically, the transition matrix Γ is a function of the parameters $\boldsymbol{\delta}_1$ governing slow-moving arbitrageur demand so we write $\Gamma = \Gamma(\boldsymbol{\delta}_1)$. Specifically, we have

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\delta)\,\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \qquad \text{(A.206)} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 \\ \delta_P & \delta_T & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

where $\Sigma \equiv Var_t[\boldsymbol{\epsilon}_{t+1}]$. Assuming for simplicity that $\varepsilon_{P,t+1}$ and $\varepsilon_{T,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_P^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.207}$$

Letting $V^{(1)} = Var_t[rx_{t+1}] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1$, denote the variance of 1-period excess returns, active demand at time t is

$$\begin{aligned} & f h_t + (1-f)\, q b_t + (1-f)(1-q)\, k^{-1} d_t \qquad \text{(A.208)} \\ &= \left[(f + (1-f)\, q)\, \tau \frac{(\alpha_0 - \bar{i})}{V^{(1)}} + (1-f)(1-q) k^{-1} \delta_0 \right] \\ &+ \left[f\tau \frac{(\boldsymbol{\alpha}_1 - \mathbf{e})'}{V^{(1)}} + (1-f)\, q\tau \frac{\frac{1}{1-\phi} \boldsymbol{\alpha}_1' (\mathbf{I} - \phi \boldsymbol{\Gamma}) - \mathbf{e}'}{V^{(1)}} + (1-f)(1-q) k^{-1} \boldsymbol{\delta}_1' \right] \mathbf{x}_t. \end{aligned}$$

Active supply is

$$\begin{aligned}
& \bar{s}-(1-f)(1-q)k^{-1}\sum\nolimits_{i=1}^{k-1} d_{t-i} && \text{(A.209)}\\
= & \left[\bar{s}-(1-f)(1-q)\frac{(k-1)}{k}\delta_0\right]+\left[-(1-f)(1-q)k^{-1}\mathbf{e}_d'\right]\mathbf{x}_t.
\end{aligned}$$

Matching constants terms, we obtain

$$\alpha_0=\bar{i}+\frac{V^{(1)}}{\tau}\frac{\bar{s}-(1-f)(1-q)\delta_0}{f+(1-f)q} \tag{A.210}$$

which is the same in the investor extrapolation model. Matching slope coefficients, we obtain:

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\frac{f(1-\phi)\mathbf{I}+(1-f)q(\mathbf{I}-\phi\boldsymbol{\Gamma}')}{f+(1-f)q}\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)k^{-1}(\mathbf{e}_d+\boldsymbol{\delta}_1)}{(f+(1-f)q)}\right] \tag{A.211}$$

In summary, an equilibrium in this model extension solves the following system of equations

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\frac{f(1-\phi)\mathbf{I}+(1-f)q\left(\mathbf{I}-\phi\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)'\right)}{f+(1-f)q}\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}(\boldsymbol{\alpha}_1)}{\tau}\frac{(1-f)(1-q)k^{-1}(\mathbf{e}_d+\boldsymbol{\delta}_1)}{(f+(1-f)q)}\right] \tag{A.212}$$

and

$$\boldsymbol{\delta}_1'=\tau\frac{\left((\boldsymbol{\alpha}_1-\mathbf{e})'(\mathbf{I}-\boldsymbol{\Gamma}(\boldsymbol{\delta}_1))^{-1}+\frac{\theta}{1-\theta}\boldsymbol{\alpha}_1'\right)}{V^{(k)}(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)}\left(\mathbf{I}-\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)^k\right), \tag{A.213}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)$ to emphasize that the transition matrix depends on $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)$ to emphasize that the k-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. Unlike in our baseline model, the first two elements of $\boldsymbol{\delta}_1$ are going to be positive, since slow-moving arbitrageurs will buy more long-term bonds when interest rates rise. Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for α_0 and δ_0 using

$$\alpha_0=\bar{i}+\frac{V^{(1)}}{\tau}\frac{\bar{s}-(1-f)(1-q)\delta_0}{f+(1-f)q}\text{ and }\delta_0=\tau\frac{k\left(\alpha_0-\bar{i}\right)}{V^{(k)}}, \tag{A.214}$$

which yields

$$\begin{aligned}
\alpha_0 &= \bar{i}+\frac{\bar{s}/\tau}{(f+(1-f)q)\frac{1}{V^{(1)}}+(1-f)(1-q)\frac{k}{V^{(k)}}}\text{ and} && \text{(A.215)}\\
\delta_0 &= \frac{\frac{k}{V^{(k)}}}{(f+(1-f)q)\frac{1}{V^{(1)}}+(1-f)(1-q)\frac{k}{V^{(k)}}}\times\bar{s}.
\end{aligned}$$

D.3.2 Proof of Proposition 4

Proposition 4. ***Investor reaching for yield model.*** *Suppose $\rho_T=\rho_P$. When $f>0$, long rates are excessively sensitive to short rates. However, this excess sensitivity is only horizon-dependent when arbitrage capital is slow moving ($q<1$).*

Proof: To demonstrate this result, it suffices to consider two special cases. We suppose throughout that $\rho_T = \rho_P \equiv \rho_i$. First, we consider the case where there is no slow-moving capital ($q = 1$). Next, we study the case where $q < 1$ and $k = 2$. The arguments given in this special $k = 2$ case generalize naturally to the case where $k > 2$.

Special case #1: $q = 1$ **and** $\rho_T = \rho_P \equiv \rho_i$ In this case, we have

$$i_{t+1} - \bar{i} = \rho_i \left(i_t - \bar{i}\right) + \varepsilon_{i,t+1}, \tag{A.216}$$

where $Var_t\left[\varepsilon_{i,t+1}\right] = \sigma_i^2$. Yields take the form $y_t = \alpha_0 + \alpha_{1,i}\left(i_t - \bar{i}\right)$. Thus, we have

$$E_t\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \left(\frac{1-\phi\rho_i}{1-\phi}\alpha_{1,i} - 1\right)\left(i_t - \bar{i}\right) \tag{A.217}$$

and

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \alpha_i^2 \sigma_i^2. \tag{A.218}$$

Market clearing condition is:

$$\begin{aligned}
\bar{s} &= fh_t + (1-f)\,b_t \qquad \text{(A.219)} \\
&= f\tau\frac{y_t - i_i}{V^{(1)}} + (1-f)\,\tau\frac{E_t\left[rx_{t+1}\right]}{V^{(1)}} \\
&= f\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\alpha_{1,i} - 1\right)\left(i_t - \bar{i}\right)}{V^{(1)}} + (1-f)\,\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\frac{1-\phi\rho_i}{1-\phi}\alpha_{1,i} - 1\right)\left(i_t - \bar{i}\right)}{V^{(1)}}.
\end{aligned}$$

Thus, we have $\alpha_0 = \bar{i} + \tau^{-1}V^{(1)}\bar{s}$ and

$$\alpha_i = \frac{1-\phi}{f(1-\phi) + (1-f)(1-\phi\rho_i)} \geq \frac{1-\phi}{1-\phi\rho_i}, \tag{A.220}$$

with strict inequality when $f > 0$. In other words, we have $\partial\alpha_i/\partial f > 0$, so there is excess sensitivity when $f > 0$. We also have

$$E_t\left[rx_{t+1}\right] = \tau^{-1}V^{(1)}\bar{s} + \frac{f\phi(1-\rho_i)}{f(1-\phi) + (1-f)(1-\phi\rho_i)}\left(i_t - \bar{i}\right).$$

Thus, we have $\partial E_t\left[rx_{t+1}\right]/\partial i_t > 0$. Thus, in this case where $q = 1$, we have

$$\beta_h = \alpha_i^* = \frac{1-\phi}{f(1-\phi) + (1-f)(1-\phi\rho_i)}, \tag{A.221}$$

which is independent of horizon h.

Special case #2: $q < 1$, $k = 2$**, and** $\rho_T = \rho_P \equiv \rho_i$

Solution: In this case, we have $i_{t+1} - \bar{i} = \rho_i \left(i_t - \bar{i}\right) + \varepsilon_{i,t+1}$ where $Var_t\left[\varepsilon_{i,t+1}\right] = \sigma_i^2$. Yields take the form

$$y_t = \alpha_0 + \alpha_i \left(i_t - \bar{i}\right) + \alpha_d \left(d_{t-1} - \delta_0\right) \tag{A.222}$$

Demands of active slow-moving investors are given by

$$d_t = \delta_0 + \delta_i \left(i_t - \bar{i}\right) + \delta_d \left(d_{t-1} - \delta_0\right). \tag{A.223}$$

The state variable dynamics are given by

$$\mathbf{x}_{t+1} = \begin{bmatrix} \rho_i & 0 \\ \delta_i & \delta_d \end{bmatrix} \begin{bmatrix} i_t - \bar{i} \\ d_{t-1} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \end{bmatrix} \tag{A.224}$$

Thus, we have

$$E_t\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \left(\frac{1-\phi\rho_i}{1-\phi}\alpha_i - \frac{\phi}{1-\phi}\alpha_d\delta_i - 1\right)\left(i_t - \bar{i}\right) + \frac{1-\phi\delta_d}{1-\phi}\alpha_d\left(d_{t-1} - \delta_0\right). \tag{A.225}$$

and

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \alpha_i^2\sigma_i^2. \tag{A.226}$$

The market clearing condition is:

$$\begin{aligned} & \bar{s} - (1-f)(1-q)\frac{1}{2}d_{t-1} \qquad \text{(A.227)} \\ = \; & f\tau\frac{y_t - i_i}{V^{(1)}} + (1-f)\,q\tau\frac{E_t\left[rx_{t+1}\right]}{V^{(1)}} + (1-f)(1-q)\frac{1}{2}d_t \\ = \; & f\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\alpha_i - 1\right)\left(i_t - \bar{i}\right) + \alpha_d\left(d_{t-1} - \delta_0\right)}{V^{(1)}} \\ & + (1-f)\,q\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\frac{1-\phi\rho_i}{1-\phi}\alpha_i - \frac{\phi}{1-\phi}\alpha_d\delta_i - 1\right)\left(i_t - \bar{i}\right) + \frac{1-\phi\delta_d}{1-\phi}\alpha_d\left(d_{t-1} - \delta_0\right)}{V^{(1)}} \\ & + (1-f)(1-q)\frac{1}{2}\left(\delta_0 + \delta_i\left(i_t - \bar{i}\right) + \delta_d\left(d_{t-1} - \delta_0\right)\right) \end{aligned}$$

Matching constant and slope terms, we obtain

$$\alpha_0 = \bar{i} + \frac{1}{\tau}\frac{V^{(1)}}{f + (1-f)\,q}\left[\bar{s} - (1-f)(1-q)\,\delta_0\right] > 0 \tag{A.228}$$

$$\alpha_i = \frac{\left[f + (1-f)\,q\right](1-\phi) + (1-f)\,q\phi\alpha_d\delta_i - (1-\phi)(1-f)(1-q)\frac{1}{2}\tau^{-1}V^{(1)}\delta_i}{f(1-\phi) + (1-f)\,q\left(1-\phi\rho_i\right)} > 0 \tag{A.229}$$

$$\alpha_d = -\tau^{-1}V^{(1)}\frac{(1-f)(1-q)\frac{1}{2}\left(1+\delta_d\right)}{f + (1-f)\,q\frac{1-\phi\delta_d}{1-\phi}} < 0. \tag{A.230}$$

Characterizing the solution: As above, we can show that

$$0 < \frac{1-\phi}{1-\phi\rho_i} < \alpha_i < 1, \tag{A.231}$$

$0 < \delta_i$, $\alpha_d < 0$, and $-1 < \delta_d < 0$. To see this, note that

$$\begin{aligned} d_t - \delta_0 &= \delta_i\left(i_t - \bar{i}\right) + \delta_d\left(d_{t-1} - \delta_0\right) \qquad \text{(A.232)} \\ &= \frac{\tau}{V^{(2)}}\left(\begin{array}{c} \left[\alpha_i\left(i_t - \bar{i}\right) + \alpha_d\left(d_{t-1} - \delta_0\right)\right] \\ + \left[\alpha_i\rho_i\left(i_t - \bar{i}\right) + \alpha_d\left[\delta_i\left(i_t - \bar{i}\right) + \delta_d\left(d_{t-1} - \delta_0\right)\right]\right] \\ -\frac{\phi}{1-\phi}\left(\begin{array}{c} \left[\alpha_i\rho_i^2\left(i_t - \bar{i}\right) + \alpha_d\left[\delta_i\left(\delta_d + \rho_i\right)\left(i_t - \bar{i}\right) + \delta_d^2\left(d_{t-1} - \delta_0\right)\right]\right] \\ -\left[\alpha_i\left(i_t - \bar{i}\right) + \alpha_d\left(d_{t-1} - \delta_0\right)\right] \\ -\left(i_t - \bar{i}\right) - \rho_i\left(i_t - \bar{i}\right) \end{array}\right) \end{array}\right) \end{aligned}$$

Thus, we have the four conditions in four unknowns:

$$\alpha_i = \frac{\left[f + (1-f)\,q\right](1-\phi) + (1-f)\,q\phi\alpha_d\delta_i - (1-\phi)(1-f)(1-q)\,\frac{1}{2}\tau^{-1}V^{(1)}\delta_i}{f(1-\phi) + (1-f)\,q\,(1-\phi\rho_i)} \quad \text{(A.233a)}$$

$$\alpha_d = -\tau^{-1}V^{(1)}\frac{(1-f)(1-q)\,\frac{1}{2}\,(1+\delta_d)}{f + (1-f)\,q\frac{1-\phi\delta_d}{1-\phi}} \quad \text{(A.233b)}$$

$$\delta_i = \frac{\tau}{V^{(2)}}(1+\rho_i)\left(\alpha_i\frac{1-\phi\rho_i}{1-\phi} - 1\right) + \frac{\tau}{V^{(2)}}\frac{\alpha_d}{1-\phi}\left((1-\phi)\,\delta_i - \phi\left(\delta_i\left(\delta_d + \rho_i\right) + \delta_d^2\right)\right) \quad \text{(A.233c)}$$

$$\delta_d = \frac{\tau}{V^{(2)}}\frac{\alpha_d}{1-\phi}\left(\delta_d + 1\right)\left(1 - \phi\delta_d\right) \quad \text{(A.233d)}$$

- Combining (A.233b) and (A.233d) we have

$$\delta_d = -\frac{1}{2}\frac{V^{(1)}}{V^{(2)}}\frac{(1-f)(1-q)}{f + (1-f)\,q\frac{1-\phi\delta_d}{1-\phi}}\frac{1-\phi\delta_d}{1-\phi}\left(1+\delta_d\right)^2. \quad \text{(A.234)}$$

  Thus, we have $-1 < \delta_d < 0$.

- Using (A.233b) and the fact that $(1+\delta_d) > 0$, we then have $\alpha_d < 0$.

- From (A.233a), we have

$$0 = f\frac{\tau}{V^{(1)}}(\alpha_i - 1) + (1-f)\,q\frac{\tau}{V^{(1)}}\left(\frac{1-\phi\rho_i}{1-\phi}\alpha_i - 1\right) + (1-f)\left((1-q)\,\frac{1}{2} - q\frac{\tau}{V^{(1)}}\frac{\phi}{1-\phi}\alpha_d\right)\delta_i. \quad \text{(A.235)}$$

  Thus, in the stable equilibrium, we have $\delta_i > 0$ and $\frac{1-\phi}{1-\phi\rho_i} < \alpha_i < 1$. Thus, an increase in short-term rates leads (i) yield-seeking investors to sell bonds and leads both fast- and slow-moving expected-return oriented investors to buy bonds.

Computing β_h: Using the fact that $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma}\otimes\mathbf{\Gamma})^{-1}vec(\mathbf{\Sigma})$, we have

$$\mathbf{V} = \begin{bmatrix} \sigma_i^2\frac{1}{1-\rho_i^2} & \sigma_i^2\frac{\rho_i\delta_i}{(1-\rho_i^2)(1-\delta_d\rho_i)} \\ \sigma_i^2\frac{\rho_i\delta_i}{(1-\rho_i^2)(1-\delta_d\rho_i)} & \sigma_i^2\frac{\delta_i^2(1+\delta_d\rho_i)}{(1-\delta_d^2)(1-\rho_i^2)(1-\delta_d\rho_i)} \end{bmatrix} \quad \text{(A.236)}$$

Using the fact that

$$\mathbf{\Gamma}^h = \begin{bmatrix} \rho_i^h & 0 \\ \delta_i \frac{\rho_i^h - \delta_d^h}{\rho_i - \delta_d} & \delta_d^h \end{bmatrix} \tag{A.237}$$

we have

$$\begin{aligned} & Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] \quad \text{(A.238)} \\ = \; & 2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h \\ = \; & \begin{bmatrix} 2Var\left[i_t\right]\left(1-\rho_i^h\right) & \left(2-\rho_i^h-\delta_d^h\right)Cov\left[i_t, d_{t-1}\right] - \delta_i \frac{\rho_i^h - \delta_d^h}{\rho_i - \delta_d} Var\left[i_t\right] \\ \left(2-\rho_i^h-\delta_d^h\right)Cov\left[i_t, d_{t-1}\right] - \delta_i \frac{\rho_i^h - \delta_d^h}{\rho_i - \delta_d} Var\left[i_t\right] & 2Var\left[d_{t-1}\right]\left(1-\delta_d^h\right) - 2\delta_i \frac{\rho_i^h - \delta_d^h}{\rho_i - \delta_d} Cov\left[i_t, d_{t-1}\right] \end{bmatrix} \end{aligned}$$

Thus, we have

$$\beta_h = \frac{Cov\left[i_{t+h} - i_t, y_{t+h} - y_t\right]}{Var\left[i_{t+h} - i_t\right]} = \alpha_i + \alpha_d \times R_{i,d}\left(h\right) \tag{A.239}$$

where $\alpha_i > 0$, $\alpha_d < 0$, and

$$\begin{aligned} R_{i,d}\left(h\right) &= \frac{Cov\left[i_{t+h} - i_t, d_{t+h-1} - d_{t-1}\right]}{Var\left[i_{t+h} - i_t\right]} \quad \text{(A.240)} \\ &= \frac{\left(2-\rho_i^h-\delta_d^h\right)\sigma_i^2 \frac{\rho_i\delta_i}{(1-\rho_i^2)(1-\delta_d\rho_i)} - \delta_i \frac{\rho_i^h-\delta_d^h}{\rho_i-\delta_d}\sigma_i^2 \frac{1}{1-\rho_i^2}}{2\frac{1-\rho_i^h}{1-\rho_i^2}\sigma_i^2} \\ &= \frac{\rho_i\delta_i}{1-\delta_d\rho_i} - \left[\frac{1}{2}\delta_i \frac{1-\rho_i^2}{\rho_i + \delta_d^2\rho_i - \delta_d\rho_i^2 - \delta_d}\right] \frac{\rho_i^h - \delta_d^h}{1-\rho_i^h} \end{aligned}$$

Proof that $\beta_2 < \beta_1$ and that $\lim_{h\to\infty}\beta_h < \beta_1$. Since $\alpha_d < 0$, it suffices to show that $R_{i,d}\left(2\right) > R_{i,d}\left(1\right)$ and that $\lim_{h\to\infty} R_{i,d}\left(h\right) > R_{i,d}\left(1\right)$. These results follow from the facts that

$$R_{i,d}\left(1\right) = -\frac{1}{2}\delta_i \frac{1-\rho_i}{1-\delta_d\rho_i} < 0, \tag{A.241}$$

$$R_{i,d}\left(2\right) = \frac{1}{2}\delta_i \frac{\rho_i - \delta_d}{1-\delta_d\rho_i} > 0, \tag{A.242}$$

$$\lim_{h\to\infty} R_{i,d}\left(h\right) = \frac{\rho_i\delta_i}{1-\delta_d\rho_i} > 0. \tag{A.243}$$

References

Adrian, T., R. K. Crump, and E. Moench (2013): "Pricing the term structure with linear regressions," *Journal of Financial Economics*, 110, 110–138.

Backus, D. and J. H. Wright (2007): "Cracking the conundrum," *Brookings Papers on Economic Activity*, 1, 293–316.

Becker, B. and V. Ivashina (2015): "Reaching for yield in the bond market," *Journal of Finance*, 70, 1863–1902.

Berger, D. W., K. Milbradt, F. Tourre, and J. Vavra (2018): "Mortgage prepayment and path dependent effects of monetary policy," NBER Working Paper 25157.

Bordalo, P., N. Gennaioli, and A. Shleifer (2017): "Diagnostic expectations and credit cycles," *Journal of Finance*, 73, 199–227.

Brooks, J., M. Katz, and H. Lustig (2019): "Post-FOMC announcement drift in U.S. bond markets," Working Paper.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay (1996): *The Econometrics of Financial Markets*, Princeton, New Jersey: Princeton University Press.

Campbell, J. Y. and R. J. Shiller (1988): "Stock prices, earnings, and expected dividends," *Journal of Finance*, 43, 661–676.

——— (1991): "Yield spreads and interest rate movements: A bird's eye view," *The Review of Economic Studies*, 58, 495–514.

Cieslak, A. (2018): "Short-rate expectations and unexpected returns in Treasury bonds," *Review of Financial Studies*, 31, 3265–3306.

Cochrane, J. H. and M. Piazzesi (2005): "Bond risk premia," *American Economic Review*, 95, 138–160.

D'Arienzo, D. (2020): "Maturity increasing over-reaction and bond market puzzles," Working paper, Bocconi University.

Drechsler, I., A. Savov, and P. Schnabl (2018): "A model of monetary policy and risk premia," *Journal of Finance*, 73, 317–373.

Duffee, G. (2002): "Term premia and interest rate forecasts in affine models," *Journal of Finance*, 57, 405–443.

——— (2011): "Information in (and not in) the term structure," *Review of Financial Studies*, 24, 2895–2934.

Duffie, D. (2010): "Asset price dynamics with slow-moving capital," *Journal of Finance*, 65, 1238–1268.

Duffie, D. and R. Kan (1996): "Yield factor models of interest rates," *Mathematical Finance*, 64, 379–406.

Eichenbaum, M., S. Rebelo, and A. Wong (2018): "State dependent effects of monetary policy: The Refinancing Channel," NBER Working Paper 25152.

Fama, E. and R. R. Bliss (1987): "The information on long-maturity forward rates," *American Economic Review*, 77, 680–692.

Giglio, S. and B. Kelly (2018): "Excess volatility: Beyond discount rates," *Quarterly Journal of Economics*, 133, 71–127.

Greenspan, A. (2005): "Federal Reserve Board's semiannual Monetary Policy Report to the Congress," Testimony on February 16, 2005 before the Senate Committee on Banking, Housing, and Urban Affairs.

Greenwood, R., S. G. Hanson, and G. Y. Liao (2018): "Asset price dynamics in partially segmented markets," *Review of Financial Studies*, 31, 3307–3343.

Greenwood, R. and D. Vayanos (2014): "Bond supply and excess bond returns," *Review of Financial Studies*, 27, 663–713.

Gürkaynak, R. S., B. Sack, and E. T. Swanson (2005): "The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models," *American Economic Review*, 95, 425–436.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007): "The U.S. Treasury yield curve: 1961 to the present," *Journal of Monetary Economics*, 54, 2291–2304.

Hansen, L. P. and R. J. Hodrick (1980): "Forward exchange rates as optimal predictors of future spot rates: An econometric analysis," *Journal of Political Economy*, 88, 829–853.

Hanson, S. G. (2014): "Mortgage convexity," *Journal of Financial Economics*, 113, 270–299.

Hanson, S. G. and J. C. Stein (2015): "Monetary policy and long-term real rates," *Journal of Financial Economics*, 115, 429–448.

Joslin, S., A. Le, and K. J. Singleton (2013): "Gaussian macro-finance term structure models with lags," *Journal of Financial Econometrics*, 11, 589–609.

Joslin, S., M. Preibsch, and K. J. Singleton (2014): "Risk premiums in dynamic term structure models with unspanned macro risks," *Journal of Finance*, 69, 1197–1233.

Kiefer, N. M. and T. J. Vogelsang (2005): "A new asymptotic theory for heteroskedasticity-autocorrelation robust tests," *Econometric Theory*, 21, 1130–1164.

Lu, L., M. Pritsker, A. Zlate, K. Anadu, and J. Bohn (2019): "Reach for yield by U.S. public pension funds," .

Maddaloni, A. and J.-L. Peydró (2011): "Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the euro-area and the US lending standards," *Review of Financial Studies*, 24, 2121–2165.

Malkhozov, A., P. Mueller, A. Vedolin, and G. Venter (2016): "Mortgage risk and the yield curve," *Review of Financial Studies*, 29, 1220–1253.

Maxted, P. (2020): "A macro-finance model with sentiment," .

Newey, W. K. and K. D. West (1987): "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix," *Econometrica*, 55, 703–708.

Piazzesi, M., J. Salomao, and M. Schneider (2015): "Trend and cycle in bond premia," Working Paper, Stanford University.

Vayanos, D. and J.-L. Vila (2020): "A preferred-habitat model of the term structure of interest rates," *Econometrica*, Forthcoming.