Family Law Effects on Divorce, Fertility and Child Investment\textsuperscript{1}

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July 2011

\textsuperscript{1}This research was supported in part by grants from the National Science Foundation, Collegio Carlo Alberto and the C.V. Starr Center for Applied Economics at NYU. We are grateful to the CHRR for access to NLSY 1979 cohort Geocode data, and to Hugette Sun for sharing her extensive child support guideline library. Steven Laufer contributed indispensible components of the programming framework, and James Mabli provided excellent research assistance. David Blau, Hanming Fang, Ariel Pakes, Ken Wolpin and seminar participants at the Institute for Research on Poverty, NYU, Virginia, UNC-Greensboro, Wisconsin, Torino, the Society for Economic Dynamics, ESPE, the American Economic Association Meetings, the UNC/Duke Conference on Labor, Health and Aging, the Minnesota Applied Micro Workshop and the Stanford Institute for Theoretical Economics provided valuable comments. The views and opinions offered in this paper do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.
Abstract

In order to assess the child welfare impact of policies governing divorced parenting, such as child support orders, child custody and placement regulations, and marital dissolution standards, one must consider their influence not only on the divorce rate but also on spouses’ fertility choices and child investments. We develop a continuous time model of marriage, fertility and parenting, with the main goal being the determination of how policies toward divorce influence outcomes for husbands, wives and children. Estimates are derived for model parameters of interest using the method of simulated moments, and simulations based on the model explore the effects of changes in custody allocations and child support standards on outcomes for intact and divided families. Simulations indicate that, while a small decrease in the divorce rate may be induced by a significant child support hike, the major effect of child support levels for both intact and divided households is on the distribution of welfare between parents. Simulated divorce, fertility, test scores and parental welfare all increase with a move toward shared physical placement. Finally, the simulations indicate that children’s interests are not necessarily best served by minimizing divorced parenting.

JEL codes: J12, J13, J18
1 Introduction

Divorced parenting in the U.S. is regulated through a combination of laws controlling marital dissolution, child custody and placement, and the assignment and enforcement of child support obligations. The primary objective of these activities is to increase the well-being of children and parents, and the divorce rate is often regarded as a first order measure of the success of family law. The rationale for this focus is the preponderance of empirical evidence that suggests that children living in households without both biological parents are more likely to suffer from behavioral problems and have lower levels of a broad range of achievement indicators measured at various points over the life cycle (see, e.g., Haveman and Wolfe 1995). Recent empirical studies of unilateral divorce laws and child support enforcement have isolated the effects of changes in such legal structures on divorce rates (e.g., Friedberg 1998 and Gruber 2004, Wolfers 2006 and Nixon 1997). A complete picture of the influence of family law on family members' welfare would include an understanding of the mechanisms by which family law changes influence fertility, child outcomes, and the distribution of resources within the family, in addition to divorce rates. Toward that end, this paper models the interaction of married couples in the shadow of existing divorce regulations in terms of decisions regarding fertility, child investment and divorce.

A standing problem for research on parents' dynamic child investment activities is the frequent absence of data on fathers. Much of what we have learned about parents' dynamic decision-making, therefore, has been in the context of a mother's (or mother and father's, assuming a unitary objective) individual dynamic optimization problem, as in Bernal (2008), Bernal and Keane (2011), Blau and van der Klaauw (2008) and Liu, Mroz and van der Klaauw (2010).

Where the subject is the influence of divorce regulations on the family, however, the distinct choices of mothers and fathers are paramount. It is virtually impossible to understand the influence of potential child support, for example, on fertility, investment and divorce decisions by studying the mother's perspective in isolation. Hence we model the choices of mothers and fathers as an ongoing, simultaneous-move game. Our model and data begin from the date of marriage, which, while excluding a substantial and non-random segment of parents, has the benefit of granting access to similar information on the mother and father when early fertility, investment and divorce choices are being made.

In taking this approach, we draw on an extensive empirical literature on marriage dynamics, including Aiyagari, Greenwood and Guner (2000), Brien, Lillard and Stern (2006), Chiappori, Fortin and Lacroix (2002), and others. This literature emphasizes the repeated interaction of a husband and wife in deciding whether to continue a marriage and the allocation of household resources. What we seek to add to existing analyses is the impact of the (endogenous) arrival and development of a shared child and its role, as well as the role of exposure to divorced parenting.

\footnote{Caucutt, Guner and Knowles (2002) is a rare example of an existing study of marriage dynamics (including a marriage market), fertility and child expenditures. However, their framework is a three period overlapping generations model, and their object of interest is the life-cycle timing of fertility, where we model spouses' decisions in continuous time throughout the fertility and childrearing process, and our ultimate interest is in child outcomes.}
regulation, in marital status dynamics.

A closely related paper is Tartari (2007). It includes a substantially enriched child quality production technology relative to the one that we estimate, and it focuses less on capturing the institutional structure and effects of family law.

The model allows spouses to make (simultaneous) choices regarding marriage continuation, fertility, and, where relevant, individual investments in children, in a continuous time framework. A match value of marriage is drawn from a population distribution and evolves stochastically over time. Fertility choices are influenced by both the expected benefit from the presence of the child and expectations regarding the duration of the marriage, given the state of the marriage quality process. Child quality, reflected by cognitive ability in the empirical implementation of the model, progresses as a result of both endogenous parental investment and marital status choices and exogenous productivity factors. Marital dissolution may result from changes in marriage match quality, child presence and quality, and when the child reaches “independence” (in the sense of the model, which is explained below). Thus the full history of marriage values and child investments determines current marital status and child investment levels. If the history of child investments and marriage values is poorer for the marginal marriage than it is for the representative marriage, then, all else equal, the child welfare gain associated with the continuation of the marginal marriage is smaller than that associated with the continuation of the representative marriage. An important objective of our analysis is to study the welfare impacts of variations in family law, which are possible to assess under our assumptions regarding the determination of the utility levels of husbands, wives, and (potential) children.

The model is estimated utilizing data from the National Longitudinal Study of Youth’s 1979 cohort (NLSY-79) using the method of simulated moments (MSM). Computational issues arise, and we employ a technique similar to the one independently developed by Imai, Jain and Ching (2009) to permit simultaneous solution and estimation of this complex dynamic game. Variation across state and intertemporal variation in marital dissolution standards and child support guidelines aids the identification of model parameters, and allows us to rely on more than model structure when estimating parameters used in our comparative statics exercises and policy analysis. While prior research has made extensive use of variation in laws regulating the granting of divorce across states, the use of states’ widely varying child support guidelines to generate informative variation in child support orders is a relatively new practice.2

The model we estimate is extremely parsimonious (which is a positive way to say ‘stylized’). Nevertheless, we find that the model is able to fit most of the many features of the data used in estimation to a very satisfactory degree, with a few notable exceptions. That gives us some confidence in using the estimated model to perform comparative statics exercises and welfare analysis.

An important feature of our model is the incorporation of a fertility decision. In the comparative statics exercises we find that family law potentially has an important impact not only the

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2Existing analysis of individual exposure to child support liability using the guidelines is, to our knowledge, limited to Sun (2005). We are very grateful to Huette Sun for sharing her child support guideline library and for her advice on working with the guideline data.
achievement levels of children from intact and nonintact households, but even more fundamentally on the number of children born and the characteristics of the households having them. The effects of variations in child contact time allocations in the divorce state have a particularly strong impact on fertility, with 20 percent fewer households having children within 10 years from the date of the marriage when the father is given no time as opposed to when he is allocated 50 percent of the time. These variations also impact the (final) distribution of child quality not only through their impact on the incentive to invest in children but also due to the fact that only parents expecting high quality children and a stable marriage will have children in such an “extreme” family law environment. Conversely, we find that the effect of wide variation in child support orders has a modest impact on fertility decisions and child quality. This seems largely due to the diminishing marginal utility of consumption and the perfect substitutability of parental investments in the stochastic child quality production function.

Our analysis concludes with an attempt to determine optimal family law parameters using a Benthamite social welfare function. The main problem with employing such an approach in our modeling framework is that the set of agents is endogenous due to the presence of the fertility decision. We are able to make some progress by merging the seminal approaches to this question found in Blackorby et al. (1995) and Golosov et al. (2007). Our simulation exercise adopts an ex ante welfare criterion (evaluated at the time of marriage) which only involves the agents always present, the husband and wife. We find that the welfare objective is optimized under 50/50 physical placement, a 20 percent child support rate and a bilateral divorce standard. Custody arrangements generally dominate the welfare ordering, with the divorce standard being of minimal net welfare consequence.

The plan of the paper is as follows. In Section 2 we describe some of the “marriage life-cycle” patterns in the data with which we hope our model is consistent. In Section 3 we develop the details of the model. Section 4 describes our rather complex estimation method and presents a fairly rigorous discussion of the manner in which primitive parameters are identified. In Section 5 we describe the data in detail and present descriptive statistics for our sample. The estimates of the primitive parameters and assessment of model fit are found in Section 6. In Section 7 we describe comparative statics results and our attempt to determine optimal family law. Section 8 concludes.

2 Divorce and Fertility Dynamics in the NLSY-79

The difficulty of isolating policy effects on marriage, fertility and child investment is perhaps best illustrated in data on families’ dynamic choices. We turn to our estimation sample of families, described in more detail in Section 5. In broad terms, the sample includes all of the women of the NLSY-79 cohort who ever marry and who have no children at the date of first marriage. To this we add the requirement that we observe complete marriage and fertility histories for the women, some income data, and a few other variables required to carry out the estimation of the model. Figure
1 describes the rate at which first children arrive to the NLSY-79 women in this sample. On the horizontal axis is the number of years since the date of marriage; on the vertical is the proportion of women who experience a birth at the given number of years since the marriage date. We look at fertility trajectories from the marriage date separately by the number of years left in the marriage. The darkest, dashed line represents fertility rates among couples who will remain married 10 years from the date in question. With the exception of the first year of marriage, their fertility rate lies everywhere above the fertility rates of couples who are approaching divorce. The irregularly dashed line represents couples who will be divorced by the next year. Their fertility rate is the lowest of all of the groups in most years. The solid and dotted lines represent couples who will divorce in two and three years, respectively. Their fertility rates lie below the married-in-10-years rates, and, for the most part, above the fertility rates of those who will divorce in one year. Overall we see a strong negative association between remaining time in the marriage and the fertility rate. If these briefer marriages are fundamentally of lower quality, or marginal, marriages, then it would appear that marginal marriages produce fewer children.

The children of stable marriages in the sample demonstrate substantially higher cognitive ability than the children of marriages headed for divorce. Figure 2 depicts children's average age-normed math score on the Peabody Individual Achievement Test (PIAT) as a function of years since their parents were married. Test score averages for children whose parents will still be married in three years lie above those for children of divorce and those for children whose parents will be divorced in three years (with the single exception of the sixth year of marriage), and the rate of increase in their average test scores outpaces those of the other groups. One interesting fact that emerges from this exercise is that the average test scores for children of marriages approaching divorce are everywhere the same as or worse than the scores for children of divorce. It appears that, in this NLSY-79 sample, weak marriages are associated not just with child attainments that resemble those in divorce more than they resemble child attainment in stable marriages, but instead with child attainments that are frequently worse than those associated with divorce.

Of course, this in no way implies that low marriage quality causes either low fertility or low child attainment. Low fertility, for example, could cause an otherwise stable marriage to end. Figure 3 depicts divorce rates beginning from the date of marriage for those with and without children. For the first seven years of marriage, the divorce rate for couples without children lies far above the divorce rate for couples with children. At around 8 years, the divorce rates for both groups stabilize at roughly 2 percent per year, and the two groups' divorce rates remain comparable thereafter. Hence we see a strong negative association between fertility and divorce, and the direction of causation is far from clear. One hypothesis is that some type of fundamental heterogeneity in marriage quality influences both marital status and fertility decisions, leading to

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3 Note that these are rolling categorizations, so that the group of couples who will divorce in two years as of the second year of marriage is the same group as those who will divorce in one year as of the third year of marriage, and so on.

4 The positive association between the presence of children and marital stability at the early stages of marriage and the eventual diminution of the relationship is consistent with the findings of Waite and Lillard (1991).
this strong negative association between fertility and divorce. Another is that marital status and fertility are simultaneously determined, in that the return to producing and investing in children as marriage-specific capital increases with the duration of marriage, and at the same time the presence of a child increases the return to marriage, contributing to its stability.\(^5\)

In such an environment, the influence of policies that alter the costs and benefits of divorce for the two spouses depends heavily on the relative importance of basic heterogeneity in marriage quality and the interdependence of fertility and marital stability in families’ decisions. For example, a marriage that is marginal in some fundamental sense may be held together by a policy adjustment, and still this marginal marriage may produce less in the way of children or child investment than the average marriage might. Conversely, if the heterogeneity in underlying marriage quality is not large and if fertility and divorce decisions are responsive to economic and policy incentives, then policy changes may have extensive welfare-improving effects on fertility and divorce rates. A model that permits both fundamental heterogeneity in marriage quality and feedback among marriage stability, fertility and child investment is therefore required in order to understand the influence of divorce policy on the well-being of wives, husbands and children.

### 3 A Model of Child Investment and Divorce Decisions

There exist two decision-making agents in our model, spouses \(s = 1, 2\). The model is set in continuous time, and the instantaneous utility function of spouse \(s\) is given by

\[
u_s(c_s, k, \theta, d) = \alpha_s \ln(c_s) + (1 - \alpha_s)\tau_s(d)p(\ln(k) + \zeta) + (1 - d)\theta, \quad s = 1, 2; \tag{1}\]

where \(c_s\) is the consumption of a private good by spouse \(s\), \(d\) is an indicator variable that takes the value 1 if the spouses are divorced, \(\theta\) is a marriage-specific match value, the value of which can change over time, \(p\) is an indicator taking the value 1 if a child is present, \(k\) is the value of child quality, which is weakly greater than 1 when the spouses have had a child, \(\tau_s(d)\) is the amount of contact that the parent has with the child given the divorce status of the parents, \(\alpha_s \in (0, 1)\) is the preference weight on private consumption and \(\zeta\) is a constant welfare cost or benefit of child presence.\(^6\) We assume throughout that the price of private consumption is fixed at 1 for both parents.

In the absence of a child, if the spouses are married their instantaneous utilities are given by

\[
u_s(c_s, 1, \theta, 0) = \alpha_s \ln(c_s) + \theta, \quad s = 1, 2;\]

\(^5\)See Lillard and Waite (1993) for a statistical model of the simultaneity of divorce and fertility choices.

\(^6\)The parameter \(\zeta\) will be free to take positive or negative values. It may be interpreted as a welfare cost or benefit of child presence or, equivalently in this specification, as a scaling factor relating the value of child quality to the value of consumption.
and if divorced their utilities are given by

\[ u_s(c_s, 1, \theta, d) = \alpha_s \ln(c_s), \ s = 1, 2. \]

In the presence of a child, the utility derived from current child quality by each parent is modified according to the amount of contact the parent has with the child in each marriage state. We assume that when married the parents enjoy complete and concurrent access to the child’s time; without loss of generality, \( \tau_1(0) = \tau_2(0) = 1 \). Though their intrinsic valuation of the child remains the same in the divorce state, the fact that the child becomes an “excludable” good after divorce reduces the utility flows that parents receive from any given level of child quality.\(^7\) We assume that parents share time with the child in divorce, implying \( \tau_1(1) + \tau_2(1) = 1 \) and \( \tau_s(1) \geq 0, s = 1, 2, \) and that physical custody and visitation allocations are fully anticipated and set exogenously with respect to parental behaviors.\(^8\)

Each spouse has a baseline income flow at any moment in time, which is denoted by \( y_s \). The actual income under the control of individual \( s \) is state-dependent in the following sense. Spouses receive incomes of \( Y_s(y_1, y_2, a), s = 1, 2 \) and where \( a = d \times p \). When \( a = 0 \), each spouse has his or her own income, so that \( Y_s(y_1, y_2, 0) = y_s \), while when \( a = 1 \) the income of ex-spouse 1 is \( Y_1(y_1, y_2, 1) = (1 - \pi)y_1 \) and the income of ex-spouse 2 is \( Y_2(y_1, y_2, 1) = y_2 + \pi y_1 \). We assume that spouse 1 bears the child support obligation.\(^9,10\)

The dynamics of the model are as follows.

1. The model begins at the time of marriage. Spouses are initially childless. If both spouses agree to attempt to have a child, then a child arrives at rate \( \psi > 0 \). We define the state variable \( f \in \{0, 1\} \) to indicate whether this fertility process is active, with \( f = 1 \) representing an active fertility process. The fertility process may only be active in the married state.

2. There are \( M \) possible values of marriage quality, with \( \theta \in \Theta = \{\theta_1, ..., \theta_M\} \), where \( \theta_1 < \theta_2 < \ldots < \theta_M \). At the onset of marriage, there is an initial marriage quality draw. During the marriage, there may occur changes to the marriage quality value, which we model as a (continuous time) random walk. Match quality increases arrive at rate \( \tilde{\gamma}^+ \), as long as current match quality \( \theta_m \) is less than \( \theta_M \). The arrival of a match quality increase leads with certainty to a new match quality of \( \theta_{m+1} \). Symmetrically, match quality decreases arrive at rate \( \tilde{\gamma}^- \), as long as \( \theta_m > \theta_1 \), and the arrival of a decrease in match quality leads to a drop from \( \theta_m \) to

\(^7\)This, in fact, is the only way in which our model reflects losses of economies of scale after divorce. Other quasi-fixed costs, such as housing, utilities, etc., which are significant sources of scale economies when three individuals live under one roof, are not included in the this modeling set up.

\(^8\)See, for example, Fox and Kelly (1995) for details on custody determination.

\(^9\)The empirical analysis assigns spouse 1 as the male and spouse 2 as the female in the observed marriage. We find that a small minority of child support payments flow from mothers to fathers in the NLSY-79.

\(^10\)By assuming that there is no transfer ordered after a divorce if the couple is childless, we are essentially assuming away alimony. Alimony is increasingly uncommon in U.S. divorce cases. According to Case et al. (2003), for example, 5 (4.2) percent of 1977 PSID (in 1997) mothers received alimony.
\( \theta_{m-1} \). For convenience of notation we define

\[
\gamma^+(\theta_m) = \begin{cases} 
\tilde{\gamma}^+ & \text{where } 1 \leq m < M \\
0 & \text{otherwise}
\end{cases}
\quad \text{and} \quad
\gamma^- (\theta_m) = \begin{cases} 
\tilde{\gamma}^- & \text{where } 1 < m \leq M \\
0 & \text{otherwise}
\end{cases}
\]

The values of \( \gamma^+(\theta_m) \) and \( \gamma^- (\theta_m) \) determine the degree of persistence in marriage quality over any given time interval.

3. There are \( B \) possible baseline income flow levels for each spouse, with \( y_s \in \Theta^y_s = \{y^1_s, \ldots, y^B_s\} \), where \( 0 < y^1_s < \ldots < y^B_s \). Each spouse begins marriage with an own-income flow of \( y_s \). Over time, there may occur shocks to each spouse’s income state. Spouse \( s \) receives positive income shocks at rate \( \tilde{\xi}^+_s \) as long as current income \( y^b_s \) is less than \( y^B_s \), and receives negative shocks at rate \( \tilde{\xi}^-_s \) as long as \( y^b_s > y^1_s \). Analogously to the case of marriage quality, a negative income shock leads to a decrease from \( y^b_s \) to \( y^{b-1}_s \) and a positive income shock leads to an increase from \( y^b_s \) to \( y^{b+1}_s \). We define

\[
\xi^+_s(y^b_s) = \begin{cases} 
\tilde{\xi}^+_s & \text{where } 1 \leq b < B \\
0 & \text{otherwise}
\end{cases}
\quad \text{and} \quad
\xi^-_s(y^b_s) = \begin{cases} 
\tilde{\xi}^-_s & \text{where } 1 < b \leq B \\
0 & \text{otherwise}
\end{cases}
\]

4. There are \( T \) possible values of child quality, with \( k \in \Theta^k = \{k_1, \ldots, k_T\} \), where \( 1 < k_1 < \ldots < k_T \). Current child quality \( k_t \) will be interpreted in the analysis that follows as a measure of the child’s achievements relative to her or his age cohort. The empirical analog to \( k_t \) that we consider is an age-normed measure of academic performance. When born, the child has an initial child quality draw of \( k_0 \). Costly investments in child quality made by the parents increase the rate at which improvements in child quality arrive. The child quality improvement rate is described by the function

\[
\delta(k_t, i_1, i_2, \theta) = \begin{cases} 
\tilde{\delta}(i_1, i_2, \theta) & \text{where } 1 \leq t < T \\
0 & \text{otherwise}
\end{cases}
\]

Here \( i_s \) denotes the child quality investment of spouse \( s \). The presence of marriage quality in the child quality production function is meant to capture the impact of the home environment on the effectiveness of a given level of parental investments. Child quality improvements may only arrive when \( k_t < k_T \); an arriving improvement increases child quality from \( k_t \) to \( k_{t+1} \).

We assume that divorced and married parents share the same child quality production function, and that when in the divorce state marriage quality is equal to 0 in terms of its “productive” value. Since the mean of the (symmetric) marriage quality distribution is normalized to zero, this implies that parents in intact marriages with marriage quality less than 0 are at a productive disadvantage with respect to when they are divorced (for fixed values of \( i_1 \) and \( i_2 \)), while those with positive match quality values are in a comparatively advantageous position.
5. Child quality setbacks occur at exogenous rate $\bar{\sigma}$, and lead to a decline in child quality from $k_t$ to $k_{t-1}$ whenever $1 < t \leq T$. We define

$$
\sigma(k_t) = \begin{cases} 
\bar{\sigma} & \text{where } 1 < t \leq T \\
0 & \text{where } t = 1.
\end{cases}
$$

6. Finally, the child may attain functional independence at the current age-normed child quality, in which case the child quality improvement process ends. The parents enjoy a terminal value that increases with the current child quality level and continues to depend on the parents’ marital status.\(^{11}\) Termination of the investment process occurs at exogenous rate $\eta$; state variable $e \in \{0, 1\}$ indicates the current investment condition, and equals 1 when the investment process has been terminated.

The child quality production function described by dynamic elements 4-6 is of necessity peculiar to our continuous-time, simultaneous investment modelling approach. However, it can be related to leading models of child investment. Cunha and Heckman (2007) and Cunha, Heckman, Lochner, and Masterov (2006) argue that a variety of skills that children must develop are subject to "critical periods" early in life, and hence much of intellectual development is accomplished by the time the child reaches school age. Hopkins and Bracht (1975), for example, demonstrate that IQ is stable by the age of 10 or so, suggesting that the critical period for intellectual development occurs by this time. Further, Cunha and Heckman, Cunha et al., and Cunha, Heckman and Schennach (2010) emphasize the importance of both cognitive and non-cognitive skill acquisition to child outcomes, along with the importance of "dynamic complementarity" and "self-productivity" of skill levels in ongoing skill production. Todd and Wolpin (2003, 2007) consider cognitive skill formation, and argue from a different perspective for the importance of both current and lagged inputs to the ongoing production process. They demonstrate the importance of allowing for unobserved endowment effects and the endogeneity of inputs to child skill production.

Like Todd and Wolpin, we restrict attention to cognitive skill.\(^{12}\) Our empirical work defines $k_t$ based on an age-normed measure of child attainment. Hence, the manner in which we allow for self-productivity and the role of lagged investments is very particular, in that prior investments and attainment determine the child’s current place among a population of children, each of whom has a history of investments and attainment that may contribute to his subsequent progress. Growth in the child’s outcome measure in this instance will depend on the relative productivity of own and peers’ current and lagged investments, and past attainments, allowing for the possibility, in a somewhat circumscribed sense, of nonlinearities in the dynamic production of absolute skill levels. The initial conditions that we specify when estimating the model directly address the need\(^{11}\) An alternative approach to finalizing the child investment process would be to impose a fixed time horizon of 18 or 21 years, after which children achieve independence. The drawback to this approach is that it generates strategic manipulations by parents approaching the date of independence that we find unrealistic.\(^{12}\) Our empirical measure, discussed below in Section 5, is in fact more narrow than theirs in the space of cognitive skills.
to account for unobserved endowment heterogeneity, and the model accounts for endogeneity of investments in determining absolute skill level in a specific manner. Finally, the investment period that we model begins at birth. Our empirical implementation focuses on progress from birth through a set of tests that are completed for most sample children before the age of ten, befitting an analysis of cognitive skill production under the prescriptions of the literature.

In modeling the behavior of married and divorced parents an important specification choice is the manner in which spouses interact. One may assume that spouses interact cooperatively or noncooperatively.\textsuperscript{13} It is unclear that ex-spouses are able to interact in a manner that achieves the Pareto frontier. In a model that moves through married and divorced states, if cooperation is ever attained in marriage it is unclear how spouses’ mode of interaction might transition from such cooperation in marriage to the potential cooperation failures of divorce, or how the presence of children might influence interactions in divorce. One might assume cooperation throughout, though this is certainly unsatisfying for childless divorced spouses and, moreover, rules out any effect of marital dissolution standards on divorce rates under conventional specifications. One might assume noncooperative interaction throughout, though this may be unsatisfying for the case of young spouses starting a family. More complex approaches include allowing spouses to choose the current mode of interaction as events progress, following Flinn (2000), or specifying population heterogeneity in spouses’ mode of interaction, following Eckstein and Lifshitz (2009). Though the latter approaches are appealing, they would add a great deal of complexity to an already complex model. For the above reasons, and given standing evidence of marriage dissolution standard effects on divorce rates in Friedberg, Gruber and elsewhere, we choose to assume noncooperative interaction throughout. In our discussion of the theoretical results we dedicate some attention to the effects of this modeling choice. Finally, we assume that spouses’ investment strategies constitute a Markov Perfect Equilibrium.\textsuperscript{14}

### 3.1 Divorced Parents

Given the absence of a remarriage market, divorce is an absorbing state. An ex-spouse $s$ who has a child of quality $k_t$ at the termination of the investment process enjoys terminal value

$$V_s(y_1^b, y_2^b, k_t, p = 1, d = 1, e = 1) = (\rho + \xi(y_1^b, y_2^b))^{-1} \{\alpha_s \ln(Y_s(y_1^b, y_2^b, 1)) + (1 - \alpha_s)\tau_s(1)(\ln(k_t) + \zeta) + V_s^*(y_1^b, y_2^b, k_t, 0, p = 1, d = 1, e = 1)\},$$

where $\rho$ is the instantaneous discount rate and $\xi(y_1^b, y_2^b) = \xi_1^+(y_1^b) + \xi_1^-(y_1^b) + \xi_2^+(y_2^b) + \xi_2^-(y_2^b)$ represents the total rate of income change arrivals. Further, define $V_s^*(y_1^b, y_2^b, k_t, \theta_m, p, d, e)$ as the sum of the values of all possible income shocks to spouse $s$ starting from state $\{y_1^b, y_2^b, k_t, \theta_m, p, d, e\}$

\textsuperscript{13}For examples of the cooperative and non-cooperative approaches, respectively, see Browning and Chiappori (1998), Lundberg and Pollak (1994), and Del Boca and Flinn (2011).

\textsuperscript{14}See, for example, Pakes and McGuire (2000).
multiplied by the shocks’ instantaneous probabilities, so that

\[
V_s^c(y_1^b, y_2^b, k_t, 0, p = 1, d = 1, e = 1) = \xi_1^+(y_1^b) V_1(y_1^{b+1}, y_2^b, k_t, p = 1, d = 1, e = 1)
+ \xi_1^+(y_1^b) V_1(y_1^{b-1}, y_2^b, k_t, p = 1, d = 1, e = 1) + \xi_2^-(y_2^b) V_2(y_1^b, y_2^{b-1}, k_t, p = 1, d = 1, e = 1).
\]

In the case of divorce with an ongoing child quality improvement process, each parent’s only decision is how much to invest in the child. We therefore look for an equilibrium in parental investments, which is determined by the state of child quality and the parental income distribution. To find the equilibrium, we first solve for the reaction function of parent \(s\); this is the decision rule used by parent \(s\) in determining his or her investment level conditional on the investment level of the other parent. The conditional value of the future to divorced parent \(s\) is given by

\[
V_s(y_1^b, y_2^b, k_t, p = 1, d = 1, e = 1 | i_{s'}) = \max_{i_s} \left( \rho + \delta(k_t, i_s, i_{s'}, 0) + \sigma(k_t) + \eta + \xi(y_1^b, y_2^b) \right)^{-1} \left\{ \alpha_s \ln(V_s(y_1^b, y_2^b, 1) - i_s) + (1 - \alpha_s) \tau_s(1) \ln(k_t + \zeta) + \delta(k_t, i_s, i_{s'}, 0) V_s(y_1^b, y_2^b, k_t+1, p = 1, d = 1, e = 0) + \sigma(k_t) V_s(y_1^b, y_2^b, k_t-1, p = 1, d = 1, e = 0) + \eta V_s(y_1^b, y_2^b, k_t, p = 1, d = 1, e = 1) + V_s^c(y_1^b, y_2^b, k_t, 0, p = 1, d = 1, e = 0) \right\}^{15}.15
\]

To find the equilibrium investment levels we solve the dynamic reaction functions. Let the function \(i_s^*(i_{s'}, y_1^b, y_2^b, k_t, d = 1)\) denote the optimal level of investment by divorced parent \(s\) given current incomes, current child quality level \(k_t\) and investment by the other parent of \(i_{s'}\). For parent \(s\), this function is the argument \(i_s\) that maximizes the right hand side of the above expression. Given the reaction functions \(i_1^s(i_2, y_1^b, y_2^b, k_t, 1)\) and \(i_2^s(i_1, y_1^b, y_2^b, k_t, 1)\), an equilibrium is a pair of investment values \((i_1, i_2)(y_1^b, y_2^b, k_t, d = 1)\) such that

\[
\hat{i}_1 = i_s^*(\hat{i}_2, y_1^b, y_2^b, k_t, 1) \\
\hat{i}_2 = i_s^*(\hat{i}_1, y_1^b, y_2^b, k_t, 1).
\]

The properties of this reaction function depend critically on the properties of the improvement rate function \(\delta\). Along with \(\frac{\partial \delta}{\partial k_t} > 0, s = 1, 2\), we assume that \(\delta\) is twice continuously differentiable and concave, and add to these the restriction that \(i_1\) and \(i_2\) behave as (weak) substitutes. Under these assumptions, \(\frac{d i_1^s(i_{s'}, y_1^b, y_2^b, k_t, d = 1)}{d i_{s'}} < 0\) and the reaction function is negatively sloped for each parent \(s\) and for all values of \(k_t < k_T\).

The expressions in [2] do not fully characterize the equilibrium of the model, since the reaction functions themselves depend upon the equilibrium values \(V_s(y_1^b, y_2^b, k_t', p = 1, d = 1, e = 0), \forall t' \neq t\). Equilibrium in the divorce state for a family with an active child investment process is therefore determined over the \(2T\) parent and child quality-specific values as well as the \(2T\) parent and child quality-specific investments. The solution is obtained numerically, and the numerical technique employed is simplified by restrictions on the relationships among equilibrium
values arising from the theory and the use of the $2T$ values of terminal child qualities. Given the ordering of child qualities and the possibility of setbacks when the investment process is active, we know that $V_s(y_1^b, y_2^b, k_T, p = 1, d = 1, e = 1)$ dominates the divorce-state values of (a) all terminal child qualities $k_t$ such that $t < T$ and (b) all non-terminal child qualities. Additionally, $V_s(y_1^b, y_2^b, k_t, p = 1, d = 1, e)$ increases monotonically with $k_t$ for both $e = 0$ and 1. The numerical solution produces equilibrium investment levels $\{i_1(y_1^b, y_2^b, k_t, d = 1), i_2(y_1^b, y_2^b, k_t, d = 1)\}_{t=1}^T$ and value functions $\{V_1(y_1^b, y_2^b, k_t, p = 1, d = 1, e = 0), V_2(y_1^b, y_2^b, k_t, p = 1, d = 1, e = 0)\}_{t=1}^T$.

3.2 Married Parents

The experiences they will have if they enter the divorce state can meaningfully affect the investment decisions of forward-looking married parents. In particular, currently married parents who believe that divorce is likely in the near future will make investment decisions that look more like those made by divorced parents than will couples who believe that divorce is a remote possibility.

We must specify the manner in which divorce decisions are made. Under our assumption of noncooperative behavior, these decisions are not, in general, efficient. The nature of the decisions depends critically on legal statutes. We consider two different cases: one in which it is enough for one of the parents to ask for a divorce for the couple to enter the divorce state and the second in which both parents must agree to the divorce for it to occur. These cases are commonly termed unilateral and bilateral divorce regimes. In the empirical component, we link this solution standard to prevailing state-year divorce laws. Given a divorce standard, we define $Q_s(y_1^b, y_2^b, k_t, p, e)$ as the value to spouse $s$ of the marital status chosen in equilibrium by both spouses in state $(y_1^b, y_2^b, k_t, p, e)$. For ease of exposition we suppress any indication of the state of divorce law in the remainder of this section, but note that the full equilibrium computation includes solution for both divorce law states.

The derivation of the married parents’ equilibrium is similar to that of the divorced parents’ equilibrium, with one major difference being the search for an equilibrium in divorce decisions as well as investments and values. As before, we begin with the value of a terminated child investment

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16 We find that solutions of the model assuming either bilateral or unilateral divorce laws, with or without allowing side payments, generate only small differences in predicted behavior. This appears to be in large part the result of the narrow range of child and marriage quality levels at which parents disagree over the divorce decision for reasonable values of the primitive parameters. Since previous research documents non-negligible effects on divorce rates of states’ adoption of unilateral divorce laws, we estimate under the assumptions of noncooperative behavior and no side payments in order to let the model accommodate any important behavior differences by divorce law existing in the data to the extent possible.
process at $k_t$ for spouse $s$:

$$V_s(y_1^b, y_2^b, k_t, \theta_m, p = 1, d = 0, e = 1) = (\rho + \gamma^+(\theta_m) + \gamma^-(\theta_m) + \xi(y_1^b, y_2^b))^{-1} \left\{ \alpha_s \ln(y_s^b) + (1 - \alpha_s)(\ln(k_t) + \zeta) + \theta_m + \gamma^+(\theta_m)Q_s(y_1^b, y_2^b, k_t, \theta_{m+1}, p = 1, e = 1) + \gamma^-(\theta_m)Q_s(y_1^b, y_2^b, k_t, \theta_{m-1}, p = 1, e = 1) + V_s^e(y_1^b, y_2^b, k_t, \theta_m, p = 1, d = 0, e = 1) \right\}. \tag{3}$$

In this case, the only possible arriving updates are to the marriage match value, $\theta$, and the spouses’ income levels. Since both spouses’ welfare levels are increasing in child and marriage quality, an increase in marriage quality (at rate $\gamma^+(\theta_m)$) cannot lead to a divorce, so that $Q_s(y_1^b, y_2^b, k_t, \theta_{m+1}, p = 1, e = 1)$ corresponds to the value of marriage at those state variables. However, a decrease in marriage quality (at rate $\gamma^-(\theta_m)$) may lead to a divorce or to marriage continuation.

Next, given the current child quality level and match value, we solve for the equilibrium investment levels and associated values for each parent conditional on the continuation of the marriage. As in the divorce case, using the reaction functions we can define a pair of equilibrium investment levels and parent-specific state values associated with marriage that are given by

$$(\hat{i}_1, \hat{i}_2)(y_1^b, y_2^b, k_t, \theta_m, d = 0); \ (V_1, V_2)(y_1^b, y_2^b, k_t, \theta_m, p = 1, d = 0, e = 0). \tag{4}$$

The investment equilibrium depends on the current marriage quality both through its direct influence on the productivity of child investment and through its effect on the anticipated duration of the parents’ marriage, which partially determines the expected gain associated with an increase in child quality.

With the spouses’ equilibrium investments in the child found as in [4], the value to spouse $s$ of marriage, a child with an ongoing child improvement process, and child quality $k_t$ is

$$V_s(y_1^b, y_2^b, k_t, \theta_m, p = 1, d = 0, e = 0) = (\rho + \gamma^+(\theta_m) + \gamma^-(\theta_m) + \delta(k_t, \hat{i}_s, \hat{i}_{s'}, \theta_m) + \sigma(k_t) + \eta$$

$$+ \xi(y_1^b, y_2^b))^{-1} \left\{ \alpha_s \ln(y_s^b - \hat{i}_s) + (1 - \alpha_s)(\ln(k_t) + \zeta) + \theta_m + \gamma^+(\theta_m)Q_s(y_1^b, y_2^b, k_t, \theta_{m+1}, p = 1, e = 0) + \gamma^-(\theta_m)Q_s(y_1^b, y_2^b, k_t, \theta_{m-1}, p = 1, e = 0) + \delta(k_t, \hat{i}_s, \hat{i}_{s'}, \theta_m)V_s(y_1^b, y_2^b, k_{t+1}, \theta_m, p = 1, d = 0, e = 0) + \sigma(k_t)Q_s(y_1^b, y_2^b, k_{t-1}, \theta_m, p = 1, e = 0)$$

$$+ \eta Q_s(y_1^b, y_2^b, k_t, \theta_m, p = 1, e = 1) + V_s^e(y_1^b, y_2^b, k_t, \theta_m, p = 1, d = 0, e = 0) \right\}. \tag{4}$$
3.3 Childless Couples and the Fertility Decision

Since divorce is an absorbing state and the fertility process is only active in the married state, divorced childless ex-spouse \( s \) makes no decisions and enjoys terminal value

\[
V_s(y^b_s, p = 0, d = 1) = (\rho + \xi_s^+(y^b_s) + \xi_s^-(y^b_s))^{-1}\{\alpha_s \ln(y^b_s) + \xi_s^+(y^b_s) V_s(y^b_{s-1}, p = 0, d = 1) + \xi_s^-(y^b_s) V_s(y^b_{s-1}, p = 0, d = 1)\}.
\]

Childless married couples, one the other hand, must jointly choose to continue in the marriage and attempt to conceive a child, to continue in the marriage and not attempt to conceive a child, or to divorce. The solution to this decision problem corresponds to the maximum of the set

\[
\{V_s(y^b_{1,2}, \theta_m, p = 0, f = 1, d = 0), V_s(y^b_{1,2}, \theta_m, p = 0, f = 0, d = 0), V_s(y^b_s, p = 0, d = 1)\},
\]

where

\[
V_s(y^b_{1,2}, \theta_m, p = 0, f = 1, d = 0) = (\rho + \psi + \gamma^+(\theta_m) + \gamma^-(\theta_m) + \xi(\theta_1, y^b_{1,2}))^{-1}\{\alpha_s \ln(y^b_1) + \theta_m \\
+ \psi E_k \xi Q_s(y^b_{1,2}, k_1, \theta_m, p = 1, e = 0) + \gamma^+(\theta_m) Q_s(y^b_{1,2}, 1, \theta_m+1, p = 0, e = 0) \\
+ \gamma^-(\theta_m) Q_s(y^b_{1,2}, 1, \theta_m-1, p = 0, e = 0) \\
+ V_s(y^b_{1,2}, 1, \theta_m, p = 0, d = 0, e = 0)\},
\]

which requires taking the expectation of the realized value of an arriving child with respect to the initial child quality distribution, and

\[
V_s(y^b_{1,2}, \theta_m, p = 0, f = 0, d = 0) = (\rho + \gamma^+(\theta_m) + \gamma^-(\theta_m) + \xi(\theta_1, y^b_{1,2}))^{-1}\{\alpha_s \ln(y^b_1) + \theta_m \\
+ \gamma^+(\theta_m) Q_s(y^b_{1,2}, 1, \theta_m+1, p = 0, e = 0) + \gamma^-(\theta_m) Q_s(y^b_{1,2}, 1, \theta_m-1, p = 0, e = 0) \\
+ V_s(y^b_{1,2}, 1, \theta_m, p = 0, d = 0, e = 0)\}.
\]

Hence the married childless couple solves a discrete, three point problem that depends on their expectations of initial child quality, the future of their income and marriage quality processes and the equilibrium investments they would make should a child arrive.

To find equilibrium fertility, investments, values, and divorce decisions over the marriage quality distribution and for all child quality levels, we again make use of the restrictions on the relative values of the possible child and marriage quality states implied by the theory. The solution is obtained numerically, with equilibrium in the married parents’ case occurring over all \( 2T \) parent-and child quality-specific values and investments across all \( M \) possible values of \( \theta \). Computation of the equilibrium is simplified by the presence of the terminal values represented in \([3]\) and \([5]\). Having followed the above steps, we have the complete solution for the marriage state,

\[
\left\{(\check{\nu}_1, \check{\nu}_2)(y^b_{1,2}, k_t, \theta_m, d = 0), (V_1, V_2)(y^b_{1,2}, k_t, \theta_m, p = 1, d = 0, e) T_{t=1}^{T}\right\}_{m=1}^{M}, e = 0, 1,
\]
and fertility and divorce decisions for childless married couples, along with divorce decisions for every value of the state variables.

### 3.4 Characterizing the Equilibrium of the Model

Given the relatively large number of state variables, strategic interactions between parents, and the complicated exogenous and endogenous dynamics of the fertility, child quality and marital status processes, it is not an easy task to characterize the equilibrium of the model and conduct comparative statics exercises. In this subsection we depict some patterns in the equilibrium behavior predicted by the model described above when we evaluate the model at the parameter estimates reported in Section 5. By presenting and discussing two figures, we hope to give the reader a feel for some of the more important characteristics of the equilibrium of the model. This will aid in interpreting the parameter estimates and in understanding the outcomes of the welfare exercises reported below.

First consider the fertility decision of a childless married couple. The decision depends on current income and marriage quality states, along with the exogenous parameters of the problem. Let spouse 1 be the husband and spouse 2 be the wife. Assume further that, should the couple both have a child and divorce, under the state child support guidelines in effect in the couple’s state at the date of marriage, the husband would be required to transfer 20 percent of his income to the wife in child support payments. A 20 percent rate is the median (and modal) child support rate calculated based on state guidelines for families in our NLSY sample, discussed below. All parameters of the problem are chosen to match the estimates in Tables 4-5, the tables of MSM estimates in Section 5, and the divorce standard is assumed to be the unilateral standard. In solving and estimating the model, we set $B = 5$ and we take as the 5 discrete income values of spouse s’s income process the midpoints of the 5 quintiles of the NLSY income distribution for spouses of s’s gender. We choose $M = 5$ exogenously fixed values of marriage, which we center at zero. Hence the third match value of marriage yields the same utility contribution and child quality productivity as the divorce state. Finally, we choose $T = 10$ and map the ten child quality levels to deciles of an age-normed test score distribution discussed in Section 5.

Table 1 describes the fertility choices of such a couple over all 125 possible $(y_1, y_2, \theta_m)$ combinations. There is a positive association between the decision to start a family and the current match quality of the marriage. At this particular parameterization, for a family of the (arbitrarily) chosen types, the couple never attempts to conceive a child at the lowest marriage quality level. The number of income pairs leading to an active fertility process is positive and increasing across the second, third and fourth marriage qualities. Childless couples always attempt to conceive in

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17 Section 5 on the estimation method details the implementation of two type-based sources of family heterogeneity in the estimation of the model. The figures described in the current section are based on a couple with income process type 4 and initial child quality type 2.

18 These quintiles are determined based on pooling all annual income observations for male or female sample members over all years in which incomes are observed, up to the 24th year of marriage.

19 In fact, under this parameterization and for this family type, couples always divorce at the lowest marriage quality level.
marriages of the fourth and fifth quality levels. As higher marriage qualities stabilize the marriage, the expected future value of a shared child increases, and spouses respond accordingly in their fertility decisions.

Children in our NLSY sample seem to have the characteristics of inferior goods, and the model estimates have clearly been determined to reflect this phenomenon. Among families who were in the first quintile of household income at the date of marriage in our NLSY sample, 73.78 percent have a child or children by the 10th year after the marriage date. The proportion with children at ten years declines steadily through the next four quintiles of the household income distribution, with 69.51 percent with children by 10 years in the middle income quintile and 65.24 percent with children by 10 years in the top income quintile. Only 53.65 percent of the top 5 percent of families by household income have children in 10 years. For the particular parameter estimates and assumed family types depicted in Table 1, married couples always attempt to conceive when the husband’s income belongs to one of the lower two quintiles. Though married couples with two of the three highest income pairings also always attempt to conceive, the majority of moderate and high income pairings result in no conception attempts at the second and third marriage quality levels, and overall the figure indicates that fertility decreases modestly with household income.20

Figure 4 graphs the total child investment of our representative couple, under the model estimates in Tables 5-6, across the ten possible child quality levels. We focus on a couple with median incomes at the marriage date, based on our NLSY sample, of $y_1 = 21,825 and $y_2 = 17,632. Financial variables throughout the paper are expressed in 2004 dollars. The figure shows investment profiles in the marriage state for each possible value of $\theta$. Investments decrease with child quality, as parents experience diminishing returns from child attainment through the parental objective, and as parents work to avoid lower child quality levels that may destabilize marriage. Given the boundary conditions imposed on our child quality production process, parents invest nothing at the top child quality.21 In the divorce state, as a result of the combined effects of maternal custody and child support, the mother’s child investments are substantially larger than the father’s. The reverse is true in marriage. In fact, at these parameter values, for these types and these incomes, the model predicts the mother’s investments in marriage to be quite similar to the father’s investments in divorce, and the father’s investments in marriage to be quite similar to the mother’s investments.20 Mumford (2007) finds a u-shaped total fertility pattern in family income for the NLSY, driven in part by elevated fertility rates in the lowest and highest income quartiles. The fertility pattern in Table 1, based on estimates for NLSY married couples, would seem to align with his result.

21 The relatively arbitrary choice of an endogenous rate of increase and exogenous rate of decrease of child quality contributes heavily to the zero investment for top quality children and the slope of investment as a function of child quality. An alternative specification that we could not reject in favor of the current form, given the aliasing problem, is one in which child quality increases exogenously and decreases endogenously. Such a profile would generate positive investments at the top child quality and flatten the investment profile.
One interesting feature of the marriage quality-investment relationship is that the worst continuing marriages produce the lowest level of child investment, when compared not only with the other continuing marriages but also with divorce. At this parameterization and income, parents of the depicted type divorce at $\theta_1$ but remain married at $\theta_2$.\textsuperscript{23} Hence the marriages of quality $\theta_2$ are the marginal marriages, and are most likely to enter divorce in the near future. The model fits the low test scores of children in marriages heading for divorce that we see in Figure 2 by allowing the second marriage quality to damage child investment productivity, and parents react by investing less in children when in second quality continuing marriages. Total equilibrium child investment at $\theta_2$ is below total investment at all other viable marriage qualities and is even substantially below total equilibrium investment in the divorce state. This is one example of the model’s ability to match relevant empirical phenomena.

4 Estimation Method

We turn now to the question of how the dynamic equilibrium is mapped to the behavior of our sample of NLSY families. Let $j = 1, ..., J$ index the sample families. The endogenous variables utilized in the estimation procedure consist of the following: (i) the time to the arrival of the first child, $n(j)$, as measured from the date of marriage, and which may be censored at final observation date $A(j)$, (ii) the child’s score on a mathematics examination administered as part of the NLSY Child survey at $G_j$ points in time, indexed by $g = 1, ..., G_j$ with $G_j \geq 1 \forall j$, and (iii) the elapsed time from marriage to divorce, $d(j)$, which may also be censored at the end of the observation period. Though many of the $J$ NLSY families that we observe will have second children and more, we model fertility and investment decisions only for the first child, and the estimation data track only the arrival and test scores of first children. This is clearly a strong simplification of the problem. While parents’ interactions over the allocation of investments across multiple children are certainly of interest, we feel that the simultaneous, multi-agent choices of marital status, whether to begin a family and what early investments to make in the family given a particular divorce policy universe are of primary concern. Tracking only the arrival and progress of a first child, along with spouses’ divorce decisions, allows us to study the crucial family formation stage as a two agent problem with many simultaneous decisions, while abstracting from the excessive complexity that arises where one allows two spouses to make choices regarding ongoing fertility and simultaneous investment allocations to multiple children. We believe that this simplification of the problem permits a clearer understanding of the mechanisms that link the inherent strength of a marriage,

\textsuperscript{22}This is reminiscent of the Warr 1983 result that expenditures on a public good in a voluntary contributions game are invariant with respect to a redistribution of income in which all parties contribute under both income distributions. The difference in this case is that there are a number of households in which contributions from one or both parties would be zero under either or both income distributions, and the fact that divorce essentially changes the value of the public good to each party, thereby changing the equilibrium contributions, even when both parties contribute positive amounts to investment in the married and divorced states.

\textsuperscript{23}Other types may divorce at $\theta_2$. 

16
spouses’ differing income processes, expected costs of divorce and child access in the divorce state, and spouses’ family building activities. Note that our sample includes married NLSY couples with any number of children, from childless couples to large families, and therefore tracking fertility only through the first child’s arrival does not impose meaningful sample restrictions within the set of ever-married respondents.

As the model clearly demonstrates, outcome variables \((i)-(iii)\) are functions of realizations of exogenous and endogenous stochastic processes. The exogenous stochastic processes include those that describe spouses’ incomes, the termination date of the “window” for child quality improvement and the trajectory of the marriage quality characteristic \(\theta\). The endogenous stochastic processes include the arrival of the first child, if any, and the timing of improvements in child quality. Because the stochastic processes generating these outcomes are rather complicated due to the endogeneity of fertility and investment behaviors, and due to the modifications to the processes around divorce and fertility, we turn to the method of simulated moments to estimate the model. Implementation of this procedure requires access to a large number of simulated sample paths for each sample household \(j\), which terminate at variable final observation dates \(A(j)\), and which produce realizations of \(\{n^t(j), k^t(j, g), d^t(j)\}_{g=1}^G\).

While the general estimation strategy we outline can be used with any number of functional form assumptions on the investment process that satisfy our conditions for uniqueness of the Nash equilibrium investment choices, in the results reported below we assume that

\[
\delta(i_1, i_2, \theta) = \delta_0(\theta)[i_1 + i_2]^{\nu},
\]

where \(\nu \in (0, 1)\) and \(\delta_0\) is a parametric function that is increasing in \(\theta\) and takes values on the nonnegative real line. This form of the \(\delta\) function satisfies the requirement that \(\frac{\partial^2 \delta((i_1, j, \nu, \theta))}{\partial i_1 \partial j_1} \leq 0\), for all \(\theta\). The specific functional form of \(\delta_0(\theta)\) used in the estimation is \(\delta_0(\theta) = \delta_0 \Phi\left(\frac{\theta - \mu_0}{\sigma_0}\right)\), where \(\delta_0\) is a scalar to be estimated.

We assume a direct mapping from the \(g\)th test score of the child of family \(j\) to her underlying child quality \(k_t\). Underlying child quality \(k_t\) takes cardinal values 1 through 10, and the child’s observed test score, denoted \(o(j, g)\), maps into these cardinal values as follows: if \(o(j, g)\) lies in the \(t^{th}\) decile of the age-normed test score distribution, then child \(j\)’s inferred quality is \(k(o(j, g)) = k_t\). In short, the cardinal value of child quality used in the estimation is equal to the number of the child’s decile in the age-normed test score distribution.

A spouse’s income follows the process described in the discussion of model dynamics in Section 3. The free parameters associated with spouse \(s\)’s income process are \(\xi_s^{+}\) and \(\xi_s^{-}\). To improve the fit of the model, and to introduce some realistic heterogeneity, we allow \(\gamma_s^b\) to be driven by two distinct, type-specific parameter vectors in the population of husbands \((s = 1)\) and two distinct, type-specific parameter vectors in the population of wives \((s = 2)\). This leads to four distinct type \(l\)-specific parameter vectors, \(\{\xi_s^{+}(l), \xi_s^{-}(l)\}_{s=1,2, l=1,2}\). Spouse \(s\)’s probability of belonging to income type 1 is determined based on the logistic expression \(1/(1 + \exp(Z_s\beta_s))\), where \(Z_s \subset Z_j\) is a vector of exogenous family characteristics influencing spouse \(s\)’s probability of being of income type 1.
Finally, we observe substantial income movements around the arrival of the first child for wives in our sample. As a result we allow the income of the wife to realize a setback following the birth with probability \( \epsilon \), where \( \epsilon \) is an additional income process parameter to be estimated. In total we estimate nine distinct income process parameter vectors. The income process plays out over a set of \( B = 5 \) exogenously set discrete income levels each for husbands and wives, defined by income quintiles as described in Section 3.

Initial conditions clearly play an important role in determining the endogenous fertility, divorce and child quality outcomes produced by simulation of the model for a given family. We have a mix of observable and unobservable initial conditions. Spouses’ incomes at the date of marriage are observable, and are therefore simply mapped to our discretized income scale in determining the initial income values from which we begin simulating a family’s history.

Initial marriage quality and the realized quality of an arriving child, on the other hand, are not directly observable in the NLSY data. For each simulated history, initial marriage quality \( \theta(0) \) is determined as follows: Each spousal pair draws a match value from a common support of \( \{\theta_1, ..., \theta_M\} \). Marriage quality values \( \{\theta_1, ..., \theta_M\} \) are located so that \( \{\Phi(\theta_1), ..., \Phi(\theta_M)\} = \{0.1, 0.3, 0.5, 0.7, 0.9\} \). Note that this implies a set of match values centered at zero, with with \( \theta_1 < 0 \) and \( \theta_M > 0 \). However, the mass of the initial marriage quality distribution need not be centered at zero and is free to favor either positive or negative marriage values. Define \( Z_\theta \subset Z_j \) as a set of household characteristics that affect the match value distribution, and define

\[
\omega_\theta(\theta(0) = \theta_m | Z_\theta) = \begin{cases} 
0.5[\Phi(\frac{\theta_1 - Z_\theta \beta_\theta}{\sigma_\theta}) + \Phi(\frac{\theta_2 - Z_\theta \beta_\theta}{\sigma_\theta})] & m = 1 \\
0.5[\Phi(\frac{\theta_{m+1} - Z_\theta \beta_\theta}{\sigma_\theta}) - \Phi(\frac{\theta_{m-1} - Z_\theta \beta_\theta}{\sigma_\theta})] & m = 2, ..., M - 1 \\
1 - 0.5[\Phi(\frac{\theta_M - Z_\theta \beta_\theta}{\sigma_\theta}) + \Phi(\frac{\theta_1 - Z_\theta \beta_\theta}{\sigma_\theta})] & m = M 
\end{cases}
\]  

where \( \Phi \) is the standard normal c.d.f. The probability distribution of the marriage quality value is parametric, and is completely determined by \( \{\theta_1, ..., \theta_M\}, \beta_\theta, \) and \( \sigma_\theta \).

Similarly, in any simulation in which spouses’ fertility choices result in the arrival of a child, child quality at birth \( k(0) \) is drawn from a discrete initial child quality distribution. Let

\[
\omega_k(k(0) = t) = \begin{cases} 
0.5[\Phi(\frac{1 - \mu_k}{\sigma_k}) + \Phi(\frac{2 - \mu_k}{\sigma_k})] & t = 1 \\
0.5[\Phi(\frac{(t+1) - \mu_k}{\sigma_k}) - \Phi(\frac{(t-1) - \mu_k}{\sigma_k})] & t = 2, ..., T - 1 \\
1 - 0.5[\Phi(\frac{(T-1) - \mu_k}{\sigma_k}) + \Phi(\frac{T - \mu_k}{\sigma_k})] & t = T 
\end{cases}
\]

Further, suppose that families can be of two initial child quality types, and that these type populations are characterized by mean initial child qualities \( \mu_{k1} \) and \( \mu_{k2} \), to be estimated. Household \( j \)'s probability of belonging to income type 1 is determined based on the logistic expression \( 1/(1 + \exp(Z_k/\beta_k)) \), where \( Z_k \subset Z_j \) is a vector of exogenous family characteristics influencing child quality. Spouses in family \( j \) have full information regarding the distribution of initial child quality given their characteristics, and this information enters their fertility decisions as described in Section 3.3.
We have access to a random sample of $J$ NLSY-79 families. For each family observation we perform $R$ replications of the following process. We begin by drawing an initial marriage quality level from distribution (6), conditioning on family characteristics $Z_j$. From there the dynamic aspects of the simulated history operate as follows. The “base draws” for the random number generation used in the dynamic simulation are kept constant across iterations of the estimation algorithm to facilitate the convergence process. For any given individual, we draw a total of $R \times S$ values from a uniform pseudo-random number generator for use in generating the timing of changes in the child quality improvement process and denote the draws $u^{(1)}$. Similarly, we draw $R \times S$ uniform random number matrices $u^{(2)}$ for the generation of the timing of decreases in child quality, $u^{(3)}$ for the timing of increases in marriage quality, $u^{(4)}$ for the timing of decreases in marriage quality, $u^{(5)}$ through $u^{(8)}$ for the (type-specific) timing of income increases and decreases for the two spouses, and $u^{(9)}$ for generating the duration to the arrival of a child given an active fertility process. Finally, we draw an $R \times 1$ vector, $u^{(10)}$, to determine the duration of the “window” for child quality improvement.

Initial incomes $y_{b1}^h(j, 0)$ and $y_{b2}^h(j, 0)$ are determined by mapping the observed incomes of the husband and wife at the date of marriage, in 2004 dollars, to the closest income level available in the relevant discrete income grids. Given $\theta(0)$, $y_{b1}^h(j, 0)$ and $y_{b2}^h(j, 0)$, we use decision rules calculated from the model regarding whether to divorce and whether to attempt to conceive a child to determine the relevant processes for couple $j$. For example, if couple $j$ decides to remain married but not to attempt to conceive, then they may still experience any one of four income shocks, or they may experience a marriage quality improvement or setback.

Using the negative exponential distribution of wait times to updates in our various processes, we define the implicit length of time in replication $r$ until a first improvement in spouse 1’s income by

$$\bar{q}_5(r, 1) = -\frac{\ln(1 - u^{(5)}(r, 1))}{\xi_1^{+}(y_{b1}^h(j, 0))}.$$ 

The time to an income improvement for spouse 2, and to setbacks for spouses 1 and 2, are defined similarly using $\{\xi_s^{+}(y_{b1}^h), \xi_s^{-}(y_{b2}^h)\}_{s=1,2}$. They are labelled $\bar{q}_6(r, 1)$ through $\bar{q}_8(r, 1)$. Times to marriage quality improvements and setbacks, similarly, are

$$\bar{q}_3(r, 1) = -\frac{\ln(1 - u^{(3)}(r, 1))}{\gamma^{+}(\theta_m)}$$

and

$$\bar{q}_4(r, 1) = -\frac{\ln(1 - u^{(4)}(r, 1))}{\gamma^{-}(\theta_m)}.$$ 

In this particular case the probability of the remaining events, child arrival and child quality improvement, setback and termination, are all zero, so that $\{\bar{q}_l(r, 1)\}_{l=1,2,9,10}$ are all arbitrarily large. Which event is actually observed is determined using a competing risks framework, namely,

\[24\text{Note we suppress type indicator } l.\]
cause $\varphi$ is observed if

$$\widehat{q}_\varphi(r, 1) = \min(\widehat{q}_1(r, 1), ..., \widehat{q}_{10}(r, 1)).$$

Before the second event is generated the state variables are updated as follows. If the observed event is an increase in spouse $s$’s income, then $y_{b, s}^h(j, 0)$ is updated to $y_{b, s}^{b+1}$. If the event is an income setback for $s$, $y_{b, s}^h(j, 0)$ is updated to $y_{b, s}^{b-1}$. Similarly, if the first event is a marriage quality improvement, then $\theta_m$ is updated to $\theta_{m+1}$, and if the event is a marriage quality setback then $\theta_m$ is updated to $\theta_{m-1}$. All other state variables remain at their initial levels.

With the resulting state vector, we begin update round 2 of replication $r$. Note that at the new state vector spouses may choose to attempt to conceive, activating update process 9 with seed values $u^{(9)}$, or they may divorce, ending the relevance of update processes 3 and 4 to replication $r$. We calculate all relevant event arrival times for the couple’s new marital state and fertility decisions, $\{\widehat{q}(r, 2)\}_{i=1}^{10}$, and apply the competing risks standard to determine the observed second event. Note that the elapsed time so far is

$$a(r, 2) = \min(\widehat{q}_1(r, 1), ..., \widehat{q}_{10}(r, 1)) + \min(\widehat{q}_1(r, 2), ..., \widehat{q}_{10}(r, 2)).$$

We continue to build a simulated history for the couple by alternating as above between the event simulation and the state vector updating steps. The simulated history may eventually include the arrival of a child, requiring an initial child quality level draw and triggering the child investment processes. Conditional on family characteristics $Z_j$, we draw $k(0)$ from initial child quality distribution (7). Given the state vector at the arrival of the child, we determine the parents’ equilibrium investments in the child, $(\hat{i}_1, \hat{i}_2)(y_{1, b}^h, y_{2, b}^h, k(0), \theta_m, d = 0)$, and from them the arrival rate of child quality improvements $\delta(k(0), \hat{i}_1, \hat{i}_2, \theta_m)$. Supposing that the child’s arrival is event number $v$, the time to a child quality improvement is then calculated as

$$\widehat{q}_1(r, v) = -\frac{\ln(1 - u^{(1)}(r, v))}{\delta(k(0), \hat{i}_1, \hat{i}_2, \theta_m)}.$$

The time to a child quality setback is

$$\widehat{q}_2(r, v) = -\frac{\ln(1 - u^{(2)}(r, v))}{\sigma(k(0))}.$$

Finally, the total duration of the active investment process is determined one time for replication $r$ for family $j$, as

$$\widehat{q}_{10}(r, v) = \widehat{q}_{10}(r) = -\frac{\ln(1 - u^{(10)}(r))}{\eta}.$$
window, so that
\[ \sum_{v=1}^{V} \min(\hat{q}_1(r,v), ..., \hat{q}_{10}(r,v)) = a(r,V) = A(j). \]

From each simulated history for family \( j \) we are able to extract values of endogenous variables \( \{n^r(j), k^r(j,g), d^r(j)\}^{G_j}_{g=1} \), which we then compare via a series of moments to the analogous values \( \{n(j), k(j,g), d(j)\}^{G_j}_{g=1} \) observed in the NLSY-79. Consider conditional expectation \( w \)

\[ E(\hat{f}_w(Z_j; n(j), d(j), k(j,1), ..., k(j,G_j)|Z_j; \Gamma)). \]  

(8)

We define \( W \) conditional expectations functions, where \( W \) \( \geq \) \( NP \), the dimension of the parameter vector \( \Gamma \). Given the complexity of the model there exists no closed form expression for (8) in general; we approximate the value of each conditional moment using the simulated histories. Given the \( R \) sample paths for household \( j \) the approximation to conditional moment \( w \) for family \( j \) is

\[ \frac{1}{R} \sum_{r=1}^{R} \hat{f}_w(Z_j, n^r(j), d^r(j), k^r(j,1), ..., k^r(j,G_j)|Z_j; \Gamma) \]

\[ \equiv \tilde{f}_w(Z_j, n(j), d(j), k(j); \Gamma), \]

where \( k(j) = k(j,1), ..., k(j,G_j) \). “Unconditioning” on \( Z_j \) yields unconditional moment

\[ \tilde{f}_w(\Gamma) = J^{-1} \sum_{j=1}^{J} \tilde{f}_w(Z_j, n(j), d(j), k(j); \Gamma). \]  

(9)

The analogous moment in the NLSY-79 data is

\[ f_w(\Gamma) = J^{-1} \sum_{j=1}^{J} f_w(Z_j, n(j), d(j), k(j); \Gamma), \]

replacing the averages across simulated moments for \( n(j), d(j) \) and \( k(j) \) with their actual values as observed for family \( j \) in the NLSY-79. Note that some moments computed in this way will only be defined for a subset of the sample. For example, one moment may be the difference in test scores for those who took the test twice. In this case, only the subset of observations for which two test scores are available could be included in the computation of this moment. This subsampling does not affect our interpretation of all moments as representing the population, since we are assuming, and have reason to believe, that the number of test measurements available is exogenously determined.\(^{25}\)

Estimation proceeds by iterating on the parameter vector \( \Gamma \) until a value of \( \Gamma \) is found at which a weighted distance between the data moments and the moments calculated from simulated histories based on the model is sufficiently small. However, calculation of the decision rules used by agents

\(^{25}\) This implies that the distribution of \( Z \) should be invariant among subpopulations defined in terms of the number of times the test has been taken. This can be checked using nonparametric methods and sample estimates of the subsample distribution functions of \( Z \).
with current state variables \( h \in H \) implied by the model at arbitrary parameter vector \( \Gamma \) is an extremely time-intensive task, and to compute the moments from the simulated histories requires access to these rules. We have developed a relatively efficient estimation technique for doing so, a discussion of which is contained in Appendix A. In brief, our method involves solving the model and estimating it at the same time, effectively reducing the computational burden of a dynamic model to that of a static model. We adopt a strategy to speed the convergence process which is related to the insightful work of Imai, Jain and Ching (2009). They recognized the wastefulness of recomputing decision rules “from scratch” at each new set of trial parameter values as one works through the iterative process to find the parameter estimates. The idea, as implemented here, is to compute some “exact” solutions to the household’s investment and divorce problem at a fixed set of parameter values, and to approximate the household investment rule as a convex combination of these parameter values, where the weights attached to the rules are a function of the relative distance between the current parameter guesses and the reference parameter vectors. Using the approximate investment rules and the current guesses of the parameters \( \epsilon \), we generate simulated moments. We iterate over \( \epsilon \) until we adequately approximate the observed sample moments, and call this estimator \( \hat{\Gamma}_1 \). We find investments over all states \( h \) at this value of the parameter vector, and compare these with the investments predicted from the approximation. If the divergence is sufficiently great for any \( h \in H \), we add \( \hat{\Gamma}_1 \) to our collection of parameter vectors with “exact” investment solutions, and restart the iteration process using as starting value \( \hat{\Gamma}_1 \). We repeat the process until the exact and approximate investment rules at our estimator are sufficiently close over all \( h \in H \). We find that this approach performs well in practice. It has many desirable properties, including that the precision of the approximated solution increases most over the course of the estimation procedure in the region of the parameter space in which the estimation algorithm searches most intensely.

4.1 Identification Issues

With such a complex model and using an estimator that is not likelihood-based, it is difficult to give precise identification conditions for the various model parameters. However, we will attempt to provide a partially heuristic, partially rigorous discussion of the central issues regarding identification in continuous time, point process models of this type. Since the impact of the family law environment on the child quality process is our main focus of interest, we begin our discussion of identification of the parameters characterizing this process from the time of birth of the child. Our subsidiary interest is in the impact of the family law environment on fertility, and we will discuss how the incorporation of fertility decisions into the model actually aids in the identification of model parameters.

In a discrete time modeling framework, with multiple observations per individual, it is natural to look at the transition probability matrix as a leading source of identifying information for underlying model parameters. Say that a random variable that assumes \( B \) distinct values is measured at two points in time for \( N \) independent realizations of the population stochastic process.
Then the transition probability matrix has $B(B - 1)$ independent elements, and as $N \to \infty$, 
plim_{N \to \infty} \frac{n_{ij}}{n_i} = \Psi_{ij}$, where $i$ is the origin state, $j$ the destination state, 
$n_i = \sum_{j=1}^{B} n_{ij}$, and $\Psi_{ij}$ is 
the true transition probability. Let the vector of primitive parameters be given by $\Gamma$, and write the 
vectorized transition matrix, after omitting redundant elements, as $\Psi$. Now define a mapping from 
the primitive parameters to the (vectorized, non redundant) transition probabilities by $\Psi^*(\Gamma)$, and, 
for simplicity assume that $\Psi^*(\cdot)$ is everywhere differentiable on the parameter space associated with 
$\Gamma$. As is obvious, for $\Gamma$ to possibly be identified from knowledge of $\Psi$ requires $NP \leq B(B - 1)$. 
Then we say that $\Gamma$ is uniquely identified by knowledge of $\Psi$ if $\Psi^*$ is 1-1, for in this case there exists 
a unique inverse function $\Gamma = (\Psi^*)^{-1}(\Psi)$. This is a very strict notion of identification. Typically 
we invoke sample identification criteria in practical applications. Given access to a finite amount of 
information, we only have access to an estimated value of $\Psi$, which we will denote by $\hat{\Psi}$. Then we 
may say that $\Gamma$ is uniquely identified if an objective function such as $(\hat{\Psi} - \Psi^*(\Gamma))^TW(\hat{\Psi} - \Psi^*(\Gamma))$ 
is globally convex in $\Gamma$, where $W$ is some positive definite matrix of the quadratic form (which could 
depend on $\Gamma$ as well). We then say that $\hat{\Gamma}$, the argument that minimizes the value of the quadratic 
form, is the unique estimate of the primitive parameter vector $\Gamma$. In many cases involving complex 
applied models, one may only be able to establish convexity locally.

The above sketch of an idealized problem corresponds roughly to the one we confront, except that 
the problem of estimating the child quality process that we face is more complex on several fronts. For approximately one-tenth of our sample, we only have access to one measurement on 
the child’s test score (i.e., $G_j = 1$). We begin by considering the transition from the origin states 
(at the time of birth) $(k(1), \theta(1))$ into the states associated with the first sampling point at time 2. 
For the moment, assume that this sampling time is the same for all sample members.

The primary problem is that the states of the process are imperfectly observed. Denote the 
state vector at the initial date by $S_1$ and at the subsequent observation time by $S_2$, where the states 
are the $MT$ (= $B$ in the discussion above) possible values of $(k, \theta)$ at each moment in time. There 
are no measurements available at time 1, creating the usual initial conditions problem. Thus, even 
if $\Gamma$ were identified from knowledge of $\Psi$ or $\hat{\Psi}$, it is not possible to estimate $\Gamma$ from readings only 
on the (partially observable) state vector $S_2$. In order to estimate $\Psi$ consistently, even given full 
observability of $S_2$, it is necessary to assume a prior distribution on values of $S_1$, $F_1(S_1|\Gamma, Z)$, which 
are functions of the primitive parameters $\Gamma$ and perhaps other covariates $Z$. Then estimation of the 
transition matrix parameters $\Psi$ is accomplished by writing the probability of $S_2$ as

$$p(S_2|\Gamma, Z) = \sum_{S_1} \Psi(S_2|S_1; \Gamma)F_1(S_1|\Gamma, Z).$$

Clearly, even if $\Gamma$ is identified given observability of $S_1$, identification of $\Gamma$ from $p(S_2|\Gamma, Z)$ depends 
critically on the functional form of $F_1(S_1|\Gamma, Z)$. Moreover, given the lack of observability of $S_1$, 
identification of $\Gamma$ from $p(S_2|\Gamma, Z)$ will hinge on untestable assumptions regarding $F_1$.

We have two test score measures for many of the children in our sample, and there is no reason
to believe that the number of measurements is endogenous. For some random sample of children, then, we observe a second measurement, let us say at some common time $S_3$. For these children, if $S_2$ and $S_3$ were both perfectly observable, we could eliminate the initial conditions problem introduced by not being able to observe $S_1$ by looking at the transitions between $S_2$ and $S_3$, and consistently estimating the transition matrix $\Psi(S_3|S_2; \Gamma)$. The problem is that the state vectors $S_t$, $t = 2$ and $3$, are only partially observable. For example, at measurement time 2, the state vector $S_2$ includes the indicator variables $d(2)$ and $e(2)$ indicating whether the parents are divorced and whether the investment process is on-going, as well as the child quality level $k(2)$ and the marriage quality level $\theta(2)$. If the parents are married at time 2, we do not observe the match quality $\theta(2)$, nor do we observe whether the investment process is on-going or not. If the parents are divorced at measurement time 2, then the value of the match is normalized to 0, and hence is known, while the investment process indicator $e(2)$ is unknown.

Divorced parents at time 2 have the potential to provide a large amount of information regarding the child quality production process, since the transitions of child quality observed for them are not “contaminated” by changes in the unobservable marriage quality process. For these parents, we observe child quality at times 2 and 3, but do not observe whether the investment process has ended as of time 2. If $e(2) = 1$, so the process is over at time 2, then we know that $k(2) = k(3)$. Conversely, $k(2) \neq k(3) \Rightarrow e(2) = 0$, that is, a change in child quality state between measurement times 2 and 3 implies that the child quality investment process had not ended at time 2. Recalling that the timing of the ending of the investment process is strictly exogenous, we can construct a conditional transition matrix $\pi(k(3)|k(2), d(2) = 1, k(3) \neq k(2), \Gamma)$, that is, in fact, only a function of a subset of primitive parameters, those characterizing the parental preferences and the stochastic production technology, that is, only the parameters $a_1, a_2, \nu, \tilde{\sigma}$, and $\eta$ appear in the equilibrium investment rules in this case. Given the equilibrium investment rules, and the exogenous processes as defined by $\tilde{\sigma}$ and $\eta$, the transition probability function for child quality measurements between times 2 and 3 are determined. Movements between child quality states for this set of individuals provide a large amount of information on a relatively small number of parameters.

Differences in test score transition rates between measurement times 2 and 3 for children of divorced and married parents (as of time 2) are important sources of identification of the impact of the marriage quality process on child outcomes and the relationship between child quality and divorce. This can be appreciated most directly if we condition on the event $k(3) \neq k(2)$, for in this case we know that the child investment process is still on-going at measurement time 2. For the same distribution of pre-transfer parental incomes, parental investment rules (and hence transition rates between child quality states) will differ due to (1) income transfers made in the divorced state (which result in different post-transfer income levels in the two cases), (2) different payoffs to child quality in the two marital states (due to the transformation of child-quality from a public to a semi-private good), and (3) differences in the value of $\theta$ between the two states (in the divorced state $\theta$ is fixed at 0, and in the married state we know the sample path of $\theta$ and $k$.

\footnote{See Section 5 for details on testing patterns in the survey.}
has never resulted in a divorce outcome). Since the divorce law environment is what changes the parental income distribution and parental preferences in the two states, and since the environment is assumed known, the differences in the married and divorced parents’ child quality transition rates mainly serve in identifying the role of the $\theta$ process in the child quality transition rates.

Up to this point we have ignored the fact that measurements of child quality are obtained at different ages. This nonconcurrence is essentially impossible to treat in a satisfactory manner using a discrete-time framework (see Flinn and Heckman 1982). The continuous time framework allows us to accommodate any sampling scheme within the estimation process. Moreover, the fact that measurements are taken at different ages across the children in our sample is an asset in terms of identification. To understand why, consider the example of the previous paragraph, where we now denote the transition matrix between measurement periods 2 and 3 by $\Psi(k(3)|k(2), d(2) = 1, k(3) \neq k(2), \Gamma, a(3) - a(2))$, where the additional conditioning argument $a(3) - a(2)$ is the elapsed time between the first test score measurement and the second. Variation in the timing of measurements provides identifying information for $\Gamma$ because, in general,

$$
\Psi(k(3)|k(2), d(2) = 1, k(3) \neq k(2), \Gamma, t) \neq \Psi(k(3)|k(2), d(2) = 1, k(3) \neq k(2), \Gamma)\phi(t),
$$

that is, the matrix of transition between states is not invariant with respect to the duration between measurements up to a scale normalization $\phi(t)$. Thus by varying the measurement periods, we actually gain more information regarding $\Gamma$ than when the measurement times are synchronized.

While we have devoted most of our attention to the case in which two test scores are available for the child, in approximately one-tenth of the families only one observation is available. Given the correctness of our functional form assumptions regarding the initial conditions ($(k(1), \theta(1))$, the same general argument applies regarding the information value of having varying ages of first test measurement. Since measurement time 1 for all children occurs at their birth, and is unobserved, transitions between $k(1)$ and $k(2)$ will be functions of the $\theta$ process since all parents are married at measurement time 1. Nonetheless, even if no child in the sample had more than one test score measurement, identification of parameters governing the child quality and marriage quality processes could be distinguished in large part due to the assumption that the marriage quality process is the same within all marriages, while the child quality process, being endogenous, is not.

We now turn to how endogenous fertility impacts the arguments we have made to this point. There is only one parameter solely associated with the fertility process, which is the rate of con-

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$^{27}$In the estimation we use state-level information on child support percentage-based orders. Heterogeneity in this rate across state jurisdictions further aids in identification of the parameters describing the $\theta$ process.

$^{28}$Recall that the timing of the measurements is considered to be determined in a strictly exogenous manner, so no selection issues arise from the varying number or timing of measurements across sample families.

$^{29}$Another way to think of this is in terms of the aliasing problem in standard stationary time series analysis. Aliasing occurs when a process defined on a given frequency is sampled at a “coarser” frequency (for example, when a process that changes value every month is observed every quarter). In this case, a sample path defined on the courser frequency is consistent with a large number of sample paths defined on the true frequency, thus leading to a form of nonidentification. By varying the frequencies of the coarse measurements, we have a way to rule out a potentially large class of candidate “true” processes.
ceiving given that the husband and wife are attempting to have a child. This rate parameter, \( \psi \), is essentially a biological one, and we fix its value at a consensus value found in the reproductive biology literature. Thus there are no new parameters to estimate with respect to the model which conditions on the presence of a child, so that the fertility process itself only sheds light on the relationship between the income process for the two parents, the marriage quality process, and the initial conditions distribution for the parents (all given the extant family law environment). The income processes of the parents are taken to be exogenous (except that a shift in the mother’s income process is allowed at the time of giving birth to a child), and as such these processes are precisely estimated no matter what the details of model structure. However, we know that the marriage quality process and the initial distribution of child quality are particularly difficult to precisely estimate. The marriage quality state mainly serves to move people into the divorce state, though it does also have some impact on the child quality investment process, which is indirectly observable through changes in the test score. By adding the fertility decision, we add another threshold marriage quality must cross (given parental income levels) to induce the parents to attempt to conceive a child. Thus second threshold provides some additional information on the marriage quality process.

The initial child quality distribution is key for the identification of the child quality process parameters, and would be essential if there was only one test score measure per child. Because it plays a role in the fertility decision (spouses having higher (rational) expectations regarding initial child quality will be more likely to have a child conditional on their income levels and marriage quality), making fertility endogenous allows us to gain considerably more information on these distributions than we can from backwards extrapolation of the (endogeneously-determined) test scores in the data.

5 Data and Descriptive Statistics

The estimation employs the main sample of the 1979 cohort of the National Longitudinal Survey of Youth (NLSY), and the associated Child and Young Adult Data. The latter follow any children of the female respondents from the nationally representative NLSY. NLSY women are 14 to 21 years of age at the date of the first interview. We observe them through the 2004 interview, at which point they are 39 to 46 years of age, and nearly all sample members who will ever have children have at least one child. Our sample consists of the NLSY women who ever marry within the sample window, and their families. However, our model cannot address nonmarital fertility, and so we exclude from the sample any woman who has a first birth before her first marriage. The 2002 National Survey of Family Growth shows that 19 percent of ever married US women had a first birth before first marriage.\(^{30}\)

\(^{30}\)Blau and van der Klaauw (2008), however, illustrate the importance of single motherhood in U.S. children’s family structure experiences, and demonstrate a substantial difference among demographic groups in the role of non-marital fertility. Hence our sample restriction may not be innocuous regarding the ability of the estimation to reflect the experiences of U.S. children across demographic categories.
Table 1 describes the construction of the estimation sample. Of the 12,686 original NLSY respondents, 6283 are women. Of these, 5188 ever marry, and of these ever married women, 1167, or 22.49 percent, experience a first birth before marriage, leaving 4021 sample-eligible women and their families. We drop 686 women from the sample based on insufficient marriage or fertility history data, as well as 489 women who have children but no observed child test scores, described below. This leaves 2846 sample-eligible women who ever married, had no children before the first marriage, and have sufficient data on marital status, fertility and child test scores. Finally, we require data on the wife and husband’s age, education, state of residence and income at the date of marriage, the wife’s 1980 Armed Forces Qualification Test (AFQT) score, and the child’s gender for inclusion in the estimation sample. Our final estimation sample consists of the 1643 families who meet each of these sample requirements.

The child outcome measure employed in the empirical analysis is based on the child’s score on the Peabody Individual Achievement Test (PIAT) in mathematics. The PIAT is administered to all children aged five and older in the NLSY Child sample, and is ceased when children exit the Child sample and enter the Young Adult sample at the age of 14. In order to include children who reach the age of five during the sampling widow and are born to mothers with as broad a range of ages as possible, we collect the child’s test score in the first year in which the child undergoes the PIAT mathematics assessment. A follow-up test score is available for more than nine-tenths of the families that meet all sample criteria. The Child Survey is biennial and begins in 1986, and child PIAT assessments for on the order of 10 percent of eligible children per wave are missing for a variety of reasons.31 As a result, sample children are assessed the first time at a mean and median age of 6 years, and the second time at a mean and median age of 8 years; standard deviations for test ages are non-negligible, at roughly one year in each case. As discussed above, our model estimation exploits interview- and age-based variation in test timing as a source of (arguably) exogenous variation in observation ages, which allows us to learn more about the dynamics of children’s progress than one might from a fixed testing date. The test score measure employed in the estimation and policy experiments is the child’s age-specific PIAT mathematics percentile.32

Spouses’ incomes are measured at survey frequency, starting from the date of marriage. A common difficulty faced by empirical studies of married and divorced parents’ interactions with their children is tracking divorced fathers who no longer reside with their children. The appeal of the NLSY in this regard is that it allows us to observe families from before birth of the child, and therefore we have some information on each sample father no matter how quickly the family dissolves after the birth of the child. Each spouse’s income is determined as the sum of reported

31 See the NLSY 1998 Child and Young Adult Data User’s Guide pages 64-66 and Tables 11 and 12 for details on missing PIAT assessments. Reasons include "inadvertently skipped" children, "interior completion status of their mothers", and interviewer failures to follow scoring decision rules, among others. Some of these issues improved with the introduction of CAPI techniques in 1994. In general, compliance rates are strong for a survey-administered test of ability.

32 The age norming performed in the 1979 NLSY Child Data uses an age-based norming sample from 1968. Therefore, NLSY sample children show average scores that exceed those of the norming sample. A useful reference on this point is Dunn and Markwardt (1970).
incomes in the NLSY that are attributable to the individual spouse and not to the household or to her or his partner. Attributable income sources are wage and salary, farm and business income, military income, and unemployment income. Spouses’ incomes are inflated to 2004 dollars for the purposes of reporting and estimation.

The estimation relies on marital status at various points in time, some of which depend on birth and test dates, which vary across families. Hence we require complete marital status histories for all sample families throughout their relevant observation windows. Divorce indicator \( d \) is equal to zero if parents remain married and not separated from the first interview after the marriage through the relevant date of observation. Otherwise, it is one. As discussed in Section 4, the probability distribution of initial child quality \( k(0) \) is permitted to rely on a vector of characteristics of the parents and child observed at (or, in one case, before) the date of marriage. Characteristics entering \( Z_k \) are a constant, the mother and father’s ages, the mother and father’s years of schooling and the mother’s score on the Armed Forces Qualifications Test (AFQT) administered to NLSY respondents in 1980.

Like the initial child quality, the probability distribution of the quality of the parents’ marriage at the child’s birth relies on a vector of characteristics of the parents. Characteristics entering \( Z_\theta \) are a constant and the mother’s and father’s ages at marriage. Following the initial \( \theta(0) \), we assume that marriage quality improvements and setbacks occur exogenously at rates \( \gamma^+ \) and \( \gamma^- \).

Marriage dissolution standards were gradually liberalized over the 1970s and 80s at the state level. Where previously the consent of both spouses was required for a divorce to be obtained, a regime labelled bilateral divorce law, many states adopted unilateral divorce laws under which one spouse’s request for a divorce was sufficient. Friedberg (1998), Gruber (2004) and others have examined the implications of these law changes for the divorce rate and a set of outcomes realized by children of divorce. In our sample, 44 percent of couples live in unilateral divorce states at the time of marriage. We use divorce standards at the time of marriage as a source of exogenous policy variation in our estimation; divorce decisions in the estimation follow a unilateral standard, in that only one spouse need prefer a divorce for it to occur, for couples in unilateral states, and they follow a bilateral standard, in that both spouses must consent in order to divorce, for couples in bilateral divorce states.

The estimation requires data on the child support payment that will be required of the father in the event of divorce, and works on the assumption that parents are able to predict child support arrangements while still married. The history of child support guidelines suggests that predicting child support from the vantage point of marriage was reasonably feasible for most families in our sample. Two federal reforms, the Child Support Enforcement Amendments of 1984 and the Family Support Act of 1988, established and extended the requirement that states maintain and use set guidelines for the determination of child support. Some states had guidelines in place before the reforms. Therefore most sample parents had access to state child support guidelines from the birth of their children. While state guidelines have changed little since the second reform, some variation in the application and formulas of state guidelines precedes the second reform. We find that current
formulas match 1988 formulas for most states. Therefore we determine child support rate $\pi$ for each couple based on the spouses’ incomes and the current child support guidelines for the state in which the couple resided at the date of marriage. Child support formulas are coded by hand, and so where obvious differences exist between current guidelines and the historical guidelines to which particular families would have been subject at the time of the child’s birth, appropriate corrections have been made. We apply the guidelines for one child families in which the father enjoys 20 percent of physical placement. Though the resulting assignment of projected child support rates is imperfect, we feel it provides a rich and reasonably accurate picture of the cross-state variation in the child support levels to which sample families would have been subject. The resulting mean and median child support rate is 20 percent, with masses also at 0, 17 and 25 percent. The mean annual child support dollar amount is 5819 2004 US dollars.\footnote{The authors thank Hugette Sun for sharing her extensive research on state child support guidelines and her painstakingly assembled guideline database. More details on this can be found in Sun (2005).}

Table 3 contains the descriptive characteristics of the variables used in the estimation for the two samples. At the median, husbands and wives are 25.7 and 23.2 years old at marriage. Between 1979 and 2004, 39 percent of the sample of ever married NLSY women experience divorce. Median time to divorce conditional on ever divorcing is 6.7 years. Further, 70 percent of the (ever-married) sample women have a child within the observation window. The median time from marriage to birth is 2.7 years. The 30 percent of households that we observe who are childless, as well as the parent households that we observe pre-and post-birth, are crucial to the identification of both fertility processes and the role of child quality in marital status decisions, as discussed in the previous section. Households with children that do and do not encounter divorce provide information on fertility and marital status choices, and in addition on the child quality production processes in marriage and divorce. We observe substantial populations of households in childless, married parenting and divorced parenting circumstances.

Husbands and wives’ average education levels are 13.15 and 13.30 years, respectively, and their average annual incomes at marriage are 31,723 and 18,750 2004 US dollars. By sample construction, we have first test scores on the 70 percent of households with children. We observe second test scores for 64 percent of the sample, or 91 percent of sample households with children. Sample median age-normed test scores are 64 and 62 at the first and second observation, respectively, which may be compared to a median of 50 in the 1968 US testing population used for age-norming. Further information on fertility and divorce dynamics in the sample may be inferred from the sample moments appearing in Table 6.

6 Empirical Results

In order to generate the simulated sample paths used in the MSM estimator, we must specify the family law environment each family faces. We fix the policy parameter $\tau_1(1)$ at a value intended to reflect average outcomes in terms of custody/visitation arrangements. In this exercise, it is set at 0.2, so the mother is assumed to be in contact with the child 80 percent of the time in the
divorce state. Custody averages over the period in which we observe our NLSY sample have been studied for eight states. All but California maintain approximately 80-20 custody division averages; California’s custody decisions favor fathers substantially more than those of other states, with as much as 40 percent custody going to fathers on average.\footnote{See, for example, Cancian (1998) on states’ custody averages.}

Durations are measured in years. The instantaneous discount rate $\rho$ is fixed at 0.05. We assume $\eta = 0.06$, implying an average age for the termination of investment productivity of between 16 and 17. We set $T = 10$, $M = 5$ and $B = 5$, as discussed in Section 3.4. Parents’ incomes are measured in units of $1000 2004$ dollars. Simulated moments are based on $R = 30$ replications per family per $(k(1), \theta(1))$ pair, or 1500 replications per family.

The model relates exogenous household characteristics $X = \{y_1, y_2, a, \zeta, \theta(1), k(1)\}$ to outcomes $n(j), k(j)$ and $d(j)$ for a given family. Therefore the moments we choose pertain to the relationship between parents’ incomes, children’s test ages, determinants $Z_\theta$ of marriage quality and determinants $Z_k$ of child quality and the fertility, child attainment and divorce outcomes; the outcome averages for the full sample; and higher-order interactions among the outcome measures and elements of $X$ and $Z$. Overall we have selected 86 moments as the basis for our estimator. The moments that we attempt to fit are described in Table 6. Note that we choose to measure and simulate unconditional moments in most instances, due to the complication associated with simulating and evaluating conditional moments across family-initial condition combinations that are each associated with unique weights. However, the moments chosen contain information equivalent to conditional moments where, for example, one compares moments that condition on marital status to unconditional moments involving products of $d$ and elements of $n, k, X$ or $Z$.

The vector of parameters estimated using our MSM procedure govern the parents’ preferences, the production of child quality, the income process and the relationship of characteristics of the parents observed at or before marriage to the initial marriage and child qualities. The complete vector of parameters we estimate is $\Gamma = \{\alpha_1 = \alpha_2, \tilde{\delta}, \nu, \sigma, \zeta, \beta_\theta, \mu_k, \sigma_k, \gamma^+, \gamma^-, \xi, \lambda, \beta_s\}$. In all, $\Gamma$ contains 31 free parameters, which the estimator identifies based on the 86 moments.

The parameter estimates are reported in Tables 4 and 5. The point estimate of $\nu$ is 0.498, and, given its estimated standard error, we can conclude that the stochastic production function is strongly concave in total parental investments in the child. The precisely estimated $\tilde{\delta}$ estimate of 0.532, together with our estimate of $\nu$, indicates a high degree of efficiency of parental investments in the production of child achievement. Our estimate of $\sigma$ is 0.466, which indicates a decrease in child quality every 2.15 years on average. An investment level of roughly one third of total family income is somewhat common in the solutions. At this investment level and the estimated parameters, the child quality improvement rate is enough to offset the setback rate for all but the lowest marriage quality level.

We find that marriage quality shocks are fairly frequent; on average, both improvements and setbacks hit $\theta$ about every four years. The estimated timing of these shocks is closely tied to the frequency of divorce.
The estimated spouses’ preference weights associated with own consumption are similar, at 0.488 for the father and 0.494 for the mother. Since we lack direct consumption and investment data, the similarity in these weights derives from the similarities between mothers and fathers in terms of moments reflecting the interaction between child outcomes and parent incomes and educations.

Estimates of $\mu_{k1}$ and $\mu_{k2}$ choose the second child quality type as the high child quality type. The difference between the mean of the initial child quality distributions for the two groups is quite large, with $\mu_{k1} = 4.152$ and $\mu_{k2} = 22.289$ for the type one and two groups, respectively, indicating a roughly centered child quality distribution for the first type and the majority of the probability weight over the maximum child quality for the second type. The $\beta_k$ estimates indicate that the second type is very rare, though the probability of being of initial child quality type 2 is increasing in the mother and father’s education and the mother’s AFQT score. The more common type 1 initial child quality distribution places the most probability weight on the child quality levels associated with the 40th and 50th percentiles of the test score distribution. While the point estimates produce only a limited role for parents’ abilities in the determination of (unobserved) initial child quality, they also allow parental ability to influence child attainment through the effects of income (and education- and test score-derived income types) on the child attainment process. Our estimates of $\beta_g$ indicate that spouses’ ages at marriage have a modest, precisely estimated and roughly symmetric positive effect on the stability of the marriage.

The estimated utility shifter in the presence of children is negative and differs significantly from zero at the one percent level. This free parameter entering the instantaneous utility of the spouse only in the presence of a child was free to reflect independent utility gains or losses from parenting, and is estimated as a negative 0.885. This estimated utility loss from the presence of a child is large. At the estimates, its magnitude is similar to the welfare loss for a wife associated with going from consuming all of the 70th percentile income level in our sample to consuming all of the 30th percentile income level in our sample. The reasons for this prediction are straightforward. Relative consumption and child quality weights estimated for the model generate large utility gains over the long term in the presence of a child. In order to match the roughly 70 percent 10 year fertility rate observed in the data, the model requires an offsetting utility cost of childrearing.

Turning to the income process, the $\epsilon$ estimate of 0.702 implies that roughly 70 percent of mothers experience a one unit setback in their current income level following the birth of the child. Recall that this is the only manner in which parenting is parameterized to enter the income process. This setback rate is used to fit the decline in average wife’s income after five years of marriage from $22,097 in the full sample to $18,907 among those with children by year 5, as evident from moments 7, 17 and 53, and from $22,252 for the full sample to $18,840 for mothers by the 10th year of marriage, as evident in moments 8, 22 and 57. The $\beta_1$ and $\beta_2$ vector estimates place higher probability weight on a husband being of income type 2 as his education increases, and, similarly, higher probability weight on a wife being of income type two as her education and test score increase. The type 1 income process for husbands involves income improvements and setbacks each arriving at
a rate of roughly once every two and a half years, but the slightly higher improvement rate predicts long-term real income growth for husbands of this type. Type 2 husbands experience substantially more income growth and somewhat higher income volatility, with improvements arriving every two years on average and setbacks every two and a half years. The two types’ income processes differ much more for wives. Wives with type 1 incomes experience steady income growth and low volatility. They experience an improvement every three and a half years and a setback every four and a half years. However, type 2 wives’s incomes are both volatile and declining, with improvements arriving every 11 months and setbacks arriving every 10 months, on average.

The 86 data and simulated moments listed in Table 6 give an idea of the fit of the model. Overall, the simulated moments match the patterns in the data reasonably well. The first four moments, divorce rates at 1, 2, 5 and 10 years, and the next four moments, proportion of families with children at 1, 2, 5 and 10 years, give a measure of the model’s ability, with 31 parameters, to fit the complex dynamics of marital status and fertility. Sample divorce rates are 1, 3, 15 and 26 percent in the first, second, fifth and tenth years of marriage; simulations at the estimates produce analogous divorce rates of 3, 6, 11 and 22 percent. Fertility rates at the same interval are 18, 34, 60 and 69 percent in the data and 29, 41, 63 and 76 in the simulations. From the following four interaction moments, we see that rates of divorced parenting at 3, 5, 7 and 10 years are fit even more accurately, with divorced parenting rates of 1, 4, 6 and 11 percent in the data and 1, 4, 6 and 13 percent in the simulation.

The overall average first observed test score in the NLSY sample is 4.41; we simulate an overall average test score of 4.55. The mean gain from first to second test score is 0.0943; we simulate it to be 0.0839. The level and growth of test scores being simulated in the model hence appears to represent the data well. Further, the simulations closely match the variation in test scores with parents’ marital status that we observe in the data. For example, the average actual and simulated test score gains for children whose parents are married at the first test and divorced at the second are 0.0097 and 0.0073, respectively. The ability of the model to match these differences provides some encouraging feedback regarding the assumptions on the structure of the child quality production function and its relationship to the marriage state.

With only nine parameters devoted to the income process, tasked with generating flexible income paths for both husbands and wives over many years and across substantial life events, the model appears to fit observed incomes extremely well. Moments 13 through 24, for example, demonstrate a close fit of simulated to data income level and variability at marriage years 2, 5 and 10. Higher order moments in income, such as moments 52 to 59 and moments 70 to 81, reveal similar accuracy.

Since most simulated moments accurately match their sample analogues, we turn to what appear to be the biggest misses. Moments 32, 33, 60 and 66 (test score changes × income or income changes, and year 5 divorce rate × child support dollars) appear to be off, but once one considers the sample variability in these moments as a result of the scale of incomes, their accuracy appears to be in line with the other moments. Moment 64, the squared test score difference, indicates that, while the simulations replicate the mean test score change well, they underpredict the sample’s level of test
7 Comparative Statics and Welfare Analysis

Given the estimates of the primitive parameters that characterize the model and the distributions of exogenous sample characteristics, we now turn to examining the implications of the model in terms of (i) how decisions and welfare outcomes change with respect to modifications of the family law environment and (ii) the definition of and characteristics of an “optimal” family law environment in the context of this model.

7.1 Comparative Statics Exercises

The comparative statics exercises use simulated family histories generated at the point estimates of the model parameters to establish baselines in terms of the joint divorce, fertility, and child outcome distribution. We then examine the impact of changes in fundamental and family law parameters on these outcome variables of interest. Fundamental parameters are the “true” primitive parameters of the model, those which are invariant to policy change and that are not capable of being manipulated by household members or a social planner. The family law parameters, instead, are taken as given by the spouses, but are the policy instruments of the social planner.

In particular, we examine the impacts of both the rate of arrival of marital quality innovations and the family law environment on the outcomes listed below:

- The proportion of households divorced after 10 years of marriage.
- The proportion of marriages with children 10 years from the date of marriage.
- The average child quality at the completion of the investment process.
- The proportion of (former) couples engaged in divorced parenting 10 years from the date of marriage.

For the purpose of examining the impact of marriage stability and the family law configuration on these outcomes, we take as a baseline the status quo child support orders (π) inferred from state-level child support guidelines in combination with family characteristics, status quo state-level marital dissolution standards, and an assumption, based on the above-cited references, that the father is allocated a proportion of the child’s time (τ₁(1)) approximately equal to 0.20.

In calculating these effects, we employ an initial distribution of household characteristics consistent with those in our sample. That is, for each household in our sample, we generate \( R = 1500 \) sample paths under family law regime \( F \). We examine the state of each of the \( N \times R \) sample paths at 10 years after marriage to compute the divorce and fertility proportions. When a child is born during a sample path, we examine the child’s final quality level.
First, we look at the effect of changing one primitive model parameter, the rate of arrival of marriage quality shocks (\(\gamma\)), on the outcomes listed above. The role of marriage stability in determining fertility choices, child investments and eventual child outcomes is of central interest in our analysis. Table 7 reports the simulated outcomes as we vary \(\gamma\) around its point estimate of 0.23. The directions of the effects of marriage quality update rates are reasonably predictable. A very low arrival rate of marriage quality shocks, for example 0.01 in the reported simulation, is associated with no simulated divorce 10 years after marriage. This stable marriage case is associated with considerably higher fertility and average child quality at independence than those we observe in the data or in our baseline simulations. As the marriage quality update rate increases, the divorce rate increases, fertility decreases and average child quality at independence decreases. These gradients are fairly steep. As we move from extremely rare marriage quality updates to improvements and setbacks each occurring at a frequency of 2.5 years on average, the divorce rate grows from 0 to 35 percent, the 10 year fertility rate falls by roughly ten percentage points, and average child quality at independence falls by nearly 0.6 deciles. The divorce rate change far outpaces the fertility change, and so on net the rate of divorced parenting grows substantially as the rate of marriage quality updates rises. Overall, we find that predicted outcomes for the family are quite responsive to changes in the primitive parameter of the model of greatest relevance to our policy analysis, and that these responses follow intuitively plausible patterns.

Second, we turn to the effects of parameters of the problem that may be manipulated by policymakers. The family law configurations we examine aside from the baseline involve the manipulation of either child support or custody (in the form of physical placement) while leaving all other policy parameters at their status quo levels for each family. In the child support experiments, we examine divorce, fertility and child quality outcomes when all families anticipate a child support transfer rate from the father to the mother of:

- \(\pi : 0, 0.10, 0.17, 0.20, 0.25, 0.30\).

These child support levels contain many that are relatively common in state guidelines. In the custody experiments, we examine the same divorce, fertility and child quality outcomes when the father’s share of time with the child in the divorce state is

- \(\tau_1(1) : 0, 0.1, 0.2, 0.3, 0.4, 0.5\).

In the end, we evaluate family outcomes for each of 12 different simulated policy manipulations.

Table 8 contains simulated outcomes in the custody (interpreted as physical placement share) experiment. We see a non-monotonic but reasonably clear increasing pattern in divorce rates with paternal custody share, from 10.65 percent under zero paternal custody to 23.50 percent under shared custody. Similarly, there is a not everywhere monotonic but still sizeable increase in fertility rates with paternal custody. Overall the trend in terminal child quality is also positive with paternal custody share. It is not clear what drives the large child quality jump from 40 percent paternal to shared custody. This might demonstrate some source of excess sensitivity of the investment process
to custody in the model. Finally, we note that divorced parenting is most common in the shared custody regime, where both fertility and average terminal child quality are also at their highest levels. This suggests that divorced parenting, often targeted for reduction by policymakers, may not always be damaging to children’s (and parents’) welfare.

Table 9 shows simulated outcomes in the child support experiment. Recall that, while custody enters the estimation through the imposition of a single, reasonably representative value of $\tau_1(1)$, child support variation informs the estimation via state-specific and highly nonlinear guidelines applied at the level of the individual family. Overall, the responsiveness of divorce, fertility and child attainment to child support orders is predicted to be considerably more modest. We find an extremely modest (and nonmonotonic) increase in child quality with child support levels. This may result from the offsetting effect of one parent’s investments in the child on the other parent’s investments that we observe in the solution matrix, which is somewhat evident in Figure 4. As the child support order transfers dollars from fathers to mothers, the model in many instances predicts roughly offsetting investment changes by mothers and fathers. Divorce and fertility rates are also less responsive to this source of policy variation, with perhaps a weak decrease in both divorce and fertility rates as the child support order climbs from zero to 30 percent of the father’s income.

### 7.2 Welfare Analysis

In order to perform the estimation exercise we have endowed each agent in the model with a cardinal utility function. As a result it is natural to consider collective welfare issues using a Benthamite approach in which we seek to maximize

$$W(F) = \zeta_1 V_1(F) + \zeta_2 V_2(F) + \zeta_k V_k(F),$$

where we normalize $\zeta_1 + \zeta_2 + \zeta_k = 1$, with each welfare weight $\zeta_j$ being strictly nonnegative. If each of the $V_j$ are well-defined, then given the utility functions of the agents, a set of behavioral rules they use within any family law environment $F$, their endowments, and a vector of welfare weights $\zeta$, then we will say that an optimal family law environment $F^*$ is

$$F^* = \arg \max_F W(F).$$

Of course, to give any content to the exercise we first must rigorously define the set of family law environments from which the planner can choose. The characterization of the family law environment will be the one considered throughout the paper. While admittedly very limited and stylized, it nevertheless covers some of the most important dimensions of family law.

There are a number of problems with implementing this social choice analysis in the context of our modeling framework. While welfare analysis is often conducted under the assumption of perfect certainty, our environment is highly uncertain. Since $W$ is a linear function, it is natural to simply replace known payoff values with their expectation, so that the expected welfare associated
with $\mathcal{F}$ is
\[ EW(\mathcal{F}) = \zeta_1 \text{EV}_1(\mathcal{F}) + \zeta_2 \text{EV}_2(\mathcal{F}) + \zeta_k \text{EV}_k(\mathcal{F}). \]

In our case, however, these expectations are not representable as analytic functions of $\mathcal{F}$. The expectations can only be approximated by averaging over simulated sample family histories, a form of Monte Carlo integration.

Another practical problem arises in solving for $\mathcal{F}^*$ since not all of the dimensions of $\mathcal{F}$ are continuous. The space of divorce “types,” child support tax rates on noncustodial parents, and contact time proportions is $\mathcal{F} = fB;Ug[0;1]^2$. Clearly the divorce law type - bilateral or unilateral - is binary, so that maximum of $EW(\mathcal{F})$ would be determined by maximizing over $\pi$ and $\tau_1(1)$ for each law type, and then choosing the one system that was associated with the highest value of $EW(t, \pi^*(t), \tau_1(1)^*(t))$, where $t = B, U$. Given the fact $\text{EV}_j, j = 1, 2, k$ are extremely complex nonanalytic functions of the family law environment, there is essentially no hope that gradient-based methods can be employed to find the solutions from the pair of first order conditions given by
\[
\frac{\partial EW(\mathcal{F}^*(t))}{\partial \pi} = 0, \ t = B, U \\
\frac{\partial EW(\mathcal{F}^*(t))}{\partial \tau_1(1)} = 0, \ t = B, U.
\]

In light of these numerical difficulties, our method for finding a solution to the optimal policy problem is to conduct a grid search along the $\pi$ and $\tau_1(1)$ dimensions at intervals of 0.1. Because of this, we cannot claim to have found an optimal policy over the entire space $\Omega_\mathcal{F}$, but instead only over the discrete set $\hat{\Omega}_\mathcal{F} = \{B, U\} \times \{0, 0.1, ..., 1.0\}^2$. Because $EW(\mathcal{F})$ is finite for all values $\mathcal{F} \in \hat{\Omega}_\mathcal{F}$, and because the set of outcomes is finite, there exists a maximum $EW(\mathcal{F}^*)$ which is determined by evaluation of $EW(\mathcal{F})$ for all $\mathcal{F} \in \hat{\Omega}_\mathcal{F}$. The likelihood that there exist two or more elements of $\hat{\Omega}_\mathcal{F}$ that yield the same value of $EW(\mathcal{F})$ is arbitrarily close to 0. In this sense, we can say that there exists a unique optimal policy $\mathcal{F} \in \hat{\Omega}_\mathcal{F}$ almost surely.\(^{35}\)

The final practical problem is determining the point of evaluation of the welfare of each individual (potential) household member. Any policy environment will impact the path of payoffs to each family member over the planning horizon of the model. For example, one criterion could be the welfare of the husband and wife (and the child, assuming it exists) at $t$ years from the marriage. This obviously would yield a very limited picture of the effect of the environment on lifetime welfare. In particular, certain types of family law environments might yield a “good” distribution of outcomes at point in time $t$ but a “poor” distribution of outcomes at point in time $s, s \neq t$.

There are two general approaches to dealing with this issue. One is to utilize the expected value

\(^{35}\)Of course, the downside of this existence and uniqueness is that it is specific to the choice set $\hat{\Omega}_\mathcal{F}$, in the sense that if we defined another ‘grid’ over the $\pi$ and $\tau_1(1)$ space $\check{\Omega}_\mathcal{F}$, it may well be the case that $\arg \max_{\mathcal{F} \in \check{\Omega}_\mathcal{F}} EW(\mathcal{F}) \neq \arg \max_{\mathcal{F} \in \hat{\Omega}_\mathcal{F}} EW(\mathcal{F})$. In any practical application of this exercise this is not really problematic (given sufficient computing power), since institutional agents are likely to choose child support ‘tax’ rates and contact times with the child from the set $\{0, 0.01, 0.02, ..., 1.0\}$.
of the initial marriage quality. This is an ex ante measure of welfare, that computes the expected value of the marriage “career.” In this case, the social welfare function explicitly only considers the payoffs of the husband and wife, and not of a child which may or may not be born. This is advantageous methodologically since we do not have to explicitly include the welfare of agents that may or may not exist along different sample paths. Nevertheless, the family law environment will have an impact on the welfare of children indirectly through the welfare of their (potential) parents. In this case, the ex ante expected welfare function has a value given by

\[ W(\mathcal{F}) = \zeta_1 V_1(\mathcal{F}) + \zeta_2 V_2(\mathcal{F}), \]  

(10)

where \( \zeta_1 + \zeta_2 = 1 \) and where the \( V_j(\mathcal{F}) \) is the average beginning of marriage valuation across all of the population types (distinguished in terms of initial state variable vectors). As we have said, variations in \( \mathcal{F} \) will in general result in different fertility rates and different distributions of the terminal value of child quality. In this way, impacts on child welfare, including simply birth itself, can be evaluated.

An alternative valuation method is more ex post in nature. In this scenario, we explicitly include the child’s welfare in the social welfare function, and immediately face the problem of how to value the child’s utility at any point in time when it is not alive, as well as to take a stance on the utility payoff of children who are alive and who in the model are only differentiated in terms of their quality state \( k \). We assume that the lowest quality state is \( k = 1 \), and let us assume that the child’s utility is determined by its current quality level in the same fashion as it impacts the parent’s payoff, that is, \( u_3 = \ln(k) + \zeta \), where the child, should he or she exist, is indexed as family member 3. Then the minimum value of \( u_3 \) for a living child is \( \zeta \), while the maximum value of utility is \( u_3 = \ln(\max k) + \zeta \), which under our scaling is \( \ln(10) + \zeta \). We evaluate the welfare of parents and (potential) children over a continuum of possible sample paths that could be realized under a set of initial conditions specific to the marriage unit and a family law environment \( \mathcal{F} \). Along any sample path, let the utility flows of the spouses (agents 1 and 2) at any moment in time \( a \) be given by \( u_j(a; \mathcal{F}) \), \( j = 1, 2 \), and where it is understood that the sample path extends to infinity, that is, past the date at which the active child investment process terminates. Then

\[ EU_j(\mathcal{F}) \equiv \int_0^\infty \exp(-\rho t) u_j(t; \mathcal{F}) dt, \quad j = 1, 2. \]

The valuation of the (potential) child’s expected utility is more problematic, naturally. In the unborn state, we assign a utility to the child equal to the lowest possible value when alive, which is \( \zeta \). If the child is never born, we assume it realizes a value of \( \zeta \) at each point in time over its potential life, so that its total value is \( \zeta / \rho \). Instead, if the child is born, along a given sample path its utility flow is \( \zeta \) until the time it is born, and is then given by \( \ln(k(a, \mathcal{F})) + \zeta \) for all \( a \) in the active investment period for the child. If we let \( a_T \) denote the time at which the investment process ends along a given sample path, then we assume that the utility flow for all times past the end of the investment process is \( u_3(a) = \ln(k(a_T)) + \zeta \), \( a \geq a_T \). Given these not uncontroversial assumptions,
the computation of the (potential) child’s expected utility is accomplished in exactly the same way as it was for the parents, with the (potential) child’s expected utility given by $EU_3(F)$. Then the social welfare function in this case is given by

$$EW(F) = \zeta_1 EU_1(F) + \zeta_2 EU_2(F) + \zeta_3 EU_3(F).$$

(11)

There is no reason to believe that the family law environment that maximizes (10) will also maximize (11). A crucial difference in the social welfare functions is the way in which (potential) children are treated. This is a difficult problem, and two of the best treatments of it can be found in Blackorby et al. (1995) and Golosov et al. (2007). The Blackorby paper assumes cardinal utility, which makes interpersonal welfare comparisons possible and enables the use of a social welfare function, just as we have done here. The Golosov et al. (GJT) treatment is more general. They are able to examine efficiency issues without assuming cardinal utility by exploiting a dynastic modeling structure. When comparing two dynastic allocations, one welfare comparison only considers the welfare of agents that are born in both allocations (this the authors refer to as $A$-efficiency), while the other efficiency criterion involves the comparison of the welfare of all (potential) agents (this is referred to as $P$-efficiency). Our policy analysis combines elements of both of these papers. The cardinal utility assumption allows us to utilize a social welfare function to determine optimal policies. In terms of the specification of the social welfare function, under (10), only the welfare of agents alive in all possible states of the world (i.e., sample paths) are explicitly considered, with the welfare of born and unborn children only entering through their impact on parental welfare. Under (11) we have a situation more similar to that associated with $P$-efficiency in GJT., with all agents, including those unborn, explicitly appearing in the welfare function. Of course, this requires us making strong assumptions on the welfare of these agents in the unborn state.

Given the great degree of arbitrariness in defining ex post welfare measures, we have opted only to analyze policy choices utilizing the $A$-efficiency type measure which includes only the spouses’ welfare. We look at optimal policies for three values of the welfare weight $\zeta_1$, which are 1, 0, and 0.5. The first exercise only considers the welfare of the husband when defining optimal family law policy, the second considers the welfare of only the wife, and the third gives the spouses equal weight. We only seriously consider the configuration of family law policy when $\zeta_1 = 0.5$; the other two exercises provide us with some indication of the impact of the trade-offs in spousal welfare that lead to the results obtained in the $\zeta_1 = 0.5$ case.

We find that the expected welfare maximizing family law structure under equal weighting of spousal expected welfare is (1) bilateral; (2) an even split of time with the child in the divorce state, i.e., $\tau_1(1) = 0.5$; and (3) a child support “tax” rate of $\pi = 0.2$. We now try to provide some rationale for these findings based on our model structure and the estimated model parameters.

In terms of the finding that bilateral divorce is optimal, we note that there are extremely small differences in welfare values between the unilateral and bilateral cases. Wives tend to have a very slight preference for the bilateral standard, and husbands for the unilateral standard. Averaged spouse welfare is almost unresponsive to the marriage dissolution standard. As discussed previously,
there is little difference in outcomes under the two regimes where both spouses tend to share the same relative valuations of divorce versus marriage, which is one interpretation of our finding.

In terms of the determination of optimal custody/contact time, the 50-50 split (which is at the boundary of the choice set that we endowed the institutional agent with) is primarily produced by the estimates we obtained of the valuation of child quality by the parents, $1 - \alpha_1$ and $1 - \alpha_2$. On average, fathers have higher incomes than mothers, so that to encourage spending on the public good, the child, it will be optimal to give the greatest incentives to investment to the spouse with the higher income if the parents have the same baseline level of preference for child quality. However, if the mother were to show substantially greater valuation of child quality than the father, which is allowed in our parameterization of the model, this could result in a situation where father’s contact time is optimally set at a value considerably lower than that of the mother. Our estimates of $\alpha_1$ and $\alpha_2$ are quite similar, and this is the main reason that fathers are given so much contact time.

In Figure 5 we present the relationship between expected welfare, custody arrangements, and child support orders. We see in the figure the overriding importance of custody arrangements in the determination of optimal family law structures. For each of the custody arrangements considered, any value of the child support order that we consider produces a welfare outcome greater than the one associated with any combination of a lower contact time with the father and child support transfer. The importance of the custody arrangement is due to the fact that there is no recontracting possibility available on the time dimension. Ordered child support amounts may have little impact on the welfare of a payer, for example, if he is able to reduce his child investment accordingly with the mother increasing the amount she invests to “make up the difference.”

We see in Figure 5 that the optimal amount of the child support order is a function of the custody arrangement. When fathers are given low amounts of contact time, they have little incentive to invest in the child and hence are often found at the corner solution of zero investment. In this case, to increase the welfare of the mother and child, orders should be set at a high level to give the mother a bigger endowment from which to invest in the child when she is the sole investor. At the custody level of 0.5, the optimal child support tax rate is 0.2, while at custody level (for the father) of 0.2, the optimal order rate is 0.25.

In Figure 6 we illustrate the tension between the objectives of husbands and wives in the determination of family law. Here we look solely at the determination of the optimal child support order given a 50-50 split in the custody of the child upon divorce. We see that the husband’s expected welfare is a monotonically decreasing function of the child support tax rate, while the mother’s expected welfare is monotonically increasing in this rate. Only by averaging the expected welfares do we arrive at an optimal tax rate on the interior of the choice set of the institutional agent. This gives us some indication of the sensitivity of the optimal family law environment to the preference weights given to wives and husbands.
8 Conclusion

We have developed and estimated a continuous time model that allows for strategic behavior between parents in making fertility, child quality investment, and divorce decisions. An important component of the behavioral model is the family law environment, which has a large impact on the rewards attached to the marital states and, in turn, the returns to investment in child quality. We use data from the Mother-Child subsample of the NLSY to estimate model parameters using a Method of Simulated Moments estimation procedure. We find that the parameter estimates are roughly in accord with our priors, and that the correspondence between simulated and sample moments varies between “adequate” to “good.”

The most important contribution of our work is to the understanding of the dynamic relationship between divorce decisions and the evolution of fertility and child quality, and the dependence of this process on family law parameters. While there is a well-established empirical relationship between child outcomes and the characteristics of the household in which she or he lives, we have attempted to disentangle the simultaneous relationships between divorce, fertility, and child development using a behavioral model of these decisions. To our knowledge, this is one of the first studies to link the family law environment to the fertility decisions of intact families, and, in some instances, we find the link to be substantial.36 While our estimated model is based on a number of restrictive and ultimately untestable assumptions, our view is that this type of framework is the only way to begin to understand the complex dynamics present within married households.

We have conducted some initial investigations of how substantial changes in the parameters characterizing the family law environment - those reflecting contact time between divorced parents and the child and the child support transfers between parents - impact the parental welfare distribution and child outcomes, which include birth. To date, our experiments suggest small to moderate impacts of changing the family law environment on the average value of child quality in the population (though the impact on the number of children born can be great in some circumstances). Instead, the concurrent impact on the welfare distribution of parents is substantially greater, with custody arrangements dominating child support in terms of their impact on spouses’ welfare. Such a result may suggest a rationale for why changes in family law tend to occur very gradually over time. While “better” family law environments may favorably impact the child outcome distribution, the gains are slight compared to the shifts in the parental welfare distribution. It follows that it may be difficult to attain the wide-spread support from both mothers and fathers that radical changes in family law require.

Though complex, the model is quite stylized and it seems important to generalize it along several dimensions in order to bolster the credibility of our policy experiments. We view the most problematic feature of our modeling setup as the lack of direct measures of investment in children. Our model allows investment in children to operate solely through money expenditures as opposed to time. Time investments appear in other recent work. Tartari (2007) estimates an

36The other that we are aware of is Aizer and McLanahan (2006).
elaborate model of the trade-off between marital conflict and time with parents in the production of child attainment, which provides a different perspective on the likely effects of shoring up the marginal marriage. Del Boca, Flinn and Wiswall (2010) have estimated a child quality production function that includes as arguments detailed types of time expenditures by mothers and fathers as well as money expenditures on investment goods. Their data source allows them to observe these inputs at various stages in the child development process. While an advance over our model on the production function side, their model is not easily generalizable to include fertility and divorce decisions. Nonetheless, it would clearly be advantageous to allow some form of time input into child production, and this is the focus of our current research.
References


A Estimation Algorithm

Let the number of parameter vectors at which exact solutions are computed be given by $H$, and let the collection of these parameter vectors be given by $\Lambda = \{\Gamma_1, \Gamma_2, ..., \Gamma_H\}$, where each $\Gamma_h \in \Omega_{\Gamma}$, the parameter space associated with $\Gamma$. The Nash equilibrium investment rules for the household are given by $i^*(o; \Gamma_h) = i^*_1(o; \Gamma_h) + i^*_2(o; \Gamma_h)$ at the parameter vector $\Gamma_h$. Let the true value of the parameter vector be given by $\Gamma_0$. Both $\Gamma_0$ and $\Gamma$ are interior points in the $K$-dimensional parameter space $\Omega_{\Gamma}$. Estimation proceeds as follows.

1. Begin by selecting $H$ distinct points in the parameter space $\Omega_{\Gamma}$, which we denote by $\Gamma_h$, $h = 1, ..., H$, with the collection of these points defined as $\Lambda$. For these $H$ values of the parameter vector we solve for the investment rules for all values $o$ in the finite state space $O$.

2. Given any current guess of the values of the parameters $\Gamma$, compute the weights

$$w_{\Gamma}(h) = \frac{[D(\Gamma, \Gamma_h)]^{-1}}{\sum_{i=1}^{H}[D(\Gamma, \Gamma_i)]^{-1}},$$

where $D(x, y)$ is a distance function so that $D(x, y) = D(y, x)$, $D(x, y) > 0$ for all $x \neq y$, and $D(x, x) = 0$. As a result, $w_{\Gamma}(h) \in [0, 1]$, $\forall h$, and

$$\sum_{h=1}^{H} w_{\Gamma}(h) = 1, \forall \Gamma \in \Omega_{\Gamma}. \quad (13)$$

3. Form the approximate decision rules for every value of $o$,

$$i^*(o; \Gamma) = \sum_{h=1}^{H} w_{\Gamma}(h)i^*(o; \Gamma_h). \quad (14)$$

4. Generate the simulated moments at the parameter vector $\Gamma$ using the approximate decision rules $i^*(o; \Gamma)$.

5. Define the distance function

$$L_1(\Gamma; A^N) = (A^N - \hat{A}(\Gamma))'W(C^N - \hat{A}(\Gamma)), \quad (15)$$

where $A^N$ are the sample moments, $\hat{A}(\Gamma)$ are the analogous moments computed from the simulated sample at the parameter vector $\Gamma$, and $W$ is a positive-definite weighting matrix.

6. Using the Nelder-Mead simplex algorithm, repeat steps (2)-(5) until

$$L_1(\Gamma; A^N) < \varepsilon_N, \quad (16)$$

where $\varepsilon_N$ is a small positive number.
7. Denote the value of $\tilde{\Gamma}$ that satisfies (16) by $\tilde{\Gamma}_1^*$, where the subscript ‘1’ suggests that this is an estimator that has passed the first convergence criterion.

8. Compute the optimal investments at $\tilde{\Gamma}_1^*$ for each $o \in O$. Define

$$L_2(\tilde{\Gamma}_1^*) = \max_{s \in S} \{|i^*(o; \tilde{\Gamma}_1^*) - i^*(o; \tilde{\Gamma}_1^*)| \}^O_{o=1}. \quad (17)$$

9. If $L_2(\tilde{\Gamma}_1^*) < \zeta_N$, where $\zeta_N$ is a small positive number, then we say that the final estimator of $\Gamma$ is

$$\hat{\Gamma}_2^* = \tilde{\Gamma}_1^*. \quad (18)$$

If not, then add the point $\tilde{\Gamma}_1^*$ to the set $\Lambda$ (or $\Lambda' = \Lambda \cup \tilde{\Gamma}_1^*$) so that the cardinality of this set increases to $H + 1$. Then repeat all steps beginning with (2), keeping the current guess of the parameter vector fixed at $\tilde{\Gamma}_1^*$.

In practice we have had good success with this estimation method. At this point we cannot supply a formal proof of consistency of this estimator, but we turn to a sketch its elements.

First consider the approximation of the investment rule as a function of the parameter vector $\Gamma$. Given our $H$ element set $\Lambda$, for a given value of $\Gamma \in \Omega$, we compute

$$i^*(o; \Gamma) = \sum_{h=1}^{H} w_{\Gamma}(h) i^*(o; \Gamma_h).$$

If

$$\max |i^*(o; \Gamma) - i^*(o; \Gamma)| \geq \zeta_N,$$

then we add the point $\Gamma$ to the set $\Lambda$ as element $H + 1$ of the set. If not, we say that we have adequately approximated the decision rule.

If the convergence criterion is not satisfied, we return to recompute the weights attached to the “exact” investment rules associated with the new set of points $\Lambda' = \Lambda \cup \Gamma$. The weight attached to any arbitrary evaluation point $h$ can be expressed as

$$w_{\Gamma}(h) = \frac{[D(\Gamma, \Gamma_h)]^{-1}_{j=1}^{H+1} [D(\Gamma, \Gamma)]^{-1}_{j=1}^{H+1}, \ h = 1, ..., H + 1}{1 - \frac{1}{D_{h}(\Gamma)} + \frac{1}{D_{h}(\Gamma)} + \cdots + \frac{1}{D_{H+1}(\Gamma)}}$$

$$= \frac{1}{D_{h}(\Gamma) + D_{h+1}(\Gamma)}$$

$$= \frac{D_{-1}(\Gamma) + D_{-2}(\Gamma) + \cdots + D_{-(H+1)}(\Gamma)}{D_{-1}(\Gamma)D_{2}(\Gamma)D_{3}(\Gamma) \cdots D_{H+1}(\Gamma)}$$

$$= \frac{D_{-h}(\Gamma)}{D_{-1}(\Gamma) + D_{-2}(\Gamma) + \cdots + D_{-(H+1)}(\Gamma)}.$$
where $D_j(\Gamma)$ is shorthand for $D(\Gamma, \Gamma_j)$ and

$$D_{-j}(\Gamma) = D_1(\Gamma) \cdots D_{j-1}(\Gamma)D_{j+1}(\Gamma) \cdots D_{H+1}(\Gamma).$$

But note that in this case $\Gamma = \Gamma_{H+1}$, so $D_{H+1}(\Gamma) = 0$ and $D_j(\Gamma) > 0$, $\forall j \neq H+1$, since all points of evaluation are distinct. Then $D_{-(H+1)}(\Gamma) > 0$, while $D_{-j}(\Gamma) = 0$, $\forall j \neq H+1$. Thus $w_{\Gamma}(H+1) = 1$, and the new “approximate” decision rule is the “exact” one computed at the point $\Gamma$, or

$$i^*(\omega; \Gamma) = i^*(\omega; \Gamma), \ \forall s.$$

This completes the discussion of the ability of the investment rule approximation method to fit the actual investment rule solution for every value of $\omega$. While it is always capable of providing a perfect fit, we will not want to enforce this in practice since this would imply an indefinite number of iterations over steps (2)-(5). For consistency of the entire estimator, we will only require the critical value used for convergence to get arbitrarily small as sample size grows.

Now we need to consider the convergence of the stage one estimator, $\tilde{\Gamma}_1^*$, which is computed on the basis of a fixed collection of decision rules. Since the weights attached to the exact investment rules used in forming the approximation are functions of the current parameter guess $\tilde{\Gamma}$, the approximation is as well. As long as the distance function is a continuous function of $\tilde{\Gamma}$, then the weights are as well, which implies that the approximation is continuous in $\tilde{\Gamma}$.

Given that certain events involved in the moment computation are discrete (such as divorce), it is not possible to claim that the functions $\tilde{A}(\tilde{\Gamma})$ are continuous. However, continuity is not required for consistency, as is made clear in Pakes and Pollard (1989). We have not explicitly noted dependence of $\tilde{A}$ on $R$, but for now write $\tilde{A}_R(\tilde{\omega})$. Then we need uniform convergence of $\tilde{A}_R(\tilde{\Gamma})$, so that there exists a value $\tilde{R}$ and $\kappa > 0$ such that

$$|\tilde{A}_R(\tilde{\Gamma}) - A(\tilde{\Gamma})| < \kappa \quad (19)$$

for all $R \geq \tilde{R}$ and $\tilde{\Gamma} \in \Omega$. Standard Law of Large Numbers results yield $\text{plim}_{N \to \infty} A^N = A$. Then the key elements required for $\text{plim}(\tilde{\Gamma}_2) = \Gamma_0$ are:

1. $\varepsilon_N \to 0$ as $N \to \infty$
2. $\zeta_N \to 0$ as $N \to \infty$
3. $R \to \infty$ as $N \to \infty$
4. $A(\Gamma)$ continuous function of $\Gamma$
5. Uniform convergence of $\tilde{A}_R(\Gamma)$.

We do not attempt to characterize the requirements for deriving a well-defined limiting distribution for the estimator $\tilde{\Gamma}_2$. Although computation of the estimator is demanding, it is still feasible to construct bootstrap estimates of it sampling distribution.
### Table 1: Fertility Region of the Income-Marriage Quality Space

Fertility choice: + attempt to conceive, - do not

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<th>30th</th>
<th>50th</th>
<th>70th</th>
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Table 2: Estimation Sample Construction

<table>
<thead>
<tr>
<th>Description</th>
<th>Number</th>
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<tr>
<td>Total NLSY-79 respondents</td>
<td>12,686</td>
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<tr>
<td>Men</td>
<td>6403</td>
</tr>
<tr>
<td>Never married women with no kids</td>
<td>609</td>
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<tr>
<td>Never married women with kids</td>
<td>486</td>
</tr>
<tr>
<td>Women with kids before first marriage</td>
<td>1167</td>
</tr>
<tr>
<td>Insufficient marriage or fertility data</td>
<td>686</td>
</tr>
<tr>
<td>Women with kids but no PIAT scores</td>
<td>489</td>
</tr>
<tr>
<td>Families with missing data on income, state, age, ed, gender or AFQT</td>
<td>1203</td>
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<tr>
<td>Final sample</td>
<td>1643</td>
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</table>
Table 3: Estimation Sample Descriptive Statistics

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<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
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<tr>
<td>Ever divorce?</td>
<td>0.3944</td>
<td>0.4889</td>
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<td>0</td>
<td>1</td>
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<td>Time to divorce</td>
<td>yes</td>
<td>8.122</td>
<td>5.485</td>
<td>6.667</td>
<td>0.83</td>
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<tr>
<td>Ever child?</td>
<td>0.7005</td>
<td>0.4582</td>
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<td>0</td>
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<tr>
<td>Time to child</td>
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<td>2.308</td>
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<tr>
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<td>Child age at 1</td>
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<td>0.91</td>
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<td>Test score 2</td>
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<td>Child age at 2</td>
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<td>Wife's AFQT score</td>
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<td>Child gender (f =1)</td>
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<td>0.2000</td>
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<td>6929.24</td>
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<td>0.00</td>
<td>187279.20</td>
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</table>

\(N = 1643\). The husband and wife’s age, education and initial income are each measured at the first interview following the marriage. The wife’s AFQT score is measured in 1980. All financial variables are reported in 2004 US dollars.
### Table 4: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate (Standard Error)</th>
<th>Parameter</th>
<th>Estimate (Standard Error)</th>
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</thead>
<tbody>
<tr>
<td>$\zeta$</td>
<td>0.885 (0.035)</td>
<td>$\mu_{k2}$</td>
<td>22.289 (1.470)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.532 (0.034)</td>
<td>$\beta_{\theta_0}$ constant</td>
<td>9.326 (1.595)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.498 (0.001)</td>
<td>$\beta_{\theta_1}$ on mother’s age</td>
<td>0.057 (0.005)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.466 (0.040)</td>
<td>$\beta_{\theta_2}$ on father’s age</td>
<td>0.059 (0.004)</td>
</tr>
<tr>
<td>$\gamma^+ = \gamma^-$</td>
<td>0.230 (0.020)</td>
<td>$\sigma_{\theta}$</td>
<td>0.922 (0.004)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.488 (0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.494 (0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{k_0}$ constant</td>
<td>-230.7 (1.763)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{k_1}$ on AFQT</td>
<td>0.162 (0.014)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{k_2}$ on mother’s education</td>
<td>1.057 (0.057)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{k_3}$ on father’s education</td>
<td>0.984 (0.068)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{k1}$</td>
<td>4.152 (0.598)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$N = 1643$. Parents’ incomes are scaled to units of 1000 2004 dollars. Standard errors are based on 50 bootstrapped samples.
### Table 5: Income Process Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate (Standard Error)</th>
<th>Parameter</th>
<th>Estimate (Standard Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{\xi}_1^+ )</td>
<td>0.416 (0.006)</td>
<td>( \tilde{\xi}_1^+ )</td>
<td>0.292 (0.019)</td>
</tr>
<tr>
<td>( \tilde{\xi}_1^- )</td>
<td>0.399 (0.005)</td>
<td>( \tilde{\xi}_1^- )</td>
<td>0.218 (0.14)</td>
</tr>
<tr>
<td>( \tilde{\xi}_2^+ )</td>
<td>0.503 (0.027)</td>
<td>( \tilde{\xi}_2^+ )</td>
<td>1.098 (0.011)</td>
</tr>
<tr>
<td>( \tilde{\xi}_2^- )</td>
<td>0.411 (0.030)</td>
<td>( \tilde{\xi}_2^- )</td>
<td>1.233 (0.012)</td>
</tr>
</tbody>
</table>

| \( \eta \) | 0.702 (0.006) | \( \beta_{20} \) constant | -2.182 (0.147) |
| \( \beta_{10} \) constant | -0.868 (0.070) | \( \beta_{21} \) on mother’s education | 0.026 (0.001) |
| \( \beta_{11} \) on father’s education | 0.019 (0.002) | \( \beta_{22} \) on mother’s AFQT | 0.001 (0.000) |
The simulations are based on $R = 1500$ replications per family.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Sim</th>
<th>Moment</th>
<th>Data</th>
<th>Sim</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] $E[I(d(j) \leq 1)]$</td>
<td>0.0091</td>
<td>0.0317</td>
<td>[29] $E[k(j, 2) - k(j, 1)</td>
<td>d = 0$</td>
<td>0.0097</td>
</tr>
<tr>
<td>[2] $E[I(d(j) \leq 2)]$</td>
<td>0.0323</td>
<td>0.0629</td>
<td>at $g = 1$, $d = 1$ at $g = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[3] $E[I(d(j) \leq 5)]$</td>
<td>0.1497</td>
<td>0.1060</td>
<td>[30] $E[k(j, 1) * y_1]$ at $g = 1$</td>
<td>17.730</td>
<td>23.454</td>
</tr>
<tr>
<td>[4] $E[I(d(j) \leq 10)]$</td>
<td>0.2641</td>
<td>0.2199</td>
<td>[31] $E[k(j, 1) * y_2]$ at $g = 1$</td>
<td>186.618</td>
<td>202.607</td>
</tr>
<tr>
<td>[5] $E[I(n(j) \leq 1)]$</td>
<td>0.1765</td>
<td>0.2941</td>
<td>[32] $E[(k(j, 2) - k(j, 1)) * ((y_1)$</td>
<td>0.4670</td>
<td>-0.0008</td>
</tr>
<tr>
<td>[6] $E[I(n(j) \leq 2)]$</td>
<td>0.3366</td>
<td>0.4052</td>
<td>at $g = 2) - (y_1 at g = 1))]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[7] $E[I(n(j) \leq 5)]$</td>
<td>0.6013</td>
<td>0.6307</td>
<td>[33] $E[(k(j, 2) - k(j, 1)) * ((y_2)$</td>
<td>-0.5396</td>
<td>-2.1577</td>
</tr>
<tr>
<td>[8] $E[I(n(j) \leq 10)]$</td>
<td>0.6890</td>
<td>0.7569</td>
<td>at $g = 2) - (y_2 at g = 1))]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[9] $E[I(d(j) \leq 3) * I(n(j) \leq 3)]$</td>
<td>0.0079</td>
<td>0.0109</td>
<td>[34] $E[k(j, 1) * y_1(j, 0)]$</td>
<td>4.2018</td>
<td>4.6835</td>
</tr>
<tr>
<td>[10] $E[I(d(j) \leq 5) * I(n(j) \leq 5)]$</td>
<td>0.0359</td>
<td>0.0403</td>
<td>[35] $E[k(j, 1) * I(unilateral)]$</td>
<td>4.4072</td>
<td>4.5507</td>
</tr>
<tr>
<td>[11] $E[I(d(j) \leq 7) * I(n(j) \leq 7)]$</td>
<td>0.0621</td>
<td>0.0594</td>
<td>[36] $E[k(j, 1)*mother’s ed]$</td>
<td>59.376</td>
<td>60.345</td>
</tr>
<tr>
<td>[12] $E[I(d(j) \leq 10) * I(n(j) \leq 10)]$</td>
<td>0.1053</td>
<td>0.1304</td>
<td>[37] $E[k(j, 1)*father’s ed]$</td>
<td>59.013</td>
<td>59.640</td>
</tr>
<tr>
<td>[13] $E[y_1]$ in year 2</td>
<td>25.001</td>
<td>27.166</td>
<td>[38] $E[k(j, 1)*mother’s AFQT]$</td>
<td>337.015</td>
<td>336.092</td>
</tr>
<tr>
<td>[16] $Var[y_2]$ in year 2</td>
<td>138.460</td>
<td>130.181</td>
<td>[41] $E[I(d(j) \leq 10) * \pi]$</td>
<td>0.0513</td>
<td>0.0426</td>
</tr>
<tr>
<td>[19] $Var[y_1]$ in year 5</td>
<td>291.935</td>
<td>267.292</td>
<td>[44] $E[I(d(j) \leq 5)*m’s AFQT]$</td>
<td>10.446</td>
<td>7.8197</td>
</tr>
<tr>
<td>[20] $Var[y_2]$ in year 5</td>
<td>168.372</td>
<td>144.667</td>
<td>[45] $E[I(d(j) \leq 5)*m’s age]$</td>
<td>3.1848</td>
<td>2.4365</td>
</tr>
<tr>
<td>[22] $E[y_2]$ in year 10</td>
<td>22.252</td>
<td>20.661</td>
<td>[47] $E[I(n(j) \leq 5)*m’s ed]$</td>
<td>7.8588</td>
<td>8.3896</td>
</tr>
<tr>
<td>[23] $Var[y_1]$ in year 10</td>
<td>323.990</td>
<td>284.629</td>
<td>[48] $E[I(n(j) \leq 5)*father’s ed]$</td>
<td>7.7699</td>
<td>8.3027</td>
</tr>
<tr>
<td>[24] $Var[y_2]$ in year 10</td>
<td>161.750</td>
<td>140.061</td>
<td>[49] $E[I(n(j) \leq 5)*m’s AFQT]$</td>
<td>43.673</td>
<td>46.597</td>
</tr>
<tr>
<td>[25] $E[k(j, 1)] * E[d \leq 1st test date]$</td>
<td>0.5149</td>
<td>0.6779</td>
<td>[50] $E[I(n(j) \leq 5)*m’s age]$</td>
<td>13.572</td>
<td>14.621</td>
</tr>
<tr>
<td>[26] $E[k(j, 1)]$</td>
<td>4.4072</td>
<td>4.5507</td>
<td>[51] $E[I(n(j) \leq 5)*f’s age]$</td>
<td>14.990</td>
<td>16.258</td>
</tr>
<tr>
<td>[27] $E[k(j, 2) - k(j, 1)]$</td>
<td>0.0943</td>
<td>0.0839</td>
<td>[52] $E[y_1 in year 5</td>
<td>n(j) \leq 5]$</td>
<td>16.366</td>
</tr>
<tr>
<td>[28] $E[k(j, 2) - k(j, 1)</td>
<td>d = 0 at 1]$</td>
<td>0.0639</td>
<td>0.0777</td>
<td>[53] $E[y_2 in year 5</td>
<td>n(j) \leq 5]$</td>
</tr>
</tbody>
</table>
The simulations are based on \( R = 1500 \) replications per family.
Table 7: Comparative Statics Exercise for $\gamma$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\overline{d}$ at 10 years</th>
<th>$\overline{p}$ at 10 years</th>
<th>terminal $\overline{k}$</th>
<th>divorced parenting</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0000</td>
<td>0.8522</td>
<td>4.9103</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0714</td>
<td>0.8382</td>
<td>4.7009</td>
<td>0.0349</td>
</tr>
<tr>
<td>0.20</td>
<td>0.1784</td>
<td>0.8077</td>
<td>4.4716</td>
<td>0.1203</td>
</tr>
<tr>
<td>0.30</td>
<td>0.2825</td>
<td>0.7860</td>
<td>4.4080</td>
<td>0.2056</td>
</tr>
<tr>
<td>0.40</td>
<td>0.3528</td>
<td>0.7537</td>
<td>4.3390</td>
<td>0.2442</td>
</tr>
</tbody>
</table>

Simulated outcomes are based on $R = 1500$ replications for each family.
Table 8: Comparative Statics Exercise for Custody

<table>
<thead>
<tr>
<th>$\tau_1(1)$</th>
<th>$\bar{d}$ at 10 years</th>
<th>$\bar{p}$ at 10 years</th>
<th>terminal $\bar{k}$</th>
<th>divorced parenting</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.1065</td>
<td>0.7022</td>
<td>4.4873</td>
<td>0.0957</td>
</tr>
<tr>
<td>0.10</td>
<td>0.1785</td>
<td>0.7626</td>
<td>4.8909</td>
<td>0.1132</td>
</tr>
<tr>
<td>0.20</td>
<td>0.2199</td>
<td>0.7569</td>
<td>4.6346</td>
<td>0.1304</td>
</tr>
<tr>
<td>0.30</td>
<td>0.2575</td>
<td>0.7270</td>
<td>4.7384</td>
<td>0.1336</td>
</tr>
<tr>
<td>0.40</td>
<td>0.2259</td>
<td>0.8065</td>
<td>4.8189</td>
<td>0.1387</td>
</tr>
<tr>
<td>0.50</td>
<td>0.2350</td>
<td>0.8987</td>
<td>5.6185</td>
<td>0.1630</td>
</tr>
</tbody>
</table>

Simulated outcomes are based on $R = 1500$ replications for each family.
Table 9: Comparative Statics Exercises for Child Support

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>$\bar{d}$ at 10 years</th>
<th>$\bar{p}$ at 10 years</th>
<th>terminal $\bar{k}$</th>
<th>divorced parenting</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.2114</td>
<td>0.7702</td>
<td>4.5556</td>
<td>0.1303</td>
</tr>
<tr>
<td>0.10</td>
<td>0.2196</td>
<td>0.7796</td>
<td>4.6132</td>
<td>0.1315</td>
</tr>
<tr>
<td>0.17</td>
<td>0.2138</td>
<td>0.7669</td>
<td>4.5589</td>
<td>0.1333</td>
</tr>
<tr>
<td>0.20</td>
<td>0.2137</td>
<td>0.7695</td>
<td>4.6008</td>
<td>0.1274</td>
</tr>
<tr>
<td>0.25</td>
<td>0.1985</td>
<td>0.7658</td>
<td>4.7126</td>
<td>0.1288</td>
</tr>
<tr>
<td>0.30</td>
<td>0.1973</td>
<td>0.7643</td>
<td>4.6935</td>
<td>0.1291</td>
</tr>
</tbody>
</table>

Simulated outcomes are based on $R = 1500$ replications for each family.
Figure 1: Fertility Rate by Time to Divorce, NLSY
Figure 2: PIAT Score by Time to Divorce

- Married in 3 yrs
- M now, D in 3 yrs
- Divorced now

Mean PIAT math score vs. Year of marriage.
Figure 3: Divorce Rate With and Without Children, NLSY
Figure 4a: Mother's Equilibrium Child Investment in Marriage and Divorce

Figure 4b: Father's Equilibrium Child Investment in Marriage and Divorce
Figure 5: Ex ante average welfare by paternal custody share

Examining the figure, we observe that as the child support rate increases, the ex ante average welfare decreases. This trend is more pronounced as the paternal custody share increases, with higher values of Tau1(1) showing a steeper decline. For instance, when Tau1(1) = 0.5, the welfare is highest, and as we move to lower values of Tau1(1), the welfare decreases significantly at higher child support rates. The figure clearly illustrates the impact of child support rates on welfare, highlighting the importance of understanding these dynamics in policy-making.
Figure 6: Spouses' Welfare and Child Support, 50/50 Custody

- Husband's ex ante welfare
- Wife's ex ante welfare
- Average ex ante welfare